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THEORETICAL PREDICTION OF AIRPLANE STABILITY DERIVATIVES AT SUBCRITICAL SPEEDS

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SUMMARY

This report describes the theoretical development and application of an analysis for predicting the major static and rotary stability derivatives for a complete airplane. The analysis utilizes potential flow theory to compute the surface flow fields and pressures on any configuration that can be synthesized from arbitrary lifting bodies and nonplanar thick lifting panels. The pressures are integrated to obtain section and total configuration loads and moments due side slip, angle of attack, pitching motion, rolling motion, yawing motion, and control surface deflection. Subcritical compressibility is accounted for by means of the Gothert similarity rule.

Within the scope of predicting the total configuration stability derivatives it was necessary to study the problem of computing the spanwise variation of potential form drag due to panel lift and thickness. Included in appendix F is a solution to this problem. Also, in solving the potential form drag problem the work of Woodward and Wagner was thoroughly analyzed. A complete derivation of Woodard's influence equations is given in appendix D and a comprehensive review of Wagner's lifting surface theory is included in appendix E.

INTRODUCTION

In order to develop a procedure for predicting total configuration stability derivatives, the perturbation flow due to the vehicle had to be represented by a grid of efficient aerodynamic finite elements general enough to satisfy the boundary conditions over the surfaces of diverse shapes and also produce the correct resultant forces and moments. The quadrilateral vortex was selected to represent the perturbation velocity due to the bodies because of its numerical efficiency, net force producing capability, limited range of influence, and relationship to the horseshoe vortex which has demonstrated amazing accuracy in predicting loads on wings of arbitrary shape.

The nonplanar thick lifting panels are divided into two sections, (1) the outboard section which is defined by a locus of chord lines, and (2) the root section which is a transition region from the outboard section to the juncture of the panel and a body. If the panel is attached to another panel there is no root section. The perturbation velocity due to panel lift is represented by quadrilateral vortices in the root section and skewed horseshoe vortices in the outboard section. The perturbation velocity due to panel thickness is represented by a source lattice. The panel aerodynamic finite elements were selected because of their numerical efficiency and proven accuracy in predicting flow fields over wings of general shape.

The panel singularities are placed on a mean surface instead of the actual external surface of the panel to maintain computing efficiency. This will sacrifice surface pressure accuracy at the juncture between two panels or a panel and a body, but for this type of general analysis the savings in computer time makes the compromise practical. Second order corrections to account for the interference between lift and thickness and to account for blunt leading edge airfoil sections are included.

The source and vortex lattice influence equations are formulated in terms of the same quantities, which allows the perturbation velocity due to lift and thickness to be computed simultaneously and thereby save computing effort. Also, due to the limited range of significant influence of the quadrilateral vortex the influence of any quadrilateral vortex is only computed at those points within a given area of influence. This can save considerable computing time in developing the aerodynamic influence martix.

Computer time is also saved by reducing the number of unknowns by transforming the aerodynamic influence matrix by constraint matrices. The constraint matrices constrain the body and panel vorticity, thereby reducing the number of unknowns from that of the number of vortex elements to the number of constraint functions. This is an option in the program and can be applied in just the longitudinal direction, lateral direction, both directions, or not at all.

The pressures are integrated by numerical means and the panel section drag is computed by means of the Kutta-Joukowsky theorem. The total configuration induced drag is computed by means of a Trefftz plane analysis.

LIST OF SYMBOLS

AR	aspect ratio
A_R	reference area
$[A_x]$, $[A_y]$, $[A_z]$	aerodynamic influence matrix components
a_{B_i}	body constraint function coefficient
a_{P_K}	panel constraint function coefficient
b	panel span
C	chord or aerodynamic coefficient
$\bar{C}$	reference chord
C_S	chord line between (X_J, Y_J, Z_J) and (X_R, Y_R, Z_R)
C_f	trailing edge flap chord
C_K	leading edge flap chord
C_P	pressure coefficient
$[E]$	two dimensional influence matrix
$\hat{h}$	unit vector tangent to control surface hinge line
h	body height
$\hat{i}$, $\hat{j}$, $\hat{k}$	unit vectors in the x, y, and z directions, respectively
K	quadrilateral vortex strength
M_∞	reference mach number
$\bar{\bar{N}}$	equivalent incompressible surface normal unit vector

$[\bar{N}_x]$, $[\bar{N}_y]$, $[\bar{N}_z]$	equivalent incompressible surface normal unit vector component matrices
$[N_z]$, $[N_y]$, $[N_z]$	actual surface normal unit vector component matrices
$\vec{N}_P$	panel surface normal unit vector
p	roll rate
q	pitch rate
r	yaw rate
R_{c_x}, R_{c_y}, R_{c_z}	subarea centroid position vector components
R_B	body radius
$[R]$	transformation matrix between quadrilateral and horseshoe vortex strengths
$\vec{R}$	general position vector
S_B	body circumferential distance
$[S_x]$, $[S_y]$, $[S_z]$	source influence martix components
$\vec{\bar{T}}_M$	equivalent incompressible surface lingitudinal tangent unit vector
$[\bar{T}_{M_x}]$, $[\bar{T}_{M_y}]$, $[\bar{T}_{M_z}]$	equivalent incompressible surface longitudinal tangent unit vector component matrices
$\vec{\bar{T}}_T$	equivalent incompressible surface lateral tangent unit vector
$[\bar{T}_{T_x}]$, $[\bar{T}_{T_y}]$, $[\bar{T}_{T_z}]$	equivalent incompressible surface lateral tangent unit vector component matrices
$\vec{T}_P$	panel surface lateral tangent unit vector
$[T_{B_i}]$	body aerodynamic constraint transformation matrix

$[T_{P_K}]$	panel aerodynamic constraint transformation matrix
$[T_{M_x}]$, $[T_{M_y}]$, $[T_{M_z}]$	actual surface longitudinal tangent unit vector component matrices
$[T_{T_x}]$, $[T_{T_y}]$, $[T_{T_z}]$	actual surface lateral tangent unit vector component matrices
U	total velocity in x direction
V	total velocity in y direction
V_x, V_y, V_z	onset flow components
V_∞	reference velocity
W	total velocity in z direction
w	body width
X, Y, Z	global coordinates
X_B, Y_B, Z_B	body coordinates
X_P, Y_P, Z_P	panel coordinates
X_J, Y_J, Z_J	points along intersection of panel root section and outboard section
X_R, Y_R, Z_R	points along intersection of panel root section and a body
$X_{C.G.}$, $Y_{C.G.}$, $Z_{C.G.}$	center of gravity position vector components
X_k	trailing edge of leading edge control surface
X_f	leading edge of trailing edge control surface
X_h	hinge line location
Y_{B_M}	body multiplication factor in y direction
Z_{B_M}	body multiplication factor in z direction

Z_c	Perpendicular distance between chord line and mean camber line
Z_t	airfoil thickness

GREEK SYMBOLS

α	angle of attack
β	$\sqrt{1-M_\infty^2}$ or angle of yaw
γ	vorticity or ratio of specific heats
δ	control surface deflection
Γ	horseshoe vortex strength
$\Delta A_x, \Delta A_y, \Delta A_z$	directed subareas
ΔY_B	displacement of body cross-section in y direction
ΔZ_B	displacement of body cross-section in z direction
$\vec{\Delta S}$	incremental vector tangent to constant percent chord line
ΔY	panel surface increment in y direction
ΔZ	panel surface increment in z direction
ϵ	panel twist
η	fraction of lateral or circumferential distance
θ_B	body polar coordinate
θ	general lateral polar coordinate
Λ	sweep
ρ	density

Σ	source strength
ϕ	general longitudinal polar coordinate

SUBSCRIPTS

AVG.	average
B	body
C	camber
C_x, C_y, C_z	centroid components
C.P.	center of pressure
C.G.	center of gravity
E	edge
f	trailing edge control surface or final
h	hinge line
i	summation index or induced
j	summation index
J	root section and outboard panel juncture
k	summation index
L	lower surface or lift
L.E.	leading edge
m	longitudinal direction
MAX.	maximum
M	longitudinal subpanel number
n	lateral subpanel number
O	origin

P	panel
r	summation index
R	panel-body juncture
S	lateral or circumferential direction
T	lateral direction
t	airfoil thickness
T.E.	trailing edge
u	upper surface or number of longitudinal constraint function
w	number of lateral constraint function
X, Y, Z	directions
η	lateral or circumferential direction
∞	infinity

THEORETICAL DEVELOPMENT

Configuration Representation

The theory discussed in this report is capable of predicting the surface pressures and integrated loads on any configuration which can be synthesized from lifting bodies and thick lifting panels of arbitrary shape. These two basic elements can be attached along longitudinal subpanel edges in order to represent complete airplane configurations of arbitrary shape.

Lifting Body. - The arbitrary lifting body, as shown in figure 1, can be of the solid or flow-through type. The body external surface is divided into a grid, which represents the edges of the body subpanels. The body bound vortex lines are placed at the quarter chord point of the subpanels and the fixed trailing vortex lines along the longitudinal edges of the subpanels. The trailing vortex lines are shed from the aft end of the body and extend in the X direction to infinity unless the free wake option of the program is used, in which case the locations of the trailing vortices aft of the body are determined such that they are force-free. If the body is closed to a point at the aft end, all of the body trailing vortices aft of the body cancel and are, therefore, neglected.

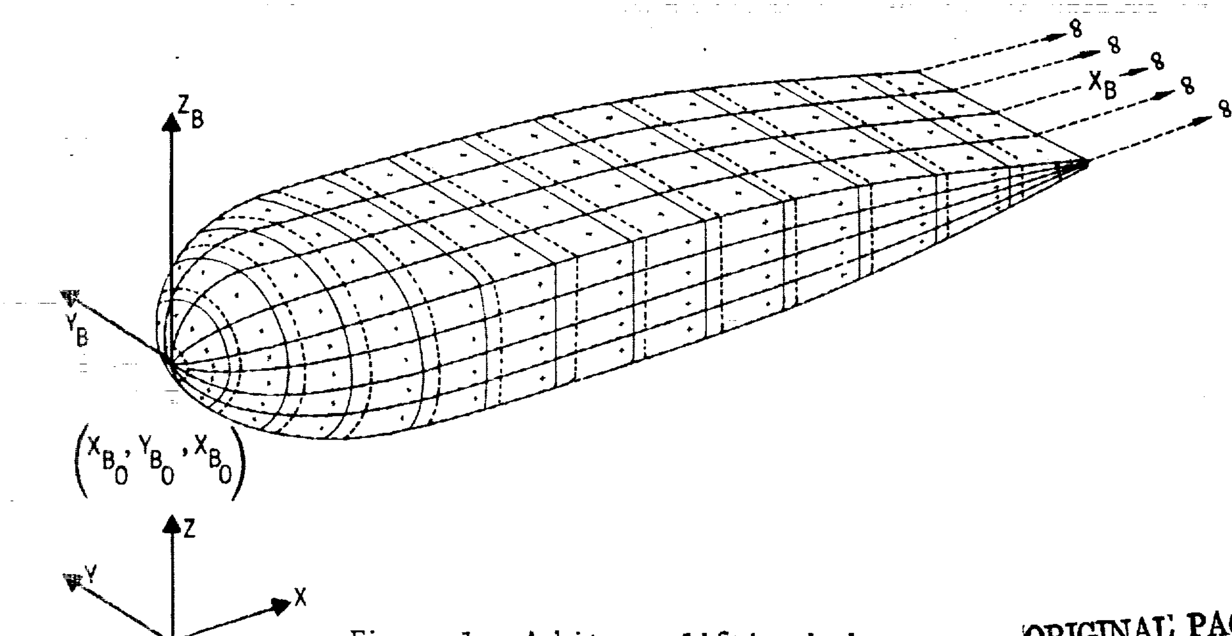

Figure 1.- Arbitrary lifting body.

The body vortex strengths are assumed constant around closed paths defined by the bound vortex lines of two subpanels, adjacent to each other in the longitudinal direction, and that portion of the longitudinal edges of these adjacent panels between the two bound vortex lines. The contribution from the vortex loop, or quadrilateral vortex, is defined as positive when the Biot Savart line integral is taken in the clockwise direction by an observer looking at the external surfaces of the two subpanels. The body subpanels and quadrilateral vortices are shown in figure 1 along with the location of the body control points.

The body control points are located at the three-quarter chord of the body subpanels. At these points the total flow is summed and forced to be a minimum in the direction normal to the body surface. If the body vorticity is not constrained by functions with unknown coefficients, a control point is placed at each subpanel and a discrete solution for the unknown body vortex strengths is obtained. If the body vorticity is constrained, control points are placed at as many subpanels as is necessary to obtain a good representation of the body shape and to insure that there is sufficient control of the constraint functions. In this case the unknown coefficients of the constraint functions are determined by the method of least squares.

The body subpanels are further subdivided in the lateral direction so that the vortex grid is mapped to the body surface more accurately. This also allows the use of only one quadrilateral vortex in the lateral direction for the case of a body of revolution in a uniform flow at zero angle of attack. In this case the two side edges of the subpanel are coincident, and therefore, the fixed trailing vortices cancel leaving a longitudinal distribution of ring vortices located at the quarter chord of each subpanel.

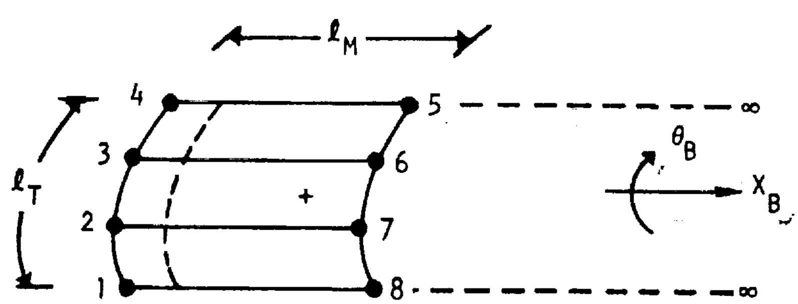

Figure 2.- Body subpanel defined by points 1, 4, 5, and 8 with subdivision points 2, 3, 6, and 7.

The coordinates of the body subpanel subdivisions, as shown in figure 2, are used to compute unit vectors tangent to the body surface in both the longitudinal and lateral directions, unit vectors normal to the body surface, directed subareas to be used in the integration of the body surface pressures, and the centroids of the subareas to compute moments about the configuration center of gravity.

The unit vector tangent to the equivalent incompressible body subpanel in the longitudinal direction is given by,

$$\bar{\bar{T}}_M = \bar{T}_{M_X}\,\hat{i} + \bar{T}_{M_Y}\,\hat{j} + \bar{T}_{M_Z}\,\hat{k} \tag{1}$$

$$\bar{T}_{M_X} = \frac{X_7 - X_2 + X_6 - X_3}{\sqrt{(X_7-X_2)^2 + \beta^2(Y_7-Y_2)^2 + \beta^2(Z_7-Z_2)^2} + \sqrt{(X_6-X_3)^2 + \beta^2(Y_6-Y_3)^2 + \beta^2(Z_6-Z_3)^2}} \tag{2}$$

$$\bar{T}_{M_Y} = \frac{\beta(Y_7 - Y_2 + Y_6 - Y_3)}{\sqrt{(X_7-X_2)^2 + \beta^2(Y_7-Y_2)^2 + \beta^2(Z_7-Z_2)^2} + \sqrt{(X_6-X_3)^2 + \beta^2(Y_6-Y_3)^2 + \beta^2(Z_6-Z_3)^2}} \tag{3}$$

$$\bar{T}_{M_Z} = \frac{\beta(Z_7 - Z_2 + Z_6 - Z_3)}{\sqrt{(X_7-X_2)^2 + \beta^2(Y_7-Y_2)^2 + \beta^2(Z_7-Z_2)^2} + \sqrt{(X_6-X_3)^2 + \beta^2(Y_6-Y_3)^2 + \beta^2(Z_6-Z_3)^2}} \tag{4}$$

The unit vector tangent to the equivalent incompressible body subpanel in the lateral direction is given by,

$$\bar{\bar{T}}_T = \bar{T}_{T_X}\,\hat{i} + \bar{T}_{T_Y}\,\hat{j} + \bar{T}_{T_Z}\,\hat{k} \tag{5}$$

$$\bar{T}_{T_X} = \frac{X_3 - X_2 + X_6 - X_7}{\sqrt{(X_3-X_2)^2 + \beta^2(Y_3-Y_2)^2 + \beta^2(Z_3-Z_2)^2} + \sqrt{(X_6-X_7)^2 + \beta^2(Y_6-Y_7)^2 + \beta^2(Z_6-Z_7)^2}} \tag{6}$$

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$$\bar{T}_{T_Y} = \frac{\beta(Y_3 - Y_2 + Y_6 - Y_7)}{\sqrt{(X_3-X_2)^2+\beta^2(Y_3-Y_2)^2+\beta^2(Z_3-Z_2)^2} + \sqrt{(X_6-X_7)^2+\beta^2(Y_6-Y_7)^2+\beta^2(Z_6-Z_7)^2}} \quad (7)$$

$$\bar{T}_{T_Z} = \frac{\beta(Z_3 - Z_2 + Z_6 - Z_7)}{\sqrt{(X_3-X_2)^2+\beta^2(Y_3-Y_2)^2+\beta^2(Z_3-Z_2)^2} + \sqrt{(X_6-X_7)^2+\beta^2(Y_6-Y_7)^2+\beta^2(Z_6-Z_7)^2}} \quad (8)$$

The unit vector normal to the equivalent incompressible body subpanel is given by,

$$\bar{\bar{N}} = \bar{N}_X \hat{i} + \bar{N}_Y \hat{j} + \bar{N}_Z \hat{k} \quad (9)$$

$$\bar{N}_X = \frac{\bar{T}_{M_Y}\bar{T}_{T_Z} - \bar{T}_{M_Z}\bar{T}_{T_Y}}{\sqrt{\left(\bar{T}_{M_Y}\bar{T}_{T_Z} - \bar{T}_{M_Z}\bar{T}_{T_Y}\right)^2 + \left(\bar{T}_{T_X}\bar{T}_{M_Z} - \bar{T}_{T_Z}\bar{T}_{M_X}\right)^2 + \left(\bar{T}_{M_X}\bar{T}_{T_Y} - \bar{T}_{M_Y}\bar{T}_{T_X}\right)^2}} \quad (10)$$

$$\bar{N}_Y = \frac{\bar{T}_{T_X}\bar{T}_{M_Z} - \bar{T}_{T_Z}\bar{T}_{M_X}}{\sqrt{\left(\bar{T}_{M_Y}\bar{T}_{T_Z} - \bar{T}_{M_Z}\bar{T}_{T_Y}\right)^2 + \left(\bar{T}_{T_X}\bar{T}_{M_Z} - \bar{T}_{T_Z}\bar{T}_{M_X}\right)^2 + \left(\bar{T}_{M_X}\bar{T}_{T_Y} - \bar{T}_{M_Y}\bar{T}_{T_X}\right)^2}} \quad (11)$$

$$\bar{N}_Z = \frac{\bar{T}_{M_X}\bar{T}_{T_Y} - \bar{T}_{M_Y}\bar{T}_{T_X}}{\sqrt{\left(\bar{T}_{M_Y}\bar{T}_{T_Z} - \bar{T}_{M_Z}\bar{T}_{T_Y}\right)^2 + \left(\bar{T}_{T_X}\bar{T}_{M_Z} - \bar{T}_{T_Z}\bar{T}_{M_X}\right)^2 + \left(\bar{T}_{M_X}\bar{T}_{T_Y} - \bar{T}_{M_Y}\bar{T}_{T_X}\right)^2}} \quad (12)$$

The directed subareas for the middle lateral subdivision for the actual body are given by;

$$\Delta A_X = \frac{1}{2}\left[(Y_6-Y_2)\ (Z_3-Z_7) - (Z_6-Z_2)\ (Y_3-Y_7)\right] \tag{13}$$

$$\Delta A_Y = \frac{1}{2}\left[(X_3-X_7)\ (Z_6-Z_2) - (Z_3-Z_7)\ (X_6-X_2)\right] \tag{14}$$

$$\Delta A_Z = \frac{1}{2}\left[(X_6-X_2)\ (Y_3-Y_7) - (Y_6-Y_2)\ (X_3-X_7)\right] \tag{15}$$

The components of the position vector to the centroid of this subdivision are given by,

$$R_{C_X} = \frac{1}{4}\ (X_2 + X_3 + X_6 + X_7) \tag{16}$$

$$R_{C_Y} = \frac{1}{4}\ (Y_2 + Y_3 + Y_6 + Y_7) \tag{17}$$

$$R_{C_Z} = \frac{1}{4}\ (Z_2 + Z_3 + Z_6 + Z_7) \tag{18}$$

Analogous directed subarea and centroidal position vector expressions are obtained for the other lateral subdivisions.

The body description is defined independent of the subpanel grid and can be input to the program in a number of different ways. Both cartesian and polar coordinates can be used to describe the body cross-sections at a set of chord stations. After the basic cross-sections have been described they can be scaled and then translated in planes perpendicular to the body chord or mean camber line. This provides a means of inputting the correct side and top views of an arbitrary body with a minimum of input data to obtain a preliminary estimate of the loads on the body. It also facilitates in inputting the exact shape of some bodies which can be represented by longitudinal segments, over which the cross-sections are mathematically similar. The actual arbitrary shape can be input directly without the use of the scaling and translation options if it is more convenient.

If polar coordinates are used to describe the body cross-sections, the lateral location of body subpanel side edges or fixed trailing vortex lines are also given in terms of angles measured from the local section Z_B axis, in a plane parallel to the $(Y_B - Z_B)$ plane, at an independent set of chordwise stations. These subpanel lateral edges are either specified at a given set of angles or defined to be at equally spaced angle locations. The subpanel longitudinal edges are specified at given X_B stations, evenly spaced in terms of X_B, or evenly spaced in terms of ϕ_B, where $\phi_B = \cos^{-1}(1 - 2 X_B/C_B)$.

The following procedure is used to determine the coordinates X_{B_E}, $(Y_{B_E} - \Delta Y_{B_E})/(Y_{B_M})$, $(Z_{B_E} - \Delta Z_{B_E})/(Z_{B_M})$ of the subpanel corners.

1. R_{B_i} versus θ_{B_i} is input at X_{B_i}.

2. θ_{B_E} (subpanel lateral edge location) versus X_{B_θ} is given as input

3. Interpolation on θ_{B_E} versus X_{B_θ} is done to obtain θ_{B_E} versus X_{B_E} (subpanel longitudinal edge location) and X_{B_i}.

4. Interpolation on R_{B_i} versus θ_{B_i} is done at each X_{B_i} to obtain R_B versus X_{B_i} at each θ_{B_E}

5. Interpolation on R_B versus X_{B_i} is done to obtain R_{B_E} versus X_{B_E} at each θ_{B_E}.

6. X_{B_E}, $(Y_{B_E} - \Delta Y_{B_E})/(Y_{B_M})$, $(Z_{B_E} - \Delta Z_{B_E})/(Z_{B_M})$ are computed from X_{B_E}, R_{B_E}, and θ_{B_E}.

If the subpanels are subdivided the coordinates for the corners of these subdivision are computed using the same procedure. There is always an odd number of subdivisions in both the longitudinal and lateral directions.

If cartesian coordinates are used to describe the body cross-sections, the lateral location of the subpanel side edges are defined by the percent circumferential length $\eta_{B_E} = S_{B_E}/S_{B_{MAX_E}}$ at independent chordwise stations. Tables of $(Y_{B_i} - \Delta Y_{B_i})/Y_{B_M}$ and $(Z_{B_i} - \Delta Z_{B_i})/Z_{B_M}$ versus percent circumferential length $\eta_{B_i} = S_{B_i}/S_{B_{MAX_i}}$ are developed at the input longitudinal stations X_{B_i}. The same procedure that was used to obtain X_{B_E}, $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$, and $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$ when the body was defined by polar coordinates is also used for this case, except that R_{B_i} is replaced by $(Y_{B_i} - \Delta Y_{B_i})/Y_{B_M}$ and $(Z_{B_i} - \Delta Z_{B_i})/Z_{B_M}$, and θ_{B_i} is replaced by $\eta_{B_i} = S_{B_i}/S_{B_{MAX_i}}$. Also, step 6 is unnecessary. The procedure is cycled through twice, first for $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$ and then for $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$.

After X_{B_E}, $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$, and $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$ have been computed, Y_{B_E} is determined by multiplying $(Y_{B_E} - Y_{B_M})/Y_{B_M}$ by the multiplication factor Y_{B_M} and then adding the translation increment ΔY_{B_E}. Z_{B_E} is determined by multiplying $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$ by the multiplication factor Z_{B_M} and then adding the translation increment ΔZ_{B_E}.

Thick lifting panel. - The thick lifting panel, as shown in figure 3, can be warped in any manner laterally, have an arbitrary distribution of chord length, thickness, twist, and camber. It can be used to represent a wing, canard, fin, pylon, horizontal or vertical tail, or be wrapped around to represent a flow through nacelle. The panels can be attached to other panels, such as in the case of a pylon on a wing, or attached to bodies. The panels can have plain leading or trailing edge flaps, ailerons, rudders, or elevators. These control surfaces can be of the full or partial span type and their hinge lines are not restricted to constant percent chord lines.

The panel is divided into two sections; (1) the outboard section where the subpanel longitudinal edges are assumed to be straight lines in the X direction, and (2) the root section which is a transition region from the outboard section to the intersection of the panel and a body. The line of intersection between the panel and a body does not have to be a straight line. Therefore the subpanel longitudinal edge lines will change in shape from that of the line of intersection at the side of the body to a straight line in the X direction at the outboard section. If the panel is not attached to a body, there is no root section.

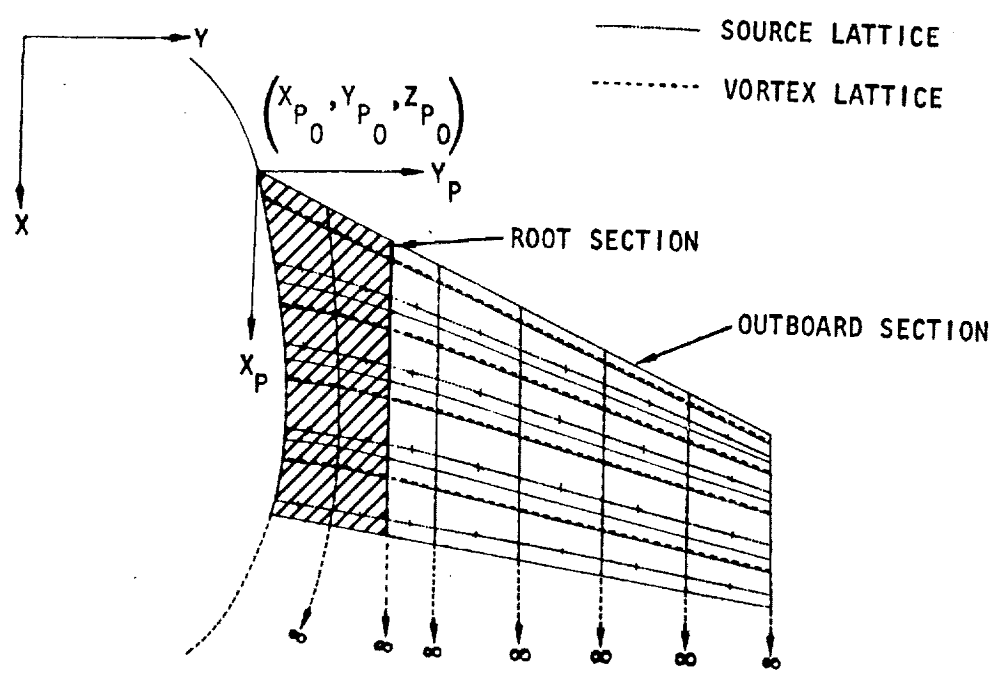

Figure 3.- Thick lifting panel.

The root section and outboard section can have any distribution of subpanel lengths and widths. The subpanel lateral edges are specified as a list of percent chord stations, evenly spaced in terms of percent chord, or evenly spaced in terms of ϕ, where

$$\phi = \cos^{-1}\left[1 - 2\left(\frac{X - X_{LE}}{C}\right)\right] .$$

The subpanel side edges are specified as a list of percent of surface semi-span η, evenly spaced in terms of percent of surface semi-span, or evenly spaced in terms of θ, where $\theta = \cos^{-1}\eta$.

The percent of surface semi-span η is defined as the percent of length of the line projected into the (y-Z) plane by the panel leading edge. A constant η line is in general curved in the root section. In the outboard section, lines of constant η are straight and in the X direction. The curved constant η lines associated with the longitudinal edges of subpanels in the root section are computed such that the sweep of the leading and trailing edges of the subpanels vary linearly from that of the sweep of the panel at the leading edge to that of the sweep of the panel at the trailing edge. Also, the corner points of the subpanels are equally spaced in the lateral direction along lines of constant percent chord. The chord at a lateral station in the root section is the length of the curved constant η line at that station. If the lateral distance along a constant percent chord line between two corner points is defined by $|\vec{\Delta S}|$, the unit vector tangent to the leading or trailing edge of a subpanel at any percent chord station is given by;

$$\left(\frac{\vec{\Delta S}}{|\vec{\Delta S}|}\right)_K = \frac{\vec{\Delta S}_{L.E._K}}{\left|\vec{\Delta S}_{L.E._K}\right|} + \left(\frac{\vec{\Delta S}_{T.E._K}}{\left|\vec{\Delta S}_{T.E._K}\right|} - \frac{\vec{\Delta S}_{L.E._K}}{\left|\vec{\Delta S}_{L.E._K}\right|}\right) (\text{percent chord}) \quad (19)$$

where

$$\left|\vec{\Delta S}_K\right| = \frac{C_S^2}{\sum_{K=1}^{K_J} \left(\frac{\vec{\Delta S}}{|\vec{\Delta S}|}\right)_K \cdot \vec{C}_S} \quad (20)$$

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and K_J equals the number of subpanels in the lateral direction in the root section.

$$\vec{C_S} = (X_J - X_R)\,\hat{i} + (Y_J - Y_R)\,\hat{j} + (Z_J - Z_R)\,\hat{k} \tag{21}$$

where (X_R, Y_R, Z_R) and (X_J, Y_J, Z_J) are the points on the curved constant percent chord line at the line of intersection of the panel and the body and at the juncture of the root and outboard sections, respectively. The equally spaced corner points along a constant percent chord line are then computed by

$$R\,(X,Y,Z)_\eta = R\,(X_R,Y_R,Z_R) + \sum_{K=1}^{K=K_\eta} \left(\frac{\vec{\Delta S}}{|\vec{\Delta S}|}\right)_K \left|\vec{\Delta S}_K\right| \tag{22}$$

where K_η is the number of subpanels, in the lateral direction, the constant η line is located from the line of intersection of the panel and the body. Since constant η lines in the outboard section are straight and in the X direction the subpanel corner points are defined directly from the input list of subpanel edge locations in terms of percent of chord and lateral η station.

The coordinates of the leading edge point $(X_{L.E.}, Y_{L.E.}, Z_{L.E.})_K$ associated with a constant η_K line are obtained from the panel perimeter description by converting the leading edge input points to tables of $X_{L.E._i}$ versus η, $Y_{L.E._i}$ versus η, and $Z_{L.E._i}$ versus η, and then interpolating in these tables to determine the leading edge coordinates at a desired lateral η_K station. Unit vectors tangent and normal to the panel at any η_K station can also be determined from these tables by evaluating the $\Delta Y_{L.E._K}$ and $\Delta Z_{L.E._K}$ about the lateral station η_K. The unit vectors are then defined by;

$$\vec{N}_{P_K} = -\frac{\Delta Z_{L.E._K}}{\sqrt{\Delta Y_{L.E._K}^{\,2} + \Delta Z_{L.E._K}^{\,2}}}\,\hat{j} + \frac{\Delta Y_{L.E._K}}{\sqrt{\Delta Y_{L.E._K}^{\,2} + \Delta Z_{L.E._K}^{\,2}}}\,\hat{k} \tag{23}$$

$$\bar{T}_{P_K} = \frac{\Delta Y_{L.E._K}}{\sqrt{\Delta Y_{L.E._K}^2 + \Delta Z_{L.E._K}^2}}\,\hat{j} + \frac{\Delta Z_{L.E._K}}{\sqrt{\Delta Y_{L.E._K}^2 + \Delta Z_{L.E._K}^2}}\,\hat{k} \tag{24}$$

With the coordinates of the subpanel corner points known, the coordinates and sweep of the vortex and source lattices used to represent the perturbation velocities due to lift and thickness, respectively, can be defined. The bound vortex lines and control points are placed at the quarter and three-quarter chord points of the subpanels, respectively. The fixed trailing vortices are placed along the subpanel side edges. In the root section the vortex lattice is a quadrilateral system, where as, in the outboard section the vortex lattice is a skewed horseshoe system. The skewed source lines are placed at both the quarter and three-quarter chord points of the subpanels.

The section geometry is input in terms of a percent thickness, percent camber, and twist. Where the percent thickness and camber are based on the local chord and measured in the $\bar{N}_{P_K}$ direction. Twist is defined in the plane described by the unit vectors $\hat{i}$ and $\bar{N}_{P_K}$. The X component of the unit vector normal to the section mean camber line and in the plane defined by $\hat{i}$ and $\bar{N}_{P_K}$ is then given by;

$$\bar{N}_{P_X} = \beta\left(-\frac{dZ_C}{dX} + \epsilon + \tan\delta\right) \tag{25}$$

where Z_C is the perpendicular distance between the section chord line and the mean camber line, ϵ is the angle of twist, and δ the deflection of any control surface. All three of these quantities are functions of both percent chord and η. The thickness is defined in the same manner as camber.

The trailing vortices aft of the trailing edge of the outboard section are straight lines in the X direction going off to infinity. The trailing vortices aft of the trailing edge of the root section lie along curved constant η lines to the end of the body and then go off to infinity in the X direction. These curved constant η lines are determined in the same manner as the constant η lines in the root section. If the free wake option of the program is utilized the location of the panel free trailing vortices are iterated for such that the wake is force free.

Discrete Influence Equations

The perturbation velocity due to the arbitrary lifting bodies is represented by quadrilateral vortices on the external surface of the bodies. The perturbation velocity due to lift on the panels is represented by quadrilateral vortices in the root section of the panel and by skewed horseshoe vortices in the outboard section of the panel. The thickness is represented by skewed source lines in both the root and outboard sections of the panel. All the panel singularities are placed on the panel chordal surface. The source strengths Σ are defined by the change in thickness over that portion of the subpanel it represents, so that;

$$\frac{\Sigma}{V_\infty} = \frac{2\,\beta\,\Delta Z_t}{\sqrt{1 + (\tan\,\Lambda)^2/\beta^2}}\;\frac{\overline{V}_X}{V_\infty} \tag{26}$$

where Λ is the sweep of the source line and $\overline{V}_X$ is the total onset velocity in the X direction. The quadrilateral vortex strengths K and the skewed horseshoe vortex strengths Γ must be solved for utilizing the boundary condition that a minimum of flow passes through the external surface of the bodies and the chordal surface of the thick lifting panels at a finite number of control points. In order to satisfy this boundary condition the total flow due to all singularities and onset flow is summed at each control point and the scalar product of this sum and the surface unit normal is minimized. This results in a set of linear aerodynamic influence equations which are solved for the unknown vortex strengths by means of Householder's method, described in appendix A.

The influence equations for the j^{th} equivalent incompressible body are given by;

$$
\begin{aligned}
&\sum_{i=1}^{N_B} \left[\left\lceil \diagdown \bar{N}_{X_{B_j}} \diagdown \right\rfloor \left[A_{X_{B_j B_i}} \right] + \left\lceil \diagdown \bar{N}_{X_{B_j}} \diagdown \right\rfloor \left[A_{Y_{B_j B_i}} \right] + \left\lceil \diagdown \bar{N}_{Z_{B_j}} \diagdown \right\rfloor \left[A_{Z_{B_j B_i}} \right] \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i} \\
&+ \sum_{K=1}^{N_P} \left[\left\lceil \diagdown \bar{N}_{X_{B_j}} \diagdown \right\rfloor \left[A_{X_{B_j P_K}} \right] + \left\lceil \diagdown \bar{N}_{Y_{B_j}} \diagdown \right\rfloor \left[A_{Y_{B_j P_K}} \right] + \left\lceil \diagdown \bar{N}_{Z_{B_j}} \diagdown \right\rfloor \left[A_{Z_{B_j P_K}} \right] \right] \left\{ \begin{matrix} \frac{K}{V_\infty} \\ \frac{\Gamma}{V_\infty} \end{matrix} \right\}_{P_K} \\
&+ \sum_{K=1}^{N_P} \left[\left\lceil \diagdown \bar{N}_{X_{B_j}} \diagdown \right\rfloor \left[S_{X_{B_j P_K}} \right] + \left\lceil \diagdown \bar{N}_{Y_{B_j}} \diagdown \right\rfloor \left[S_{Y_{B_j P_K}} \right] + \left\lceil \diagdown \bar{N}_{Z_{B_j}} \diagdown \right\rfloor \left[S_{Z_{B_j P_K}} \right] \right] \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_K} \\
&\qquad + \left\lceil \diagdown \bar{N}_{X_{B_j}} \diagdown \right\rfloor \left\{ \frac{\bar{V}_X}{V_\infty} \right\} + \left\lceil \diagdown \bar{N}_{Y_{B_j}} \diagdown \right\rfloor \left\{ \frac{\bar{V}_Y}{V_\infty} \right\} + \left\lceil \diagdown \bar{N}_{Z_{B_j}} \diagdown \right\rfloor \left\{ \frac{\bar{V}_Z}{V_\infty} \right\} = \left\{ e \right\}_{B_j}
\end{aligned}
\tag{27}
$$

where

$\bar{N}_{X_{B_j}}$, $\bar{N}_{Y_{B_j}}$, and $\bar{N}_{Z_{B_j}}$ are the components of the j^{th} equivalent incompressible body surface unit normal vectors,

$A_{X_{B_j B_i}}$, $A_{Y_{B_j B_i}}$, and $A_{Z_{B_j B_i}}$ are the components of the perturbation velocity induced by the i^{th} equivalent incompressible body unit strength vortices onto the j^{th} equivalent incompressible body,

$A_{X_{B_j P_K}}$, $A_{Y_{B_j P_K}}$, and $A_{Z_{B_j P_K}}$ are the components of the perturbation velocity induced by the k^{th} equivalent incompressible panel unit strength vortices onto the j^{th} equivalent incompressible body, and

$S_{X_{B_jP_K}}$, $S_{Y_{B_jP_K}}$, and $S_{Z_{B_jP_K}}$ are the components of the perturbation velocity induced by the k^{th} equivalent incompressible panel unit strength sources onto the j^{th} equivalent incompressible body.

Also,

$\left\{\frac{\bar{V}_X}{V_\infty}\right\}$, $\left\{\frac{\bar{V}_Y}{V_\infty}\right\}$, and $\left\{\frac{\bar{V}_Z}{V_\infty}\right\}$ are the components of the onset flow divided by the reference velocity V_∞,

$\{K\}_{B_i}$ are the quadrilateral vortex strengths on the i^{th} equivalent incompressible body, and

$\{K\}_{P_K}$, $\{\Gamma\}_{P_K}$, and $\{\Sigma\}_{P_K}$ are the root section quadrilateral vortex strengths, the outboard section skewed vortex strengths, and the skewed source strengths on the k^{th} equivalent incompressible panel, respectively. The vector $\{e\}_{B_j}$ is the flow through the surface of the j^{th} equivalent incompressible body at the control points, for a discrete body solution this vector is zero.

The influence equations for the j^{th} equivalent incompressible panel are given by;

$$\sum_{i=1}^{N_B}\left[\left[\diagdown\bar{N}_{X_{P_j}}\diagdown\right]\left[A_{X_{P_jB_i}}\right]+\left[\diagdown N_{Y_{P_j}}\diagdown\right]\left[A_{Y_{P_jB_i}}\right]+\left[\diagdown N_{Z_{P_j}}\diagdown\right]\left[A_{Z_{P_jB_i}}\right]\right]\left\{\frac{K}{V_\infty}\right\}_{B_i}$$

$$+\sum_{K=1}^{N_P}\left[\left[\diagdown\bar{N}_{X_{P_j}}\diagdown\right]\left[A_{X_{P_jP_k}}\right]+\left[\diagdown N_{Y_{P_j}}\diagdown\right]\left[A_{Y_{P_jP_k}}\right]+\left[\diagdown N_{Z_{P_j}}\diagdown\right]\left[A_{Z_{P_jP_k}}\right]\right]\left\{\begin{matrix}\frac{K}{V_\infty}\\ \frac{\Gamma}{V}\end{matrix}\right\}_{P_k}$$

$$+\sum_{K=1}^{N_P}\left[\left[\diagdown\bar{N}_{X_{P_j}}\diagdown\right]\left[S_{X_{P_jP_k}}\right]+\left[\diagdown N_{Y_{P_j}}\diagdown\right]\left[S_{Y_{P_jP_k}}\right]+\left[\diagdown N_{Z_{P_j}}\diagdown\right]\left[S_{Z_{P_jP_k}}\right]\right]\left\{\frac{\Sigma}{V_\infty}\right\}_{P_k}$$

$$+\left[\diagdown\bar{N}_{X_{P_j}}\diagdown\right]\left\{\frac{\bar{V}_X}{V_\infty}\right\}+\left[\diagdown N_{Y_{P_j}}\diagdown\right]\left\{\frac{\bar{V}_Y}{V_\infty}\right\}+\left[\diagdown N_{Z_{P_j}}\diagdown\right]\left\{\frac{\bar{V}_Z}{V_\infty}\right\}=\{e\}_{P_j} \tag{28}$$

Where

$\bar{N}_{X_{P_j}}$, $N_{Y_{P_j}}$, and $N_{Z_{P_j}}$ are the components of the j^{th} equivalent incompressible panel mean camber surface unit normal vectors,

$A_{X_{P_j B_i}}$, $A_{Y_{P_j B_i}}$, and $A_{Z_{P_j B_i}}$ are the components of the perturbation velocity induced by the i^{th} equivalent incompressible body unit strength vortices onto the j^{th} equivalent incompressible panel,

$A_{X_{P_j P_k}}$, $A_{Y_{P_j P_k}}$, and $A_{Z_{P_j P_k}}$ are the components of the perturbation velocity induced by the k^{th} equivalent incompressible panel unit strength vortices onto the j^{th} equivalent incompressible panel, and

$S_{X_{P_j P_k}}$, $S_{Y_{P_j P_k}}$, and $S_{Z_{P_j P_k}}$ are the components of perturbation velocity induced by the k^{th} equivalent incompressible panel unit strength sources onto the j^{th} equivalent incompressible panel. The vector $\{e\}_{pj}$ is the flow through the mean camber surface of the j^{th} equivalent incompressible panel at the control points. This vector is also zero for a discrete panel solution.

The onset flow velocity ratios

$\frac{\bar{V}_X}{V_\infty}$, $\frac{\bar{V}_Y}{V_\infty}$, and $\frac{\bar{V}_Z}{V_\infty}$ are given by the following expressions,

$$\frac{\bar{V}_X}{V_\infty} = 1 - q^* \frac{2\beta\,(Z-Z_{C.G.})}{\bar{c}} - \gamma^* \frac{2\beta\,(Y-Y_{C.G.})}{b} \tag{29}$$

$$\frac{\bar{V}_Y}{V_\infty} = -\beta\beta - P^* \frac{2\beta\,(Z-Z_{C.G.})}{b} + \gamma^* \frac{2\,(X-X_{C.G.})}{b} \tag{30}$$

$$\frac{\bar{V}_Z}{V_\infty} = \alpha\beta + P^* \frac{2\beta\,(Y-Y_{C.G.})}{b} + q^* \frac{2\,(X-X_{C.G.})}{\bar{c}} \tag{31}$$

Where P^*, q^*, and γ^* are the nondimensional roll, pitch, and yaw rates, respectively. These are defined as;

$$P^* = P/\frac{2V_\infty}{b}, \quad q^* = q/\frac{2V_\infty}{\bar{c}}, \quad \text{and } \gamma^* = \gamma/\frac{2V_\infty}{b} \tag{32}$$

The above onset flow equations assume that angle of attack α and angle of yaw β are small. The coordinates $X_{C.G.}$, $Y_{C.G.}$, and $Z_{C.G.}$ define the location of the center of gravity.

Equations (27) and (28) can be combined into a single matrix aerodynamic influence equation. For a completely discrete type solution the influence equation is,

$$\begin{bmatrix} [A]_{B_jB_i} & [A]_{B_jP_k} \\ & \\ [A]_{P_jB_i} & [A]_{P_jP_k} \end{bmatrix} \begin{bmatrix} \left\{\frac{K}{V_\infty}\right\}_{B_i} \\ \begin{Bmatrix} \frac{K}{V_\infty} \\ \frac{\Gamma}{V_\infty} \end{Bmatrix}_{P_K} \end{bmatrix} = \begin{bmatrix} \{\nabla S\}_{B_j} \\ \\ \{\nabla S\}_{P_j} \end{bmatrix} \tag{33}$$

where

$B_j = B_1, B_2, \ldots, B_{N_B}$

$B_i = B_1, B_2, \ldots, B_{N_B}$

$P_j = P_1, P_2, \ldots, P_{N_P}$

$P_k = P_1, P_2, \ldots, P_{N_P}$

N_B = Number of bodies

and

N_P = Number of panels

The matrices

$$[A]_{B_jB_i},\ [A]_{B_jP_k},\ [A]_{P_jB_i},\ \text{and}\ [A]_{P_jP_k}$$

are defined as;

$$[A]_{B_jB_i} = \left[\diagdown \bar{N}_{X_{B_j}} \diagdown\right]\left[A_{X_{B_jB_i}}\right] + \left[\diagdown \bar{N}_{Y_{B_j}} \diagdown\right]\left[A_{Y_{B_jB_i}}\right] + \left[\diagdown \bar{N}_{Z_{B_j}} \diagdown\right]\left[A_{Z_{B_jB_i}}\right] \tag{34}$$

$$[A]_{B_jP_k} = \left[\diagdown \bar{N}_{X_{B_j}} \diagdown\right]\left[A_{X_{B_jP_k}}\right] + \left[\diagdown \bar{N}_{Y_{B_j}} \diagdown\right]\left[A_{Y_{B_jP_k}}\right] + \left[\diagdown \bar{N}_{Z_{B_j}} \diagdown\right]\left[A_{Z_{B_jP_k}}\right] \tag{35}$$

$$[A]_{P_jB_i} = \left[\diagdown \bar{N}_{X_{P_j}} \diagdown\right]\left[A_{X_{P_jB_i}}\right] + \left[\diagdown N_{Y_{P_j}} \diagdown\right]\left[A_{Y_{P_jB_i}}\right] + \left[\diagdown N_{Z_{P_j}} \diagdown\right]\left[A_{Z_{P_jB_i}}\right] \tag{36}$$

$$[A]_{P_jP_k} = \left[\diagdown \bar{N}_{X_{P_j}} \diagdown\right]\left[A_{X_{P_jP_k}}\right] + \left[\diagdown N_{Y_{P_j}} \diagdown\right]\left[A_{Y_{P_jP_k}}\right] + \left[\diagdown N_{Z_{P_j}} \diagdown\right]\left[A_{Z_{P_jP_k}}\right] \tag{37}$$

The known quantities in the influence equation $\{\nabla S\}_{B_j}$ and $\{\nabla S\}_{P_j}$ are defined as;

$$\begin{aligned}\{\nabla S\}_{B_j} = -\sum_{k=1}^{N_P} \Bigg[&\left[\diagdown \bar{N}_{X_{B_j}} \diagdown\right]\left[S_{X_{B_jP_k}}\right] + \left[\diagdown \bar{N}_{Y_{B_j}} \diagdown\right]\left[S_{Y_{B_jP_k}}\right] \\ &+ \left[\diagdown \bar{N}_{Z_{B_j}} \diagdown\right]\left[S_{Z_{B_jP_k}}\right]\Bigg]\left\{\frac{\Sigma}{V_\infty}\right\}_{P_k} - \left[\diagdown \bar{N}_{X_{B_j}} \diagdown\right]\left\{\frac{\bar{V}_X}{V_\infty}\right\} \\ &- \left[\diagdown \bar{N}_{Y_{B_j}} \diagdown\right]\left\{\frac{\bar{V}_Y}{V_\infty}\right\} - \left[\diagdown \bar{N}_{Z_{B_j}} \diagdown\right]\left\{\frac{\bar{V}_Z}{V_\infty}\right\}\end{aligned} \tag{38}$$

$$\left\{\nabla S\right\}_{P_j} = -\sum_{k-1}^{N_p} \left[\left[\bar{N}_{X_{P_j}}\right] \left[S_{X_{P_j P_k}}\right] + \left[N_{Y_{P_j}}\right] \left[S_{Y_{P_j P_k}}\right] + \left[N_{X_{P_j}}\right] \left[S_{Z_{P_j P_k}}\right] \right] \left\{\frac{\Sigma}{V_\infty}\right\}_{P_k} - \left[\bar{N}_{X_{P_j}}\right] \left\{\frac{\bar{V}_X}{V_\infty}\right\} - \left[N_{Y_{P_j}}\right] \left\{\frac{V_Y}{V_\infty}\right\} - \left[N_{Z_{P_j}}\right] \left\{\frac{V_Z}{V_\infty}\right\} \tag{39}$$

The elements of the above matrices are computed using the influence equations derived in Appendices B and C. Each element is associated with the influence of a singularity on a control point. The singularities and control points are ordered such that all of the longitudinal stations for the first lateral station are cycled through first and then all of the longitudinal stations for the second lateral station. This process is continued until all stations have been cycled through. The longitudinal stations start at the leading edge of the panel, the nose of a solid body, and the tail end of the inside surface of a flow through body. The lateral stations start at the inside edge of the panel and go toward the tip of the panel. The lateral stations on the body start at the top of the body and progress in a clockwise direction when looking at the body from the tail to the nose.

All bodies are cycled through first and then the panels. Both the bodies and the panels are cycled in the order that they are input. The columns of the influence matrices are associated with singularities and the rows with control points. The elements of the matrices and the submatrices of the combined matrix influence equation are sequenced, for both the columns and the rows, in the same order that the singularities, points, bodies, and panels are cycled.

The influences of quadrilateral vortices in the panel root section and on the body surface are computed by equations derived in Appendix C. The influences of skewed horseshoe vortices on the panel outboard section are computed by equations derived in Appendix B. The influences of skewed source lines on the panel are computed by equations derived in Appendix B.

If the force free wake option of the program is used the influences of the free trailing vortices are computed using the influence equations in Appendix C. The locations of the force free trailing vortices are computed using the following iteration procedure.

1. A solution for the vortex strengths will be obtained first by assuming the location of the free vortices to be in the longitudinal direction and to be straight except in a thick lifting panel-body juncture region.

2. The total velocities U, V, and W in the X, Y, and Z directions, respectively, will be computed at the midpoint of each free trailing vortex division.

3. The actual mean camber surfaces of the thick lifting panels will be computed so that the correct relationship between fixed vortices and control points on the thick lifting panels and the free trailing vortices is maintained.

4. The ratios V/U and W/U are integrated in the X direction from the edges of the actual thick lifting panel mean camber surfaces and body aft ends where the free vortices are assumed to be shed, in order to obtain their new locations.

5. The influence of the free vortices on the actual thick lifting panel mean camber and body surfaces is computed and the difference between this influence and that from the free vortices at their previous location is added to the influence matrices for the complete configuration.

6. A new solution for the vortex strengths is determined and a new set of total velocities U, V, and W along the new free vortex lines is computed using quadrilateral vortices and source lines on the actual thick lifting panel mean camber surfaces and quadrilateral vortices on the body surfaces.

The above procedure is iterated between steps (4) and (6) until the surface pressures converge. When the influence matrix of any individual body or panel is no longer significantly changed due to a new positioning of the free vortices, the calculation of the perturbation to that matrix is terminated.

Constrained Influence Equations

The vortex strengths on any of the bodies or panels can be constrained by representing the vorticity by a finite series. This uncouples the number of unknowns, equations, and vortices used to represent the perturbation velocity. The series used must be capable of producing the type of perturbation velocity needed to satisfy the boundary conditions at the control points to an acceptable degree. If such a series can be found for a given body or panel the number of equations or control points can be reduced from that of the number of vortices or subpanels to that necessary to describe the geometry. This substantially reduces the computer time, since in general the number of vortices needed to represent the perturbation velocity is far more than the number of control points needed to represent the geometry of the body or panel.

This reduction in the number of required control points results primarily in reducing the computational effort necessary to set up the aerodynamic influence equations. A further reduction in computer time can be realized in the solution of the influence equations, if constraint functions are used, since the number of unknowns is reduced from that of the number of vortices to the number of terms in the constraint series. This results in a system of equations which is overdetermined and is solved by the method of least squares. In this process the vectors $\{e\}_{B_j}$ and $\{e\}_{P_j}$ are minimized. The body constraint equation is given by;

$$\left\{\frac{K}{V_\infty}\right\}_{B_i} = \left[\ T_{B_i}\ \right]\left\{a_{B_i}\right\} \tag{40}$$

where

$$\left[\ T_{B_i}\ \right] = \left[\ R\ \right]\left[\diagdown\ \left[\bar{\ell}_{M_{mn}}\right]\ \diagdown\right]\left[\left(\frac{\gamma}{V_\infty}\right)_{mn}\right] \tag{41}$$

The matrix [R] is a transformation matrix used to obtain the quadrilateral vortex strengths K from the bound horseshoe vortex strengths Γ_b.

$$\left[\ R\ \right] = \left[\diagdown\ [R]_n\ \diagdown\right] \tag{42}$$

where $[R]_n$ is made up of ones along the diagonal and in the lower triangle. The upper triangle is filled with zeros.

The indices m and n refer to longitudinal and lateral subpanel locations on the equivalent incompressible body, respectively. The vorticity ratio $(\gamma/V_\infty)_{mn}$ is the value of the vorticity series at the m^{th} subpanel from the nose of the body and the n^{th} subpanel in the lateral direction around the body. The longitudinal length $\bar{\ell}'_{M_{mn}}$ is given by;

$$\bar{\ell}'_{M_{mn}} = \frac{1}{2}\left[\frac{3}{4}\bar{\ell}_{M_{(m-1)n}} + \bar{\ell}_{M_{mn}} + \frac{1}{4}\bar{\ell}_{M_{(m+1)n}}\right] \tag{43}$$

where $\overline{\ell}_{M_{mn}}$ is the length of the m^{th} subpanel from the nose of the equivalent incompressible body at the n^{th} lateral station.

The vorticity series $(\gamma/V_\infty)_{mn}$ is the product of a longitudinal series and a lateral series. The elements of the matrix $\{a_{B_i}\}$ are the unknown coefficients associated with the terms in $(\gamma/V_\infty)_{mn}$ produced by the product of the longitudinal and lateral series. The terms in $(\gamma/V_\infty)_{mn}$ are ordered such that the products of all of the longitudinal constraint functions and the first lateral constraint function are first, then the products of all of the longitudinal constraint functions and the second lateral constraint functions are second, and etcetera. This process is continued until all combinations of longitudinal and lateral constraint functions have been cycled through.

Both the longitudinal and lateral constraint functions are defined over segments of the body. The same functions are used in all segments. The origin of the segment is designated by the subscript o and the end by f. The longitudinal and lateral constraint function segments are given in the data input array.

The longitudinal constraint functions for the body are defined as;

$$\frac{\gamma}{V_\infty}(X_B/C_B)_1 = 1 \tag{44}$$

$$\frac{\gamma}{V_\infty}(X_B/C_B)_2 = 1/\left\{1 + \left[Y_B\left(\frac{\overline{N}_{X_B}}{\overline{N}_{Y_B}}\right) + Z_B\left(\frac{\overline{N}_{X_B}}{\overline{N}_{Z_B}}\right)\right]^2 \Big/ \left(Y_B^{\,2} + Z_B^{\,2}\right)\right\}^{1/2} \tag{45}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_3 = \text{COT}\ \phi_B/_2 \tag{46}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_4 = \text{COT}\ (\pi/2 - \phi_{B/2}) \tag{47}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_5 = \text{SIN}\left[\pi\left(\frac{\phi_B - \phi_o}{\phi_f - \phi_o}\right)\right] \tag{48}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_6 = \text{COS}\left[\pi\left(\frac{\phi_B - \phi_o}{\phi_f - \phi_o}\right)\right] \tag{49}$$

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$$\frac{\gamma}{V_\infty}(X_B/C_B)_7 = \left[\frac{\left(\frac{X_B}{C_B}\right)_B - \left(\frac{X_B}{C_B}\right)_0}{\left(\frac{X_B}{C_B}\right)_f - \left(\frac{X_B}{C_B}\right)_0}\right] \tag{50}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_8 = \mathrm{SIN}\left[2\pi\left(\frac{\phi_B - \phi_0}{\phi_f - \phi_0}\right)\right] \tag{51}$$

$$\frac{\gamma}{V_\infty}(\phi_B)_9 = \mathrm{COS}\left[2\pi\left(\frac{\phi_B - \phi_0}{\phi_f - \phi_0}\right)\right] \tag{52}$$

$$\frac{\gamma}{V_\infty}(X_B/C_B)_{10} = \left[\frac{\left(\frac{X_B}{C_B}\right)_B - \left(\frac{X_B}{C_B}\right)_0}{\left(\frac{X_B}{C_B}\right)_f - \left(\frac{X_B}{C_B}\right)_0}\right]^2 \tag{53}$$

•

•

•

$$\frac{\gamma}{V_\infty}(X_B/C_B)_{3270} = L_{S_1}\ (X_B/C_B) \tag{54}$$

$$\frac{\gamma}{V_\infty}(X_B/C_B)_{3320} = L_{S_2}\ (X_B/C_B)$$

$$\frac{\gamma}{V_\infty}(X_B/C_B)_{3370} = L_{S_2}\ (X_B/C_B) \tag{55}$$

The functions to be used on the body are designated by their number in the above sequence. The last three functions listed above are special functions and are input at the locations in the data array, designated by their subscripts. The independent variable ϕ_B is given by, $\phi_B = \cos^{-1}\left[1 - 2(X_B/C_B)\right]$

The lateral constraint functions for the body are defined as;

$$\frac{\gamma}{V_\infty}(\theta_B)_1 = 1 \tag{57}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_2 = \bar{N}_{Z_B} \Big/ \left[\bar{N}_{Z_B}^{\;2} + \bar{N}_{Y_B}^{\;2}\right]^{1/2} \tag{58}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_3 = \bar{N}_{Y_B} \Big/ \left[\bar{N}_{Z_B}^{\;2} + \bar{N}_{Y_B}^{\;2}\right]^{1/2} \tag{59}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_4 = \sin\left[\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right] \tag{60}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_5 = \cos\left[\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right]$$

$$\frac{\gamma}{V_\infty}(\theta_B)_6 = \left[\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right] \quad \text{or} \quad \left[\frac{\eta_B - \eta_o}{\eta_f - \eta_o}\right] \tag{61}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_7 = \sin\left[2\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right] \tag{62}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_8 = \cos\left[2\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right] \tag{63}$$

$$\frac{\gamma}{V_\infty}(\theta_B)_9 = \left[\pi\left(\frac{\theta_B - \theta_o}{\theta_f - \theta_o}\right)\right]^2 \quad \text{or} \quad \left[\frac{\eta_B - \eta_o}{\eta_f - \eta_o}\right]^2 \tag{64}$$

$$\vdots$$

Here again, the lateral functions to be used on the body are designated by their number in the above sequence. If no constraint functions are specified the program will do a discrete solution.

The panel constraint equation is given by;

$$\left\{\begin{matrix} \frac{K}{V_\infty} \\ \\ \frac{\Gamma}{V_\infty} \end{matrix}\right\}_{P_K} = \left[\quad T_{P_K} \quad\right]\left\{a_{P_K}\right\} \tag{65}$$

The $[T_{P_K}]$ transformation matrix condenses all rows of the discrete aerodynamic influence matrix by the following procedure.

1. That portion of the row which is associated with the vortices on panel P_K is divided into the elements due to each lateral station. A new temporary matrix $[A_T]$ is developed with all of the elements due to the votrices at the first lateral station in row one, all of the elements due to the vortices at the second lateral station in row two, and etcetera.

2. This temporary matrix is then premultiplied by the spanwise constraint function transformation matrix $[T(\eta)]$ and postmultiplied by the chordwise constraint function transformation matrix $[T(NC)]$.

3. The transformed temporary matrix $[\bar{A}_T] = [T(\eta)][A_T][T(X/C)]$ is then opened up into a row again and replaces the old row in the original discrete aerodynamic influence matrix. The new row is formed by placing all of the elements from the first row of $[\bar{A}_T]$ in tha portion of the row, from the discrete matrix, due to the P_K panel first, then by placing all of the elements from the second row of $[\bar{A}_T]$ next to those elements from the first row of $[\bar{A}_T]$, and etcetera. The result will be the reduction of the number of elements in a given row due to the P_K panel from that of the number of vortices or subpanels on panel P_K to the number of chordwise constant functions times the number of spanwise constraint functions.

If only the chordwise constraint functions are used, then the number of elements in the row due to the P_K panel will be reduced from the number of vortices on the P_K panel to the number of chordwise constraint functions times the number of subpanels in the lateral direction. On panels with freestream edges, at span stations other than the tip, only the chordwise functions should be used, since the available spanwise constraint functions are not sufficient to produce the necessary perturbation velocity to satisfy the boundary conditions on a panel of this type.

The above described transformation constrains the skewed horseshoe vortex strengths by the following series developed in appendices F. and G. of reference (26).

$$\frac{\Gamma}{V_\infty}(\eta)_m = \sqrt{1-\eta^2}\sum_{w=1}^{N_w - P_w}\left\{\left(\frac{\Gamma'_{om}}{V_\infty}\right)a_{ow} + \sum_{f=1}^{N_f}\left(\frac{\Gamma'_{fm}}{V_\infty}\right)a_{fw} + \sum_{k=1}^{N_k}\left(\frac{\Gamma'_{km}}{V_\infty}\right)a_{kw} + \sum_{n=1}^{N_u - 1}\left(\frac{\Gamma'_{nm}}{V_\infty}\right)a_{nw}\right\}\eta^w + \sum_{w=(N_w - P_w + 1)}^{N_w}\left\{\left(\frac{\Gamma'_{om}}{V_\infty}\right)a_{ow} + \sum_{f=1}^{N_f}\left(\frac{\Gamma'_{fm}}{V_\infty}\right)a_{fw} + \sum_{k=1}^{N_k}\left(\frac{\Gamma'_{km}}{V_\infty}\right)a_{kw} + \sum_{n=1}^{N_u - 1}\left(\frac{\Gamma'_{nm}}{V_\infty}\right)a_{nw}\right\}P_w(\eta)_s \tag{66}$$

Where the indices m, f, k, o, n, and w indicate the number of the subpanel aft of the panel leading edge Γ/V_∞ is defined on, the number of the trailing edge flap, the number of the leading edge flap, the first term of the Birnbaum series, the number of the sine term in the Birnbaum series, and the number of the spanwise constraint function, respectively. The quantities N_u, N_f, N_k, P_w, and N_w are the number of terms in the Birnbaum series, the number of trailing edge flaps with unique hinge line locations, the number of leading edge flaps with unique hinge line locations, the number of special spanwise constraint functions, and the total number of spanwise constraint functions, respectively.

The columns of the matrix $[T(X/C)]$ are made up of the $(\Gamma'/V_\infty)_m$ values, where m indicates the number of the element in the column. There is one column for each set of $(\Gamma'/V_\infty)_m$'s associated with a unique chordwise constraint function. The $[T(X/C)]$ matrix is filled such that the (Γ'_{om}/V_∞) values are in column one, then the (Γ'_{Nn}/V_∞) values are in the next set of N_u-1 columns, then the (Γ'_{fm}/V_∞) values are in the next set of N_f columns, and then the (Γ'_{km}/V_∞) values are in the next set of N_k columns.

The $[T(X/C)]$ matrix is solved here in the same manner as is shown in appendix G. of reference (26). The $(\Gamma'/V_\infty)_m$ values are solved for such that the same downwash is obtained at the three quarter chord point of each subpanel, except at the last subpanel, due to a distribution of discrete vortices as would be obtained by integrating the vorticity distribution in the Blot-Savart integral for each chordwise constraint function. The additional condition, that the sum of the discrete vortex strengths must be equal to the integral of the vorticity distribution is also used. These conditions result in the following matrix equation which is solved for $[T(x/c)]$.

$$[E]\,[T(x/c)] = [W] \tag{67}$$

The matrix $[E]$ is the two-dimensional discrete vortex influence matrix and is defined as follows.

$$[E] = \begin{bmatrix} \dfrac{1}{\left[(x/c)_j - (x/c)_i\right]} \\ \\ \hline 1, 1, \ldots, 1 \end{bmatrix} \tag{68}$$

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Where $0 \leq (x/c) \leq 1.0$ and the indices j and i as used here indicate the location of the three quarter and quarter chord points, respectively.

There is one column in the $[w]$ matrix for each unique chordwise constraint function. In these columns is the downwash due to evaluating the vorticity for each chordwise constraint function in the two-dimensional Biot-Savart integral. The columns in $[w]$ are ordered such that the column due to a given chordwise constraint function is in the same location in $[w]$ as its corresponding $(\Gamma'/V_\infty)_m$ column is in the $[T(x/c)]$ matrix. The five basic types of columns in $[w]$ are defined as follows.

Due to the first term of the Birnbaum series;

$$\left\{w\right\}_{\cot\phi/2} = \left\{\begin{array}{c} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \hline 1/2 \end{array}\right\} \tag{69}$$

Due to the second term of the Birnbaum series;

$$\left\{w\right\}_{\sin\phi} = \left\{\begin{array}{c} -\cos\ \phi_1 \\ -\cos\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ -\cos\ \phi_{(N_i-1)} \\ \hline 1/4 \end{array}\right\} \tag{70}$$

Due to the third and higher order terms of the Birnbaum series;

$$\left\{ w \right\}_{\sin(n\phi)} = \left\{ \begin{array}{c} -\cos n\,\phi_1 \\ -\cos n\,\phi_2 \\ \cdot \\ \cdot \\ \cdot \\ -\cos n\,\phi_{(N_i-1)} \\ \text{------------} \\ 0 \end{array} \right\} \tag{71}$$

where $\phi_j = \cos^{-1}\left[1 - 2\,(x/c)_j\right]$

Due to the trailing edge flap term;

$$\left\{ w \right\}_{\log\left|\frac{\sin 1/2(\phi+\phi_f)}{\sin 1/2(\phi-\phi_f)}\right|} = \left\{ \begin{array}{c} w_f(\phi_1) \\ w_f(\phi_2) \\ \cdot \\ \cdot \\ \cdot \\ w_f(\phi_{N_i-1}) \\ \text{----------} \\ 1/2 \sin \phi_f \end{array} \right\} \tag{72}$$

where $w_f(\phi_j) = -(\pi - \phi_f)$ for $0 \le \phi_j < \phi_f$, $w_f(\phi_j) = \phi_f$ for $\phi_f \le \phi_j \le \pi$,
is the polar coordinate at the flap leading edge, and N_i equals the number of subpanels per chord.

due to the leading edge flap term;

$$\left\{ w \right\}_{\log \left| \frac{\sin 1/2(\phi + \phi_k)}{\sin 1/2(\phi - \phi_k)} \right|} = \left\{ \begin{array}{c} w_k(\phi_1) \\ w_k(\phi_2) \\ \cdot \\ \cdot \\ \cdot \\ w_k\left[\phi_{(N_i - 1)}\right] \\ ---------- \\ 1/2 \text{ SIN } \phi_k \end{array} \right\} \tag{73}$$

where $w_k(\phi_j) = -(\pi - \phi_k)$ for $0 \leq \phi_j \leq \phi_k$, $w_k(\phi_j) = \phi_k$ for $\phi_k < \phi_j \leq \pi$, and ϕ_k is the polar coordinate at the flap leading edge.

In the root section the [T X/C)] matrix is transformed to $[\overline{T}$ (X/C)], where each column of [T X/C)] is transformed as follows;

$$\overline{T}\ (X/C)_m = \sum_{i=1}^{m} T\ (X/C)_i \tag{74}$$

where i and m indicate the number of the element in the column in the [T (X/C)] and $[\overline{T}$ (X/C)] matrices, respectively.

The rows of the [T (η)] matrix are made up of the spanwise constraint functions. Each element in a row is equal to the value of the constraint function at the lateral subpanel which has the same lateral index as the number of the column in the [T (η)] matrix. There are N_W rows in the [T (η)] matrix, with the standard functions described by $\eta^W \sqrt{1 - \eta^2}$ in the first $N_W - P_W$ rows, and then the special P_W $(\eta)_S$ functions in the last P_W rows.

The special spanwise constraint functions are needed to account for flow induced by discontinuities in the sweep of the constant percent chord lines, for body-panel juncture induced flow, and for partial semi-span flaps. There are two basic special spanwise constraint functions, (1) polygonal functions which account for the discontinuities in the sweep of the constant percent chord lines and the body-panel junctures, and (2) flap functions which account for the partial semi-span flaps. These special functions are derived in Appendix F of Reference (26).

If the range of influences inboard and outboard of a discontinuity in the sweep of the constant percent chord line are defined as $\Delta\eta_i$ and $\Delta\eta_o$, respectively, and the discontinuity station η_b, then the following expressions define the special spanwise polygonal constraint function.

For $o < |\eta_b| < \Delta\eta_i$

$$P(\theta)_S = (1 - \eta_b/\Delta\eta_i)\, M(\theta, \tfrac{\pi}{2}) + \frac{1}{\Delta\eta_i} P(\theta, \tfrac{\pi}{2}) - \left[\frac{(\Delta\eta_i + \Delta\eta_o)(1 - \eta_b)}{\Delta\eta_i \Delta\eta_o} \right] P(\theta, \theta_b)$$

$$+ \left[\frac{\Delta\eta_i + \Delta\eta_o}{\Delta\eta_i \Delta\eta_o} - \frac{1}{\Delta\eta_i} \right] \left[1 - \Delta\eta_o - \eta_b \right] P\left(\theta, \cos^{-1}(\eta_b + \Delta\eta_o)\right) \tag{75}$$

For $\Delta\eta_i \leq |\eta_b| < (1 - \Delta\eta_o)$

$$P(\theta)_S = (1 - \eta_b + \Delta\eta_i)(1/\Delta\eta_i)\, P\left(\theta, \cos^{-1}(\eta_b - \Delta\eta_i)\right)$$

$$- \left[\frac{\Delta\eta_o + \Delta\eta_i}{\Delta\eta_o \Delta\eta_i} \right] \left[1 - \eta_b \right] P(\theta, \theta_b)$$

$$+ (1 - \eta_b - \Delta\eta_o)(1/\Delta\eta_o)\, P\left(\theta, \cos^{-1}(\eta_b + \Delta\eta_o)\right) \tag{76}$$

And for $(1 - \Delta\eta_o) \leq |\eta_b| < 1.0$

$$P(\phi)_S = (1 - \eta_b + \Delta\eta_i)(1/\Delta\eta_i)\, P\left(\theta, \cos^{-1}(\eta_b - \Delta\eta_i)\right)$$

$$- \left[1 + (1 - \eta_b)/\Delta\eta_i \right] P(\theta, \theta_b) \tag{77}$$

Where

$$P(\theta, \theta^*) = \frac{1}{2\pi(1-\cos\theta^*)} \left\{ (\cos\theta^* - \cos\theta) \, LOG_e \left| \frac{\sin 1/2 \, (\theta^* - \theta)}{\sin 1/2 \, (\theta^* + \theta)} \right| + (\cos\theta^* + \cos\theta)^2 \, LOG_e \left| \frac{\cos 1/2 \, (\theta^* + \theta)}{\cos 1/2 \, (\theta^* - \theta)} \right| + (4\,\theta^* \cos\theta^* - 2\sin\theta^*) \sin\theta \right\} \tag{78}$$

And

$$\theta^* = \cos^{-1} \eta$$

The special spanwise flap constraint function is given by;

$$P(\theta)_S = M(\theta, \theta_i) - M(\theta, \theta_o) \tag{79}$$

Where $\theta_i = \cos^{-1} \eta_i$, $\theta_o = \cos^{-1} \eta_o$, η_i is the inboard station where the control surface begins, and η_o is the outboard station where the control surface ends.

$$M(\theta, \theta^*) = \frac{1}{\pi} \left\{ (\cos\theta^* - \cos\theta) \, LOG_e \left| \frac{\sin 1/2 \, (\theta^* - \theta)}{\sin 1/2 \, (\theta^* + \theta)} \right| + (\cos\theta^* + \cos\theta) \, LOG_e \left| \frac{\cos 1/2 \, (\theta^* + \theta)}{\cos 1/2 \, (\theta^* - \theta)} \right| + 2\,\theta^* \sin\theta \right\} \tag{80}$$

The discrete aerodynamic influence equation (33) is transformed by substituting equations (40) and (65) into equation (33).

$$\begin{bmatrix} [A]_{B_jB_i} & [A]_{B_jP_{KC}} \\ [A]_{P_jB_i} & [A]_{P_jP_{KC}} \end{bmatrix} \begin{bmatrix} [T_{B_i}] & [0] \\ [0] & [T_{P_K}] \end{bmatrix} \begin{bmatrix} \{a\}_{B_i} \\ \{a\}_{P_K} \end{bmatrix} = \begin{bmatrix} \{\nabla S\}_{B_j} \\ \{\nabla S\}_{P_j} \end{bmatrix} + \begin{bmatrix} \{e\}_{B_j} \\ \{e\}_{P_j} \end{bmatrix} \tag{81}$$

The constrained aerodynamic influence equation is then given by;

$$\begin{bmatrix} [A]_{B_jB_i} & [A]_{B_jP_K} \\ [A]_{P_jB_i} & [A]_{P_jP_K} \end{bmatrix} \begin{bmatrix} \{a\}_{B_i} \\ \{a\}_{P_K} \end{bmatrix} = \begin{bmatrix} \{\nabla S\}_{B_j} \\ \{\nabla S\}_{P_j} \end{bmatrix} + \begin{bmatrix} \{e\}_{B_j} \\ \{e\}_{P_j} \end{bmatrix} \tag{82}$$

Where the arrays $\{a\}_{B_i}$ and $\{a\}_{P_K}$ are solved for such that the sum of the squares of the elements of the arrays $\{e\}_{B_j}$ and $\{e\}_{P_J}$ are a minimum.

Surface Velocities and Pressures

The velocity tangent to the surface of the j^{th} body in the longitudinal direction is given by;

$$
\begin{aligned}
\left\{\left(\frac{V_M}{V_\infty}\right)_{mn}\right\}_{B_j} = & \sum_{i=1}^{N_B}\left[\frac{1}{\beta^2}\left\lceil T_{M_{X_{B_j}}}\right\rfloor\left[A_{X_{B_jB_i}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Y_{B_j}}}\right\rfloor\left[A_{Y_{B_jB_i}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Z_{B_j}}}\right\rfloor\left[A_{Z_{B_jB_i}}\right]\right]\left\{\frac{K}{V_\infty}\right\}_{B_i} \\
& +\sum_{K=1}^{N_P}\left[\frac{1}{\beta^2}\left\lceil T_{M_{X_{B_j}}}\right\rfloor\left[A_{X_{B_jP_K}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Y_{B_j}}}\right\rfloor\left[A_{Y_{B_jP_K}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Z_{B_j}}}\right\rfloor\left[A_{Z_{B_jP_K}}\right]\right]\left\{\begin{matrix}\frac{K}{V_\infty}\\ \frac{\Gamma}{V_\infty}\end{matrix}\right\}_{P_K} \\
& +\sum_{K=1}^{N_P}\left[\frac{1}{\beta^2}\left\lceil T_{M_{Z_{B_j}}}\right\rfloor\left[S_{X_{B_jP_K}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Y_{B_j}}}\right\rfloor\left[S_{Y_{B_jP_K}}\right]+\frac{1}{\beta}\left\lceil T_{M_{Z_{B_j}}}\right\rfloor\left[S_{Z_{B_jP_K}}\right]\right]\left\{\frac{\Sigma}{V_\infty}\right\}_{P_K} \\
& +\left\lceil T_{M_{X_{B_j}}}\right\rfloor\left\{\frac{V_X}{V_\infty}\right\}+\left\lceil T_{M_{Y_{B_j}}}\right\rfloor\left\{\frac{V_Y}{V_\infty}\right\}+\left\lceil T_{M_{Z_{B_j}}}\right\rfloor\left\{\frac{V_Z}{V_\infty}\right\} \\
& +\left\{\left(\frac{\Delta V_M}{V_\infty}\right)_{mn}\Big/\ \beta\sqrt{\beta^2\bar{T}_{M_X}^2+\bar{T}_{M_Y}^2+\bar{T}_{M_Z}^2}\right\}_{B_j}
\end{aligned}
\tag{83}
$$

Where

$$
\begin{aligned}
\left(\frac{\Delta V_M}{V_\infty}\right)_{mn} = & \left[\frac{\Gamma_{b_{mn}}}{V_\infty}\Big/\left(\frac{3}{4}\bar{\ell}_{M(m-1)n}+\bar{\ell}_{M_{mn}}+\frac{1}{4}\bar{\ell}_{M(m+1)n}\right)\right]\left[\frac{\bar{\ell}_{M_{mn}}+\bar{\ell}_{M(m+1)n}}{3\bar{\ell}_{M_{mn}}+\bar{\ell}_{M(m+1)n}}\right] \\
& +\left[\frac{\Gamma_{b(M+1)}}{V_\infty}\Big/\left(\frac{3}{4}\bar{\ell}_{M_{mn}}+\bar{\ell}_{M(m+1)n}+\frac{1}{4}\bar{\ell}_{M(m+2)n}\right)\right]\left[\frac{2\bar{\ell}_{M_{mn}}}{3\bar{\ell}_{M_{mn}}+\bar{\ell}_{M(m+1)n}}\right]
\end{aligned}
\tag{84}
$$

And

$$
\frac{\Gamma_{b_{mn}}}{V_\infty}=\frac{K_{mn}}{V_\infty}-\frac{K_{(m-1)n}}{V_\infty}
\tag{85}
$$

$$\frac{\Gamma_{b_{(m+1)N}}}{V_\infty} = \frac{K_{(m+1)N}}{V_\infty} - \frac{K_{mN}}{V_\infty} \tag{86}$$

The onset flow ratios V_X/V_∞, V_Y/V_∞, and V_Z/V_∞ are given by the following expressions.

$$\frac{V_X}{V_\infty} = 1 - q^* \frac{2(Z - Z_{C.G.})}{\bar{c}} - \gamma^* \frac{2(Y - Y_{C.G.})}{b} \tag{87}$$

$$\frac{V_Y}{V_\infty} = \beta + p^* \frac{2(Z - Z_{C.G.})}{b} + \gamma^* \frac{2(X - X_{C.G.})}{\bar{b}} \tag{88}$$

$$\frac{V_Z}{V_\infty} = \alpha + p^* \frac{2(Y - Y_{C.G.})}{b} + q^* \frac{(X - X_{C.G.})}{\bar{c}} \tag{89}$$

The components of the unit vectors tangent to the actual body subpanels in the longitudinal direction are given by;

$$T_{M_X} = \beta\, \bar{T}_{M_X} / \sqrt{\beta^2\, \bar{T}_{M_X} + \bar{T}_{M_Y} + \bar{T}_{M_Z}} \tag{90}$$

$$T_{M_Y} = \bar{T}_{M_Y} / \sqrt{\beta^2\, \bar{T}_{M_X} + \bar{T}_{M_Y} + \bar{T}_{M_Z}} \tag{91}$$

$$T_{M_Z} = \bar{T}_{M_Z} / \sqrt{\beta^2\, \bar{T}_{M_X} + \bar{T}_{M_Y} + \bar{T}_{M_Z}} \tag{92}$$

and in the lateral direction by

$$T_{T_X} = \beta\bar{T}_{T_X} / \sqrt{\beta^2\bar{T}_{T_X}^{\,2} + \bar{T}_{T_Y}^{\,2} + \bar{T}_{T_Z}^{\,2}}$$

$$T_{T_Y} = \bar{T}_{T_Y} / \sqrt{\beta^2\bar{T}_{T_X}^{\,2} + \bar{T}_{T_Y}^{\,2} + \bar{T}_{T_Z}^{\,2}}$$

$$T_{T_Z} = \bar{T}_{T_Z} / \sqrt{\beta^2\bar{T}_{T_X}^{\,2} + \bar{T}_{T_Y}^{\,2} + \bar{T}_{T_Z}^{\,2}}$$

The velocity tangent to the surface of the jth body in the lateral direction is given by;

$$\left\{\left(\frac{V_T}{V_\infty}\right)_{mn}\right\}_{B_j} = \sum_{i=1}^{N_B}\left[\frac{1}{\beta^2}\left[\diagdown T_{T_{X_{B_j}}}\diagdown\right]\left[A_{X_{B_jB_i}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Y_{B_j}}}\diagdown\right]\left[A_{X_{B_jB_i}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Z_{B_j}}}\diagdown\right]\left[A_{Z_{B_jB_i}}\right]\right]\left\{\frac{K}{V_\infty}\right\}_{B_i}$$

$$+ \sum_{K=1}^{N_P}\left[\frac{1}{\beta^2}\left[\diagdown T_{T_{X_{B_j}}}\diagdown\right]\left[A_{X_{B_jP_K}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Y_{B_j}}}\diagdown\right]\left[A_{Y_{B_jP_K}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Z_{B_j}}}\diagdown\right]\left[A_{Z_{B_jP_K}}\right]\right]\left\{\begin{matrix}\frac{K}{V_\infty}\\ \frac{\Gamma}{V_\infty}\end{matrix}\right\}_{P_K}$$

$$+ \sum_{K=1}^{N_P}\left[\frac{1}{\beta^2}\left[\diagdown T_{T_{X_{B_j}}}\diagdown\right]\left[S_{X_{B_jP_K}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Y_{B_j}}}\diagdown\right]\left[S_{Y_{B_jP_K}}\right] + \frac{1}{\beta}\left[\diagdown T_{T_{Z_{B_j}}}\diagdown\right]\left[S_{Z_{B_jP_K}}\right]\right]\left\{\frac{\Sigma}{V_\infty}\right\}_{P_K}$$

$$+ \left\{\left(\frac{\Delta V_T}{V_\infty}\right)_{mn} / \beta\sqrt{\beta^2\bar{T}_{T_X}^{\,2} + \bar{T}_{T_Y}^{\,2} + \bar{T}_{T_Z}^{\,2}}\right\}_{B_j}$$

$$+ \left[\diagdown T_{T_{X_{B_j}}}\diagdown\right]\left\{\frac{V_X}{V_\infty}\right\} + \left[\diagdown T_{T_{Y_{B_j}}}\diagdown\right]\left\{\frac{V_Y}{V_\infty}\right\} + \left[\diagdown T_{T_{Z_{B_j}}}\diagdown\right]\left\{\frac{V_Z}{V_\infty}\right\} \tag{93}$$

Where

$$\left(\frac{\Delta V_T}{V_\infty}\right)_{mn} = -\frac{1}{2}\left[\frac{\Gamma_{T_{m(n-\Delta n/2)}}}{V_\infty} / \left(\bar{\ell}_{T_{mn}} + \bar{\ell}_{T_{m(N-1)}}\right) + \frac{\Gamma_{T_{m(n+\Delta n/2)}}}{V_\infty} / \left(\bar{\ell}_{T_{mn}} + \bar{\ell}_{T_{m(n+1)}}\right)\right] \tag{94}$$

And

$$\frac{\Gamma_{T_{m(n\,-\,\Delta n/2)}}}{V_\infty} = \frac{K_{m(n\,-\,1)}}{V_\infty} - \frac{K_{mn}}{V_\infty} - \sum_{r\,=\,1}^{N_a}\left[\frac{K_{m(n\,-\,\Delta n/2)}}{V_\infty}\right]_{P_r} \tag{95}$$

$$\frac{\Gamma_{T_m}(n + \Delta n/2)}{V_\infty} = \frac{K_{mn}}{V_\infty} - \frac{K_{m(n\,+\,1)}}{V_\infty} - \sum_{r\,=\,1}^{N_a}\left[\frac{K_{m(n\,+\,\Delta n/2)}}{V_\infty}\right]_{P_r} \tag{96}$$

Where

$$\sum_{r\,=\,1}^{N_a}\left[\frac{K_{m\,(n\,-\,\Delta n/2)}}{V_\infty}\right] \quad \text{and} \quad \sum_{r\,=\,1}^{N_a}\left[\frac{K_{m(n\,+\,\Delta\ /2)}}{V_\infty}\right]_{P_r}$$

are the contributions from the trailing legs of the panel vortices along the juncture line of the j^{th} body and r^{th} panel to be attached at the $(n - \Delta n/2)$ and $(n + \Delta n/2)$ lateral stations, respectively. N_a is the total number of panels attached at any one point along the line.

The velocity tangent to the surface of the j^{th} panel in the longitudinal direction is given by;

$$\left\{\left(\frac{V_M}{V_\infty}\right)_{mn}\right\}_{P_j} = \left[\frac{1}{\left\{1+(1+\tan^2\Lambda)_{mn}\,(dZ_T/dX \pm dZ_c/dX)^2_{mn}\right\}^{1/2}}\right]\left\{\sum_{i\,=\,1}^{N_B}\frac{1}{\beta^2}\left[A_{X_{P_jB_i}}\right]\left\{\frac{K}{V_\infty}\right\}_{B_i}\right.$$

$$+\sum_{K=1}^{N_P}\frac{1}{\beta^2}\left[\left[A_{X_{T_jP_K}}\right]\left\{\begin{matrix}\dfrac{K}{V_\infty}\\[2mm] \dfrac{\Gamma}{V_\infty}\end{matrix}\right\}_{P_K} + \left[S_{X_{P_jP_K}}\right]\left\{\frac{\Sigma}{V_\infty}\right\}_{P_K}\right]$$

$$\pm\frac{1}{\beta^2}\left[\left(\frac{\Delta V_M}{V_\infty}\right)_{mn}\right]_{P_j}\left[\left\{1\right\} + \left[\sqrt{1+\tan^2\Lambda_{mn}}\right]\left[S_{X_{P_jP_j}}\right]\left[\left\{\frac{\Sigma}{V_\infty}\right\}_{P_j} - \left\{\frac{\Sigma'}{V_\infty}\right\}_{P_j}\right]\right]$$

$$\left. + \left\{\frac{V_X}{V_\infty}\right\}\right\} \tag{97}$$

Where

$$\frac{\Sigma'}{V_\infty} = \left[\frac{(Z_T/C)}{(X/C)\ (1 - X/C)}\right] \left[\frac{\overline{V}_X/V_\infty}{\sqrt{1 + \dfrac{\tan^2\Lambda}{\beta^2}}}\right] \frac{\overline{\ell}_M}{2} \tag{98}$$

and where $(\Delta V_M/V_\infty)_{mn}$ is computed using equation (84) and X/C is the local percent chord.

The velocity tangent to the surface of the j^{th} panel in the lateral direction is given by;

$$\begin{aligned}
\left\{\left(\frac{V_T}{V_\infty}\right)_{mn}\right\}_{P_j} = {} & \left\lceil \frac{1}{\left|1 + (1 + \tan^2\Lambda)_{mn}\ (dZ_t/dX \pm dZ_c/dX)^2{}_{mn}\right|^{1/2}} \right\rfloor \Bigg\{ \sum_{i=1}^{N_B} \frac{1}{\beta} \Bigg[\left\lceil T_{T_{Y_{P_j}}} \right\rfloor \left[A_{Y_{P_j B_i}} \right] \\
& + \left\lceil T_{T_{Z_{P_j}}} \right\rfloor \left[A_{Z_{P_j B_i}} \right] \Bigg] \left\{\frac{K}{V_\infty}\right\}_{B_i} + \sum_{K=1}^{N_P} \frac{1}{\beta} \Bigg[\left\lceil T_{T_{Y_{P_j}}} \right\rfloor \left[A_{Y_{P_j P_K}} \right] + \left\lceil T_{T_{Z_{P_j}}} \right\rfloor \left[A_{Z_{P_j P_K}} \right] \Bigg] \left\{\begin{matrix} \dfrac{K}{V_\infty} \\ \dfrac{\Gamma}{V_\infty} \end{matrix}\right\}_{P_K} \\
& + \sum_{K=1}^{N_P} \frac{1}{\beta} \Bigg[\left\lceil T_{T_{Y_{P_j}}} \right\rfloor \left[S_{Y_{P_j P_K}} \right] + \left\lceil T_{T_{Z_{P_j}}} \right\rfloor \left[S_{Z_{P_j P_K}} \right] \Bigg] \left\{\frac{\Sigma}{V_\infty}\right\}_{P_K} + \left\lceil T_{T_{Y_{P_j}}} \right\rfloor \left\{\frac{V_Y}{V_\infty}\right\} \\
& + \left\lceil T_{T_{Z_{P_j}}} \right\rfloor \left\{\frac{V_Z}{V_\infty}\right\} \pm \frac{1}{\beta} \left\lceil \left(\frac{\Delta V_T}{V_\infty}\right)_{mn} \right\rfloor \Bigg[\left\{1\right\} + \left\lceil \sqrt{1 + \tan^2\Lambda_{mn}} \right\rfloor \Bigg[\left\lceil T_{T_{Y_{R_j}}} \right\rfloor \left[S_{Y_{P_j P_K}} \right] \\
& + \left\lceil T_{T_{Z_{P_j}}} \right\rfloor \left[S_{Z_{P_j P_K}} \right] \Bigg] \Bigg[\left\{\frac{\Sigma}{V_\infty}\right\}_{P_j} - \left\{\frac{\Sigma'}{V_\infty}\right\}_{P_j} \Bigg] \Bigg] \Bigg\}
\end{aligned} \tag{99}$$

where

$$\left(\frac{\Delta V_T}{V_\infty}\right)_{mn} = \frac{1}{2}\left[\frac{\dfrac{\Gamma_{T_m\left(n-\frac{\Delta n}{2}\right)}}{V_\infty}}{\left(\overline{\ell}_{T_{mn}} + \overline{\ell}_{T_{m(n-1)}}\right)} + \frac{\dfrac{\Gamma_{T_m\left(n+\frac{\Delta n}{2}\right)}}{V_\infty}}{\left(\overline{\ell}_{T_{mN}} + \overline{\ell}_{T_{m(n+1)}}\right)}\right] + \left[\left(\frac{\Delta V_M}{V_\infty}\right) \mathrm{TAN}\Lambda\right]_{mn} \tag{100}$$

and where $\dfrac{\Gamma_{T_m\left(n-\frac{\Delta n}{2}\right)}}{V_\infty}$

and $\dfrac{\Gamma_{T_m\left(n+\frac{\Delta n}{2}\right)}}{V_\infty}$

are computed the same as for the body except

$$\sum_{r=1}^{N_a} \left[\frac{K_m\left(n-\frac{\Delta n}{2}\right)}{V_\infty}\right]_{P_r}$$

and

$$\sum_{r=1}^{N_a} \left[\frac{K_{m \quad n+\left(\frac{\Delta n}{2}\right)}}{V_\infty} \right]_{P_r}$$

represent the contributions from the trailing legs of the panel vortices along the juncture line of the jth panel and the rth panel to be attached at the lateral stations,

$$\left(n - \frac{\Delta n}{2}\right) \text{ and } \left(n + \frac{\Delta n}{2}\right), \text{ respectively.}$$

The surface pressure coefficients at each of the control points on the bodies and panels are then computed using the following expression.

$$C_{p_{mn}} = \frac{2}{\gamma M_\infty^2} \left[\left\{ 1 + \frac{\gamma - 1}{2} M_\infty^2 \left[1 - \left(\frac{V_M}{V_\infty}\right)^2_{mn} - \left(\frac{V_T}{V_\infty}\right)^2_{mn} \right] \right\}^{\gamma/(\gamma-1)} \right] \tag{101}$$

where γ is the ratio of specific heats.

Note, in equations (97) and (100) the top sign is used to compute the velocity on the upper surface and the bottom sign the lower surface in those terms which have a plus and minus in front.

Section and Total Loads and Moments

The section loads are computed in coefficient form on each of the bodies by interpolating for the surface pressure coefficients at the centroid of the subareas, defined by the corner points of the divisions of the body subpanels, and then using the following equations to sum the product of the pressure coefficients and directed subareas.

$$\left(\frac{C_X W}{W_{AVG}}\right)_K = -\frac{1}{A_Z \, \Delta\left(\frac{X_B}{C_B}\right)_K} \sum_i C_{P_{iK}} \, \Delta A_{X_{iK}} \tag{102}$$

$$\left(\frac{C_Y h}{h_{AVE}}\right)_K = -\frac{1}{A_y \, \Delta\left(\frac{X_B}{C_B}\right)_K} \sum_i C_{P_{iK}} \, \Delta A_{Y_{iK}} \tag{103}$$

$$\left(\frac{C_Z W}{W_{AVG}}\right)_K = -\frac{1}{A_Z \, \Delta\left(\frac{X_B}{C_B}\right)_K} \sum_i C_{P_{iK}} \, \Delta A_{Z_{iK}} \tag{104}$$

where i is indexed over all subareas in the longitudinals segment $\Delta(X_B/C_B)_K$.

$$A_X = \frac{1}{2} \sum_K \sum_i \left|\Delta A_{X_{iK}}\right| \tag{105}$$

$$A_Y = \frac{1}{2} \sum_K \sum_i \left|\Delta A_{Y_{iK}}\right| \tag{106}$$

$$A_Z = \frac{1}{2} \sum_{K} \sum_{i} \left| \Delta A_{Z_{iK}} \right| \tag{107}$$

Also;

$$W_{AVG} = \frac{A_Z}{C_B} \tag{108}$$

and

$$h_{AVG} = \frac{A_Y}{C_B} \tag{109}$$

The total loads on a body are then obtained by summing in the longitudinal direction.

$$C_{X_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_{K} \left(\frac{C_X\ W}{W_{AVG}} \right)_K \Delta \left(\frac{X_B}{C_B} \right)_K \tag{110}$$

$$C_{Y_{B_j}} = \frac{A_Y}{A_{B_j}} \sum_{K} \left(\frac{C_Y\ h}{h_{AVG}} \right)_K \Delta \left(\frac{X_B}{C_B} \right)_K \tag{111}$$

$$C_{Z_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_{K} \left(\frac{C_Z\ W}{W_{AVG}} \right)_K \Delta \left(\frac{X_B}{C_B} \right)_K \tag{112}$$

The total moments on a body are summed about the center of gravity.

$$C_{M_{X_{B_j}}} = -\frac{1}{A_{B_j}\bar{C}} \sum_K \sum_i \left[\left(Y_{iK} - Y_{C.G.}\right) \Delta A_{Z_{iK}} - \left(Z_{iK} - Z_{C.G.}\right) \Delta A_{Y_{iK}} \right] C_{P_{iK}} \tag{113}$$

$$C_{M_{Y_{B_i}}} = -\frac{1}{A_{B_j}\bar{C}} \sum_K \sum_i \left[\left(Z_{iK} - Z_{C.G.}\right) \Delta A_{X_{iK}} - \left(X_{iK} - X_{C.G.}\right) \Delta A_{Z_{iK}} \right] C_{P_{iK}} \tag{114}$$

$$C_{M_{Z_{B_j}}} = -\frac{1}{A_{B_j}\bar{C}} \sum_K \sum_i \left[\left(X_{iK} - X_{C.G.}\right) \Delta A_{Y_{iK}} - \left(Y_{iK} - Y_{C.G.}\right) \Delta A_{X_{iK}} \right] C_{P_{iK}} \tag{115}$$

The body center of pressure position vector components divided by $\bar{C}$ are given by;

$$\left(\frac{X}{\bar{C}}\right)_{C.P.} = \left(\frac{X}{\bar{C}}\right)_{C.G.} + \frac{C_{Y_{B_j}} C_{M_{Z_{B_j}}} - C_{Z_{B_j}} C_{M_{Y_{B_j}}}}{C^2_{X_{B_j}} + C^2_{Y_{B_j}} + C^2_{Z_{B_j}}} \tag{116}$$

$$\left(\frac{Y}{\bar{C}}\right)_{C.P.} = \left(\frac{Y}{\bar{C}}\right)_{C.G.} + \frac{C_{Z_{B_j}} C_{M_{X_{B_j}}} - C_{X_{B_j}} C_{M_{Z_{B_j}}}}{C^2_{X_{B_j}} + C^2_{Y_{B_j}} + C^2_{Z_{B_j}}} \tag{117}$$

$$\left(\frac{Z}{\bar{C}}\right)_{C.P.} = \left(\frac{Z}{\bar{C}}\right)_{C.G.} + \frac{C_{X_{B_j}} C_{M_{Y_{B_j}}} - C_{Y_{B_j}} C_{M_{X_{B_j}}}}{C^2_{X_{B_j}} + C^2_{Y_{B_j}} + C^2_{Z_{B_j}}} \tag{118}$$

The panel section loads, moments, and centers of pressure relative to the leading edge are obtained by numerically evaluating the following integrals.

$$\frac{C_X\ C}{C_{AVG}} = \frac{C\ b_S\ AR}{2\ b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[C_{P_L} \tan\alpha_L - C_{P_U} \tan\alpha_U \right] \sin\phi\ d\phi - \left(\frac{C_T\ C}{C_{AVG}}\right) \tag{119}$$

$$\frac{C_Y\ C}{C_{AVG}} = \frac{C\ b_S\ AR\ N_{Y_{P_j}}}{2\ b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi\ d\phi \tag{120}$$

$$\frac{C_Z\ C}{C_{AVG}} = \frac{C\ b_S\ AR\ N_{Z_{P_j}}}{2\ b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi\ d\phi \tag{121}$$

where

$$b_S = \sum_{N=1}^{N_N} \sqrt{\Delta Y^2_{L.E._N} + \Delta Z^2_{L.E._N}} \tag{122}$$

$$\Delta S = \sqrt{\Delta Y^2_{mN} + \Delta Z^2_{mN}} = \text{local panel width} \tag{123}$$

$$\Delta S_{L.E.} = \sqrt{\Delta Y^2_{L.E._N} + \Delta Z^2_{L.E._N}} = \text{panel width at leading edge.} \tag{124}$$

and $\phi = \cos^{-1}\left[1 - 2\,(X/C)\right]$ where (X/C) is the local percent chord.

Also;

$$\tan\alpha = \frac{\tan\delta \mp \frac{dZ_t}{dX} - \frac{dZ_c}{dX} + \epsilon}{1 - \left(\mp \frac{dZ_t}{dX} - \frac{dZ_c}{dX} + \epsilon\right)\tan\delta} \tag{125}$$

where the top sign is used with the upper surface and the bottom sign is used with the lower surface. The control surface angle is equal to $-\delta_K$ along the leading edge flap and equal to δ_f along a trailing control surface. δ is equal to zero at other points on the chord.

The section normal force coefficient is obtained by taking the following scalar product.

$$\frac{C_N\,C}{C_{AVG}} = \frac{C_Y\,C}{C_{AVG}} N_{Y_{P_j}} + \frac{C_Z\,C}{C_{AVG}} N_{Z_{P_j}} \tag{126}$$

The section moment coefficients about the leading edge are computed as follows.

$$\frac{C_{M_{L.E._X}}\,C}{C_{AVG}} = \frac{AR\,b_S}{2\,b^2}\int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}}\left[\left(Y - Y_{L.E.}\right) N_{ZP_j} - \left(Z - Z_{L.E.}\right) N_{Y_{P_j}}\right]\left[C_{P_L} - C_{P_U}\right] \sin\phi \; d\phi \tag{127}$$

$$\frac{C_{M_{L.E._Y}}\,C}{C_{AVG}} = \frac{AR\,b_S}{2\,b^2}\int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}}\left[\left(Z - Z_{L.E.}\right)\left(C_{P_L}\tan\alpha_L - C_{P_U}\tan\alpha_U\right) - \left(X - X_{L.E.}\right) N_{Z_{P_j}}\left(C_{P_L} - C_{P_U}\right)\right] \sin\phi \; d\phi \tag{128}$$

$$\frac{C_{M_{L.E._Z}}\, C}{C_{AVG}} = \frac{AR\, b_S}{2\, b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[\left(X - X_{L.E.}\right) N_{Y_{P_j}} \left(C_{P_L} - C_{P_U}\right) - \left(Y - Y_{L.E.}\right) \left(C_{P_L} \tan\sigma_L - C_{P_U} \tan\sigma_U\right) \right] \sin\phi \, d\phi \tag{129}$$

The section center of pressure relative to the leading edge is given by;

$$\left(\frac{X}{C}\right)_{C.P.} = \frac{\left(\frac{C_Y\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._Z}}\, C}{C_{AVG}}\right) - \left(\frac{C_Z\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._Y}}\, C}{C_{AVG}}\right)}{\left(\frac{C_X\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Y\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Z\, C}{C_{AVG}}\right)^2} \tag{130}$$

$$\left(\frac{Y}{C}\right)_{C.P.} = \frac{\left(\frac{C_Z\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._X}}\, C}{C_{AVG}}\right) - \left(\frac{C_X\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._Z}}\, C}{C_{AVG}}\right)}{\left(\frac{C_X\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Y\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Z\, C}{C_{AVG}}\right)^2} \tag{131}$$

$$\left(\frac{Z}{C}\right)_{C.P.} = \frac{\left(\frac{C_X\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._Y}}\, C}{C_{AVG}}\right) - \left(\frac{C_Y\, C}{C_{AVG}}\right)\left(\frac{C_{M_{L.E._X}}\, C}{C_{AVG}}\right)}{\left(\frac{C_X\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Y\, C}{C_{AVG}}\right)^2 + \left(\frac{C_Z\, C}{C_{AVG}}\right)^2} \tag{132}$$

The section zero percent suction drag coefficient

$$\frac{C_{d_{T=0}}\, C}{C_{AVG}},$$

the section induced drag due to lift coefficient

$$\frac{C_{d_{L_i}} \; C}{C_{AVG}} ,$$

the section leading edge thrust coefficient $C_T \; C/C_{AVG}$, and the section induced drag due to thickness coefficient

$$\frac{C_{d_{T_i}} \; C}{C_{AVG}}$$

are derived in appendix C.

The panel total force coefficients are obtained by numerically integrating the section force coefficients in the spanwise direction.

$$C_{X_{P_j}} = \int_0^{\pi/2} \frac{C_X \; C}{C_{AVG}} \sin\theta \; d\theta \tag{133}$$

$$C_{Y_{P_j}} = \int_0^{\pi/2} \frac{C_Y \; C}{C_{AVG}} \sin\theta \; d\theta \tag{134}$$

$$C_{Z_{P_j}} = \int_0^{\pi/2} \frac{C_Z \; C}{C_{AVG}} \sin\theta \; d\theta \tag{135}$$

where

$$\theta = \cos^{-1}\eta \tag{136}$$

The panel total moment coefficients about the center of gravity are numerically integrated as follows.

$$C_{M_{X_{P_j}}} = \frac{AR\ b_S}{2\ b^2 \bar{C}} \int_0^{\pi/2} C \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[\left(Y - Y_{C.G.}\right) N_{Z_{P_j}} - \left(Z - Z_{C.G.}\right) N_{Y_{P_j}} \right] \left(C_{P_L} - C_{P_U}\right) \sin\phi \sin\theta \, d\phi \, d\theta \tag{137}$$

$$C_{M_{Y_{P_j}}} = \frac{AR\ b_S}{2\ b^2 \bar{C}} \int_0^{\pi/2} C \left\{ \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[\left(Z - Z_{C.G.}\right) \left(C_{P_L} \tan\alpha_L - C_{P_U} \tan\alpha_U\right) - \left(X - X_{C.G.}\right) N_{Z_{P_j}} \left(C_{P_L} - C_{P_U}\right) \right] \sin\phi \, d\phi - \left(Z - Z_{C.G.}\right) \left(\frac{C_T\ C}{C_{AVG}}\right) \right\} \sin\theta \, d\theta \tag{138}$$

$$C_{M_{Z_{P_j}}} = \frac{AR\ b_S}{2\ b^2 \bar{C}} \int_0^{\pi/2} C \left\{ \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[\left(X - X_{C.G.}\right) N_{Y_{P_j}} \left(C_{P_L} - C_{P_U}\right) - \left(Y - Y_{C.G.}\right) \left(G_{P_L} \tan\alpha_L - G_{P_U} \tan\alpha_U\right) \right] \sin\phi \, d\phi + \left(Y - Y_{C.G.}\right) \left(\frac{C_T\ C}{C_{AVG}}\right) \right\} \sin\theta \, d\theta \tag{139}$$

The panel total zero suction drag, near field induced drag due to lift, leading edge thrust, and near field induced drag due to thickness are given by:

$$C_{D_{T=0_{P_j}}} = \int_0^{\pi/2} \frac{C_{d_{T=0}}\ C}{C_{AVG}} \sin\theta \, d\theta \tag{140}$$

$$C_{D_{L_{i_{P_j}}}} = \int_0^{\pi/2} \frac{C_{d_{L_i}}\ C}{C_{AVG}} \sin\theta \, d\theta \tag{141}$$

$$C_{T_{P_j}} = \int_0^{\pi/2} \frac{C_T\ C}{C_{AVG}} \sin\theta \, d\theta \tag{142}$$

$$C_{D_{T_{i_{P_j}}}} = \int_0^{\pi/2} \frac{C_{d_{T_i}} \; C}{C_{AVG}} \sin\theta \; d\theta \qquad (143)$$

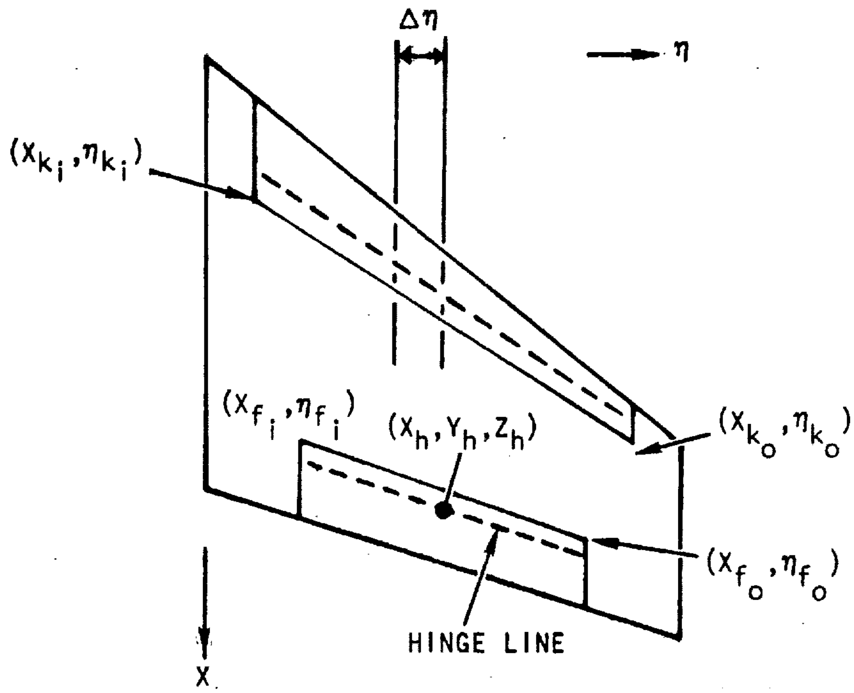
The panel center of pressure position vector components divided by $\bar{C}$ are computed as follows.

$$\left(\frac{X}{\bar{C}}\right)_{C.P.} = \left(\frac{X}{\bar{C}}\right)_{C.P.} + \frac{C_{Y_{P_j}} C_{M_{Z_{P_j}}} - C_{Z_{P_j}} C_{M_{Y_{P_j}}}}{C^2_{X_{P_j}} + C^2_{Y_{P_j}} + C^2_{Z_{P_j}}} \qquad (144)$$

$$\left(\frac{Y}{\bar{C}}\right)_{C.P.} = \left(\frac{X}{\bar{C}}\right)_{C.G.} + \frac{C_{Z_{P_j}} C_{M_{X_{P_j}}} - C_{X_{P_j}} C_{M_{Z_{P_j}}}}{C^2_{X_{P_j}} + C^2_{Y_{P_j}} + C^2_{Z_{P_j}}} \qquad (145)$$

$$\left(\frac{Z}{\bar{C}}\right)_{C.P.} = \left(\frac{Z}{\bar{C}}\right)_{C.G.} + \frac{C_{X_{P_j}} C_{M_{Y_{P_j}}} - C_{Y_{P_j}} C_{M_{X_{P_j}}}}{C^2_{X_{P_j}} + C^2_{Y_{P_j}} + C^2_{Z_{P_j}}} \qquad (146)$$

The section loads, moments, and center of pressure for the control surfaces are computed as follows.



The equation for the trailing edge of leading edge control surface;

$$X_k = X_{k_i} + \left(\frac{X_{k_o} - X_{k_i}}{\eta_{k_o} - \eta_{k_i}}\right)\left(\eta - \eta_{k_i}\right) \tag{147}$$

The equation for the leading edge of trailing edge control surface;

$$X_f = X_{f_i} + \left(\frac{X_{f_o} - X_{f_i}}{\eta_{f_o} - \eta_{f_i}}\right)\left(\eta - \eta_{f_i}\right) \tag{148}$$

The equation for a control surface hinge line;

$$X_h = X_{h_i} + \left(\frac{X_{h_o} - X_{h_i}}{\eta_o - \eta_i}\right)\left(\eta - \eta_i\right) \tag{149}$$

The unit vector in the direction of the hinge line is given by;

$$\hat{h} = \left\{ \left[X_h\left(\eta + \frac{\Delta\eta}{2}\right) - X_h\left(\eta - \frac{\Delta\eta}{2}\right)\right] \hat{i} + \left[Y_h\left(\eta + \frac{\Delta\eta}{2}\right) - Y_h\left(\eta - \frac{\Delta\eta}{2}\right)\right] \hat{j} \right.$$

$$\left. + \left[Z_h\left(\eta + \frac{\Delta\eta}{2}\right) - Z_h\left(\eta - \frac{\Delta\eta}{2}\right)\right] \hat{k} \right\} \Big/ \left\{ \left[X_h\left(\eta + \frac{\Delta\eta}{2}\right) - X_h\left(\eta - \frac{\Delta\eta}{2}\right)\right]^2 \right.$$

$$\left. + \left[Y_h\left(\eta + \frac{\Delta\eta}{2}\right) - Y_h\left(\eta - \frac{\Delta\eta}{2}\right)\right]^2 + \left[Z_h\left(\eta + \frac{\Delta\eta}{2}\right) - Z_h\left(\eta - \frac{\Delta\eta}{2}\right)\right]^2 \right\}^{1/2} \tag{150}$$

The section loads on a leading edge control surface are obtained by numerically evaluating the following integrals

$$\frac{C_{h_x} C}{C_{AVG}} = \frac{C_x AR\, b_s}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} \tan\alpha_L - C_{P_U} \tan\alpha_U\right) \sin\phi_k \, d\phi_k - \left(\frac{C_T C}{C_{AVG}}\right) \tag{151}$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_k AR\, b_s\, N_{Y_{P_j}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi_k \, d\phi_k \tag{152}$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_k AR\, b_s\, N_{Z_{P_j}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi_k \, d\phi_k \tag{153}$$

where

$$\phi_k = \cos^{-1}\left[1-2\left(\frac{C}{C_k}\right)\left(\frac{X}{C}\right)\right] \tag{154}$$

and C_k is the chord of the leading edge control surface. (X/C) is the local percent chord of the panel.

The section loads on a trailing edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{h_x} C}{C_{AVG}} = \frac{C_f AR\, b_s}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} \tan\alpha_L - C_{P_U} \tan\alpha_U\right) \sin\phi_f \, d\phi_f \tag{155}$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_f AR\, b_s\, N_{Y_{P_j}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi_f \, d\phi_f \tag{156}$$

$$\frac{C_{h_z} C}{C_{AVG}} = \frac{C_f AR\, b_s\, N_{Z_{P_j}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left(C_{P_L} - C_{P_U}\right) \sin\phi_f \, d\phi_f \tag{157}$$

where

$$\phi_f = \cos^{-1}\left[1-2\left(\frac{C}{C_f}\right)\left(\frac{X + C_f - C}{C}\right)\right] \tag{158}$$

and C_f is the chord of the trailing edge control surface.

The section normal load on the control surface is given by;

$$\frac{C_{h_N} C}{C_{AVG}} = \left(\frac{C_{h_y} C}{C_{AVG}}\right) N_{Y_{P_j}} + \left(\frac{C_{h_z} C}{C_{AVG}}\right) N_{Z_{P_j}} \tag{159}$$

The section moments about the hinge line for the leading edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{M_{h_x}} C}{C_{AVG}} = \frac{C_k \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(Y - Y_h) N_{Z_{P_j}} - (Z - Z_h) N_{Y_{P_j}}\right] \left(C_{P_L} - C_{P_U}\right) \sin \phi_k \, d\phi_k \tag{160}$$

$$\frac{C_{M_{h_y}} C}{C_{AVG}} = \frac{C_k \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(Z - Z_h) \left(C_{P_L} \tan \alpha_L - C_{P_J} \tan \alpha_U\right) - (X - X_h) N_{Z_{P_j}} \left(C_{P_L} - C_{P_U}\right)\right] \sin \phi_k \, d\phi_k - \left(\frac{Z_{L.E.} - Z_h}{\bar{C}}\right)\left(\frac{C_T C}{C_{AVG}}\right) \tag{161}$$

$$\frac{C_{M_{h_z}} C}{C_{AVG}} = \frac{C_k \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(X - X_h) N_{Y_{P_j}} \left(C_{P_L} - C_{P_U}\right) - (Y - Y_h) \left(C_{P_L} \tan \alpha_L - C_P \tan \alpha_U\right)\right] \sin \phi_k \, d\phi_k - \left(\frac{Y_{L.E.} - Y_h}{\bar{C}}\right)\left(\frac{C_T C}{C_{AVG}}\right) \tag{162}$$

The section moments about the hinge line for the trailing edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{M_{h_x}} C}{C_{AVG}} = \frac{C_f \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(Y - Y_h) N_{Z_{P_j}} - (Z - Z_h) N_{Y_{P_j}}\right] \left(C_{P_L} - C_{P_U}\right) \sin \phi_f \, d\phi_f \tag{163}$$

$$\frac{C_{M_{h_y}} C}{C_{AVG}} = \frac{C_f \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(Z - Z_h) \left(C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) - (X - X_h) \, N_{Z_{P_j}} \left(C_{P_L} - C_{P_U} \right) \right] \sin \phi_f \, d\phi_f \tag{164}$$

$$\frac{C_{M_{h_z}} C}{C_{AVG}} = \frac{C_f \, AR \, b_s}{2b^2 \bar{C}} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left[(X - X_h) \, N_{Y_{P_j}} \left(C_{P_L} - C_{P_U} \right) - (Y - Y_h) \left(C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) \right] \sin \phi_f \, d\phi_f \tag{165}$$

The section hinge moment is computed by the following equation.

$$\frac{C_{M_h} C}{C_{AVG}} = \left(\frac{C_{M_{h_x}} C}{C_{AVG}} \right) h_x + \left(\frac{C_{M_{h_y}} C}{C_{AVG}} \right) h_y + \left(\frac{C_{M_{h_z}} C}{C_{AVG}} \right) h_z \tag{166}$$

The section center of pressure, due to the control surface loading, relative to the leading edge is given by the following expressions.

$$\left(\frac{X}{C} \right)_{C.P.} = \left(\frac{X}{C} \right)_h + \frac{\bar{C}}{C} \frac{\left(\frac{C_{h_y} C}{C_{AVG}} \right) \left(\frac{C_{M_{h_z}} C}{C_{AVG}} \right) - \left(\frac{C_{h_z} C}{C_{AVG}} \right) \left(\frac{C_{M_{h_y}} C}{C_{AVG}} \right)}{\left(\frac{C_{h_x} C}{C_{AVG}} \right)^2 + \left(\frac{C_{h_y} C}{C_{AVG}} \right)^2 + \left(\frac{C_{h_z} C}{C_{AVG}} \right)^2} \tag{167}$$

$$\left(\frac{Y}{C}\right)_{C.P.} = \left(\frac{Y}{C}\right)_h + \frac{\bar{C}}{C}\,\frac{\left(\frac{C_{h_z} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_x}} C}{C_{AVG}}\right) - \left(\frac{C_{h_x} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_z}} C}{C_{AVG}}\right)}{\left(\frac{C_{h_x} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_y} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_z} C}{C_{AVG}}\right)^2} \tag{168}$$

$$\left(\frac{Z}{C}\right)_{C.P.} = \left(\frac{Z}{C}\right)_h + \frac{\bar{C}}{C}\,\frac{\left(\frac{C_{h_x} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_y}} C}{C_{AVG}}\right) - \left(\frac{C_{h_y} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_x}} C}{C_{AVG}}\right)}{\left(\frac{C_{h_x} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_y} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_z} C}{C_{AVG}}\right)^2} \tag{169}$$

The total loads and hinge moment on the control surface are given by;

$$C_{h_X} = \int_{\eta_i}^{o} \frac{C_{h_x} C}{C_{AVG}}\, d\eta \tag{170}$$

$$C_{h_Y} = \int_{\eta_i}^{o} \frac{C_{h_y} C}{C_{AVG}}\, d\eta \tag{171}$$

$$C_{h_Z} = \int_{\eta_i}^{o} \frac{C_{h_y} C}{C_{AVG}}\, d\eta \tag{172}$$

and

$$C_{M_h} = \int_{\eta_i}^{\eta_o} \frac{C_{M_h} C}{C_{AVG}} d\eta \tag{173}$$

The total loads and moments for the complete configuration are then given by the following equations.

$$C_X = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{X_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_i} C_{X_{P_j}} \tag{174}$$

$$C_Y = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{Y_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_j} C_{Y_{P_j}} \tag{175}$$

$$C_Z = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{Z_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_j} C_{Z_{1_{P_j}}} \tag{176}$$

$$C_{M_X} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{\bar{C}_{B_j}}{\bar{C}} C_{M_{X_{B_j}}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} \frac{\bar{C}_{P_j}}{\bar{C}} F_{S_j} C_{M_{X_{P_j}}} \tag{177}$$

$$C_{M_Y} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{\bar{C}_{B_j}}{\bar{C}} C_{M_Y} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} \frac{\bar{C}_{P_j}}{\bar{C}} F_{S_j} C_{M_{Y_{P_j}}} \tag{178}$$

$$C_{M_Z} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{\bar{C}_{B_j}}{\bar{C}} C_{M_Z} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} \frac{\bar{C}_{P_j}}{\bar{C}} F_{S_j} C_{M_{Z_{P_j}}} \tag{179}$$

The induced drag for the total configuration C_{D_i} is computed in the Trefftz plane with equations derived in Appendix F. The center of pressure for the total configuration is given by the following equations.

$$\left(\frac{X}{\bar{C}_R}\right)_{C.P.} = \left(\frac{X}{\bar{C}_R}\right)_{C.G.} + \frac{C_Y \, C_{M_Z} \quad C_Z \quad C_{M_Y}}{C_X{}^2 + C_Y{}^2 + C_Z{}^2} \tag{180}$$

$$\left(\frac{Y}{\bar{C}_R}\right)_{C.P.} = \left(\frac{Y}{\bar{C}_R}\right)_{C.G.} + \frac{C_Z \, C_{M_X} - C_X \, C_{M_Z}}{C_X{}^2 + C_Y{}^2 + C_Z{}^2} \tag{181}$$

$$\left(\frac{Z}{\bar{C}}\right)_{C.P.} = \left(\frac{Z}{\bar{C}_R}\right)_{C.G.} + \frac{C_X \, C_{M_Y} - C_Y \, C_{M_X}}{C_X{}^2 + C_Y{}^2 + C_Z{}^2} \tag{182}$$

F_{S_j} is a symmetry indicator which is equal to 2.0 in equations (174), (176), and (178) and equal to 0.0 in equations (175), (177), and (179) when the panel has an image.

COMPUTER PROGRAM RESULTS

Airfoil Section Velocities

In order to establish the degree of accuracy that can be obtained by the use of a source-vortex lattice procedure with second order corrections to account for the interference between lift and thickness and to account for the fact that the boundary conditions and the pertubation velocities are satisfied and computed, respectively, on the chordal plane, the results from the program have been compared with two dimensional exact solutions for a Karman-Trefftz airfoil in figures (4) and (5). Also, comparisons are made with data for a forty-five degree swept wing with an aspect ratio of five and a taper ratio of one. The airfoil section on this wing is a twelve percent thick R.A.E. 101. These comparisons are in figure (6), (7), and (8) for zero angle of attack and in figures (9), (10), and (11) at 4.2 degrees angle of attack. For all of the above cases discrete solutions were obtained with twenty subpanels in the chordwise direction and ten subpanels in the spanwise direction. Both the upper and lower surface pressures were plotted for the zero angle of attack data to indicate the degree of accuracy involved with the test data. The wing section was symmetrical.

Section Induced Drag Due to Thickness

The section induced drag $C_{d_{t_i}} C/C_{avg}$, or potential form drag due to thickness, is computed in the present program by means of a source lattice with equation (24) of Appendix F. The source lattice as shown in figure (12) agrees quite well with the exact solution by R. T. Jones for a sixty degree swept, ten percent thick biconvex section, taper ratio one, and aspect ratio six wing given in reference (51). It should be noted that Woodward's equations, derived in Appendix D, for the constant and linearly varying distributed source density panels can be superimposed to obtain the same solution, for the biconvex section, as obtained by Jones. Even though Woodward, in reference (40), only computes induced velocities at the centroid of trapezoidal panels, it was shown that the correct solution for the longitudinal perturbation velocity is also obtained at any spanwise location, including the edges.

If two taper ratio one semi-infinite swept panels are joined at their side edges to form a wing center section or kink the Woodward distributed source equations will give the same center section solution as obtained by Kuchemann and Weber in reference (2). In addition, they will also give the correct spanwise variation of the kink effect due to thickness, which must be obtained by semi-empirically determined interpolation curves in the solution by Kuchemann and Weber. The Woodward equations also treat the spanwise variation of thickness and other planform induced effects such as taper ratio, tip, and crank effects equally well. These same effects are also correctly treated by the distributed constant density trapezoidal panel equations

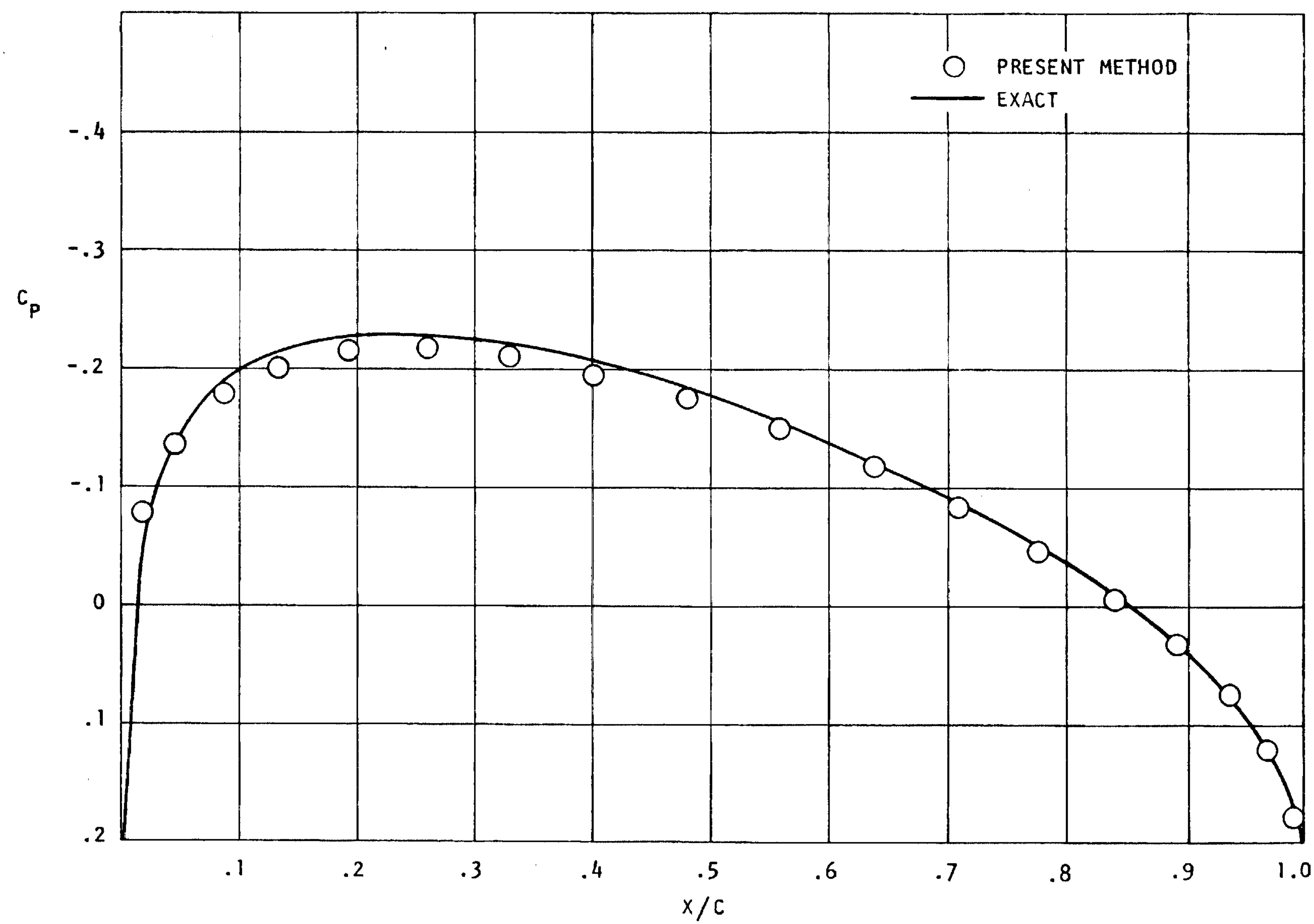


Figure 4.- Karman-trefftz airfoil pressure distribution at 0.0 degree angle of attack.

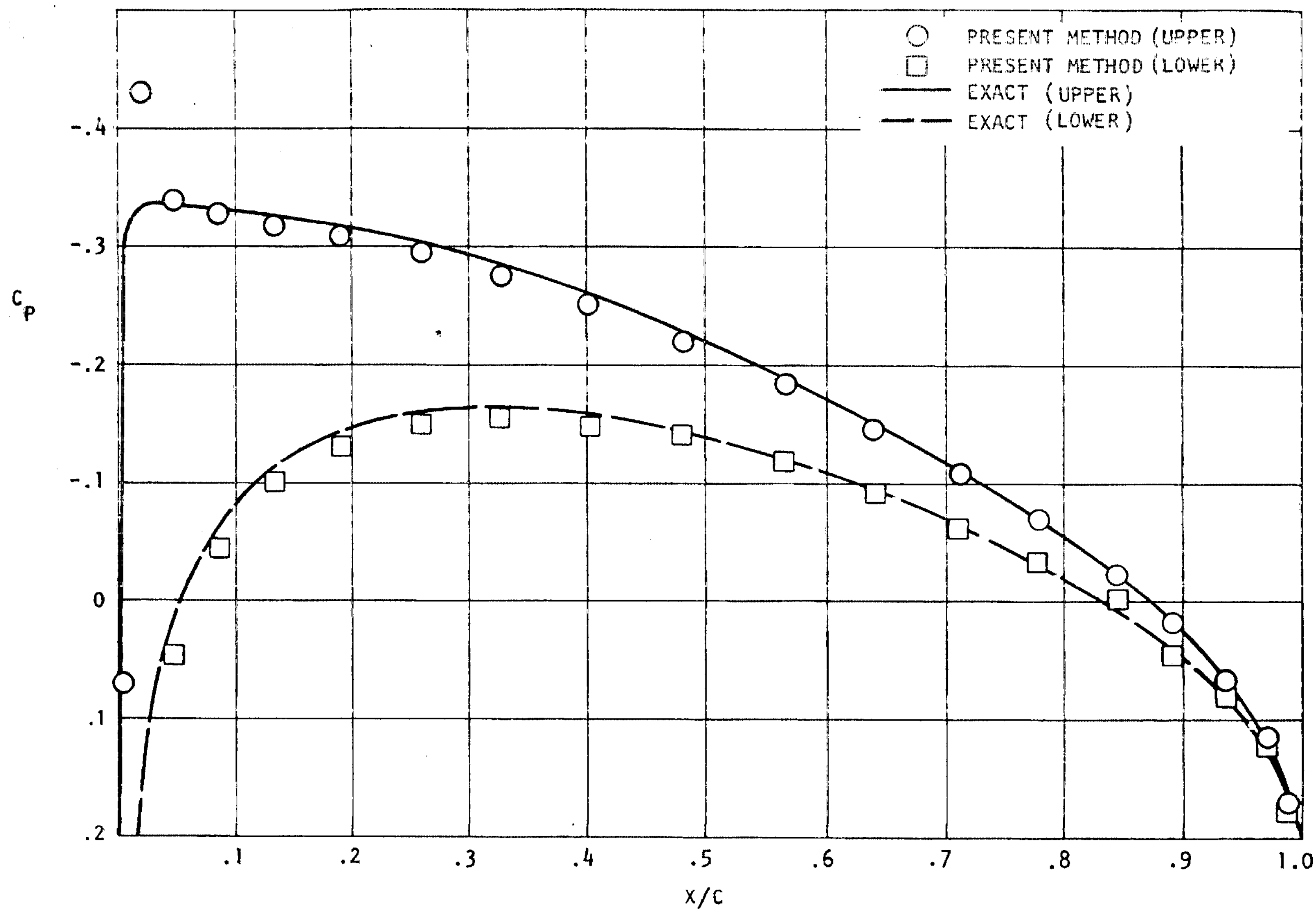


Figure 5.- Karman-trefftz airfoil pressure distribution at 1.0 degree angle of attack.

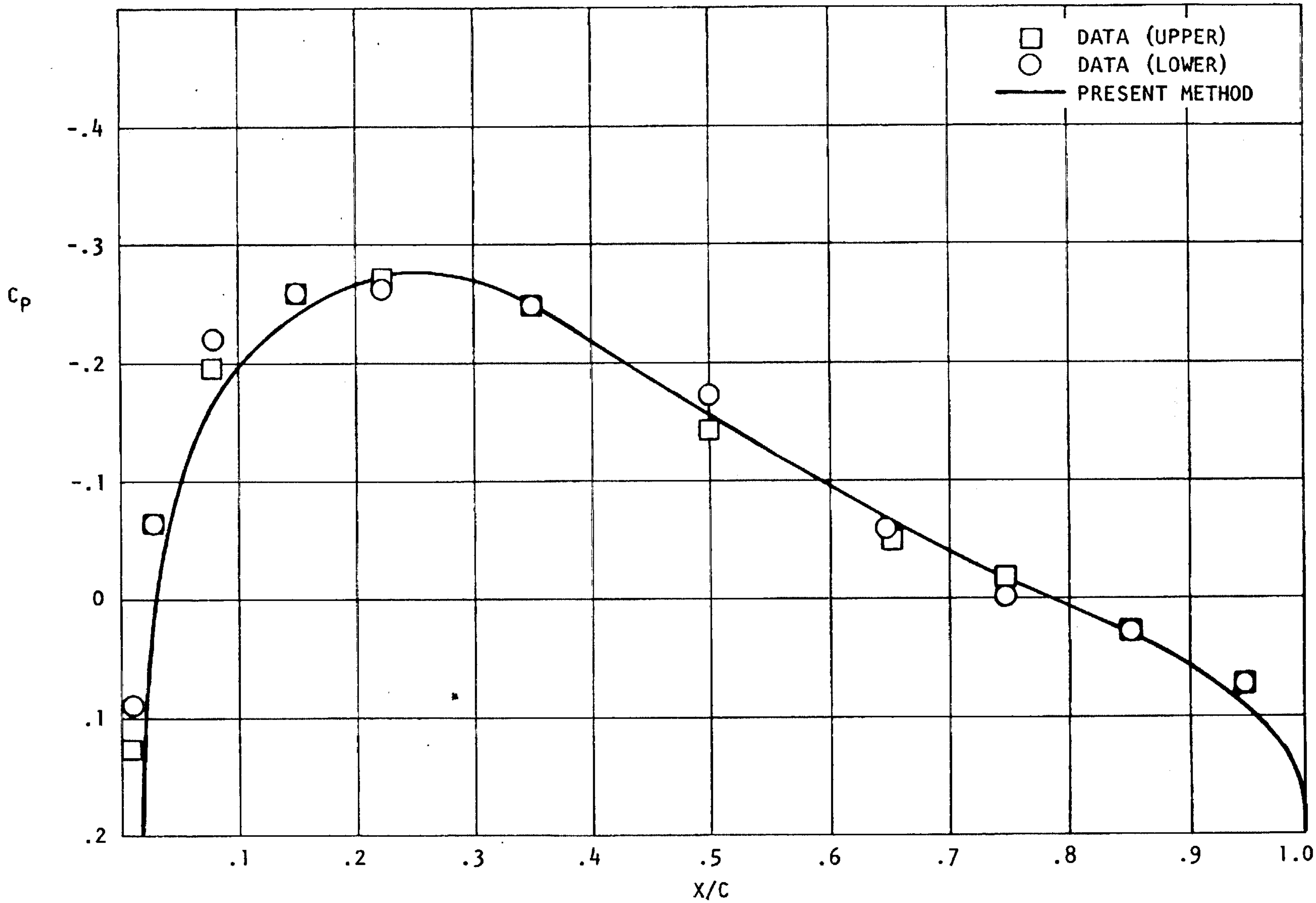


Figure 6.- Swept wing alone pressure distribution at 24.5 percent semi-span, 0.0 degree angle of attack.

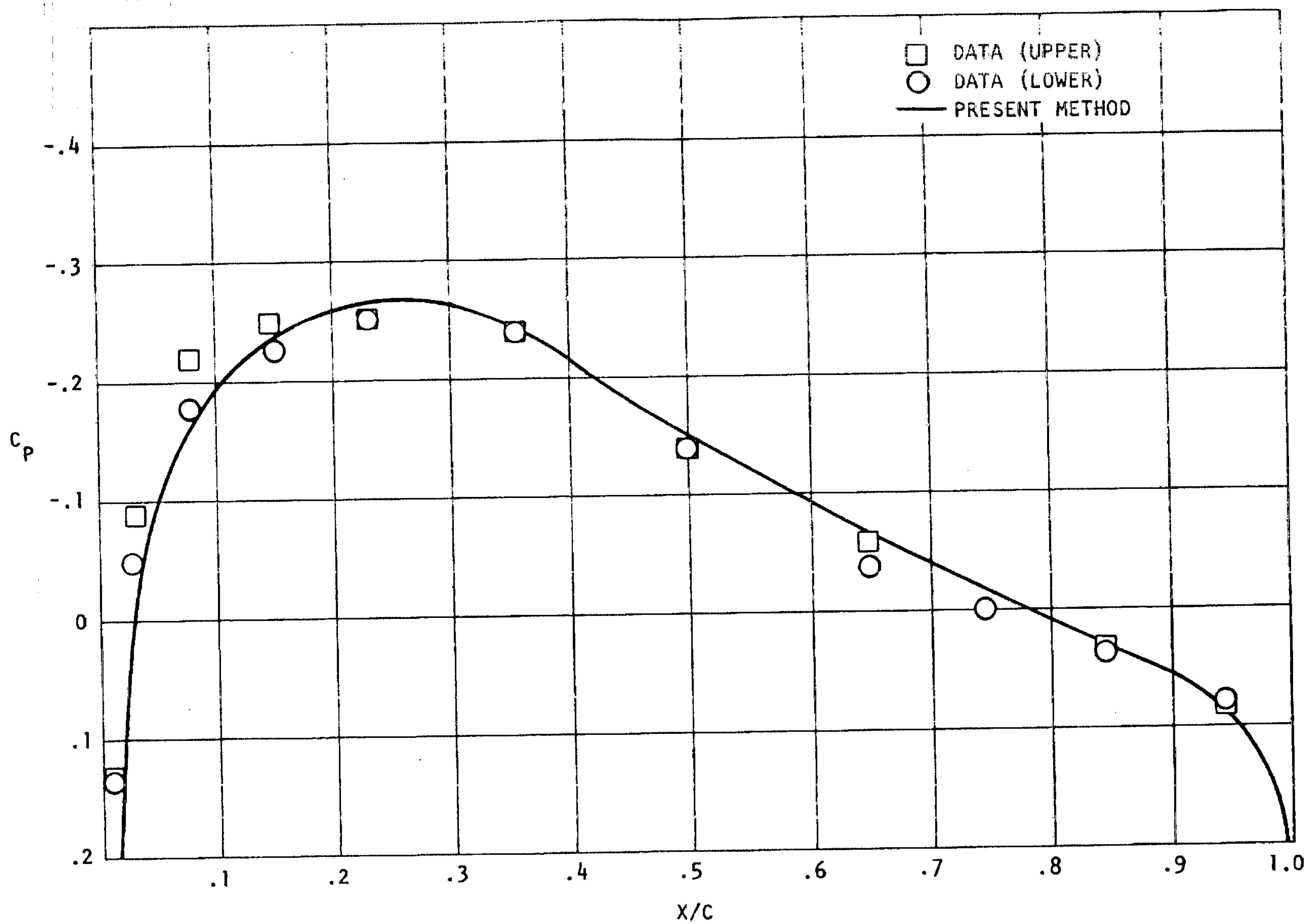


Figure 7.- Swept wing alone pressure distribution at 65.3 percent semi-span, 0.0 degree angle of attack.

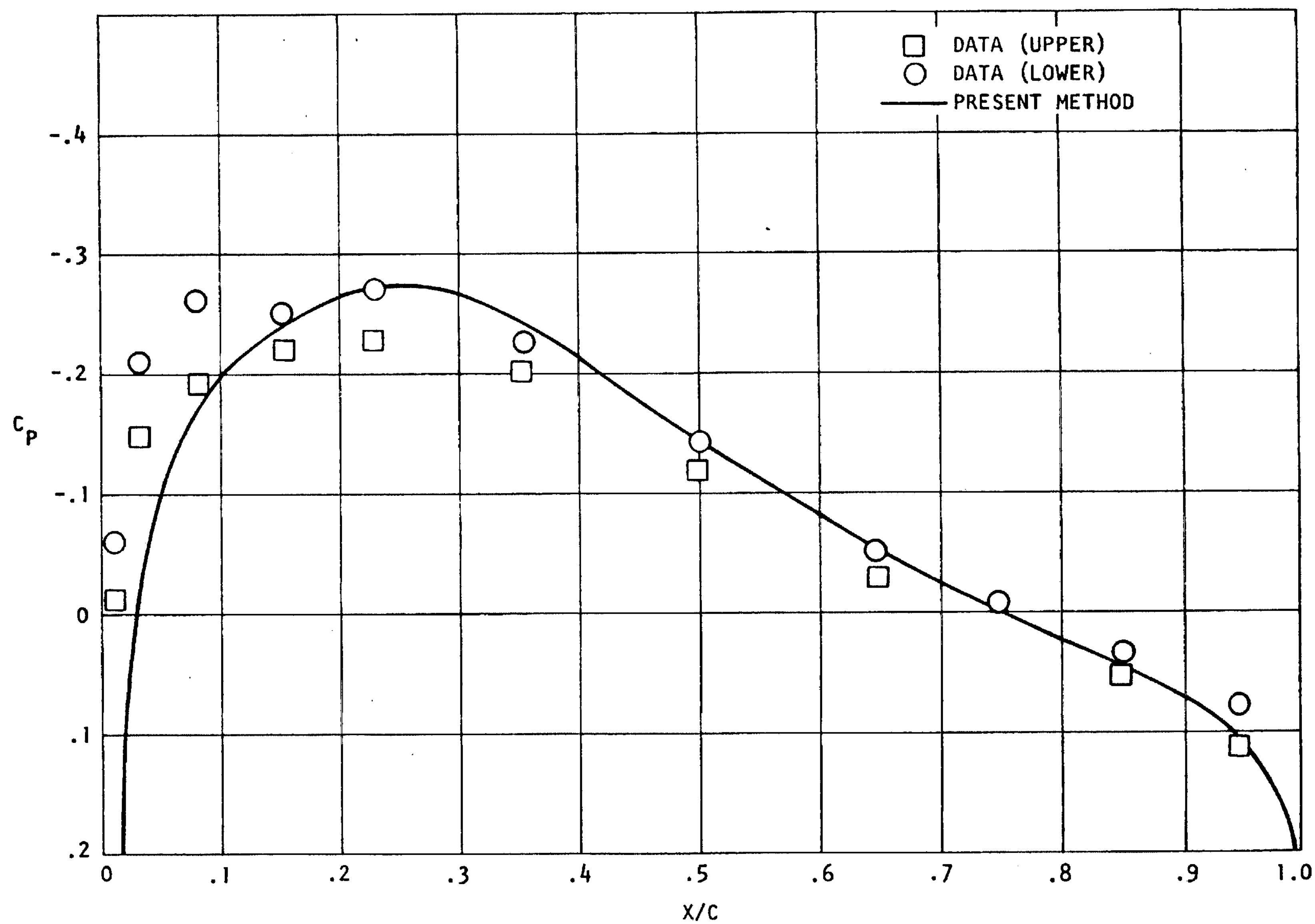


Figure 8.- Swept wing alone pressure distribution at 89.8 percent semi-span, 0.0 degree angle of attack.

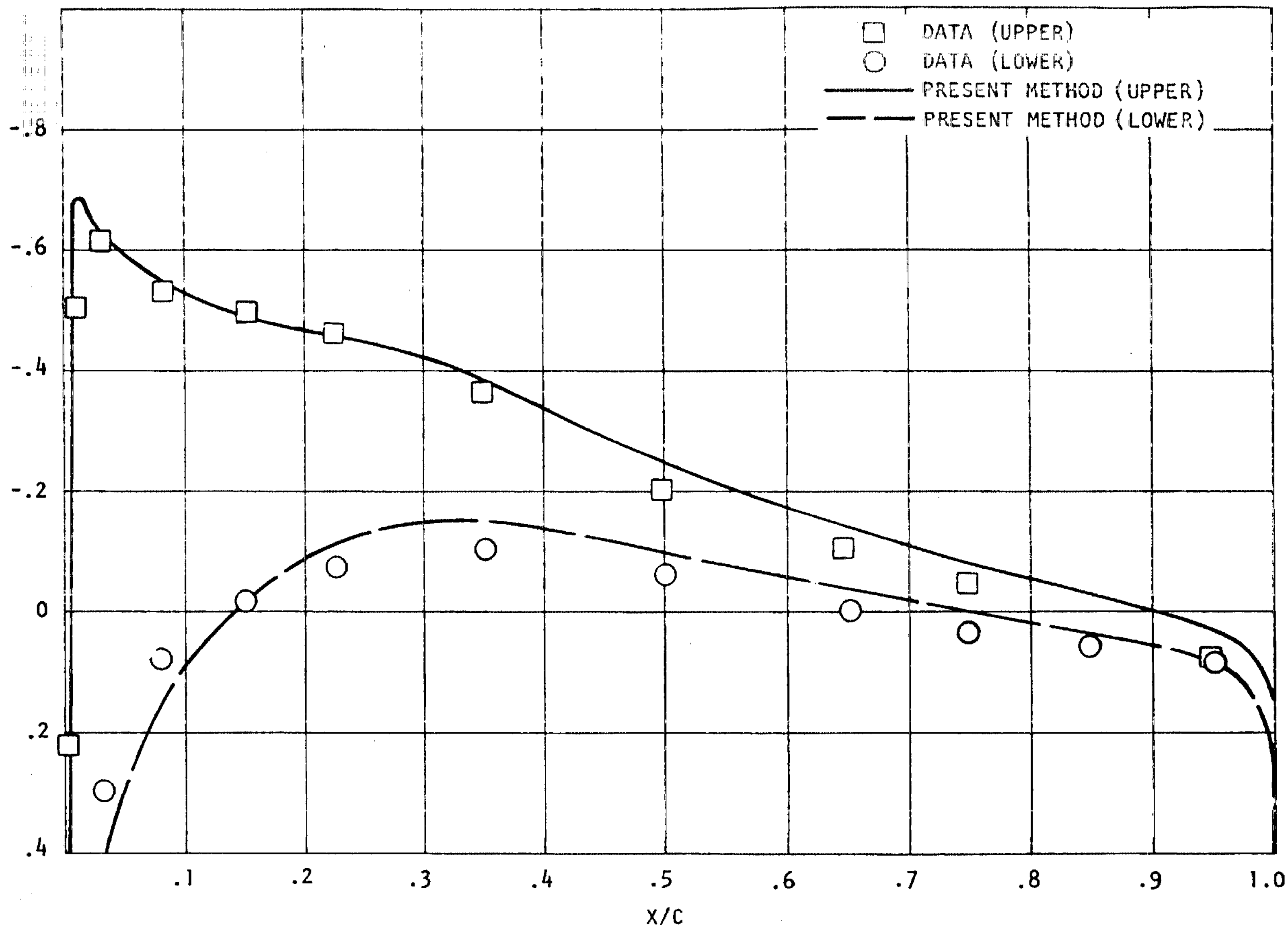


Figure 9.- Swept wing alone pressure distribution at 24.5 percent semi-span, 4.2 degrees angle of attack.

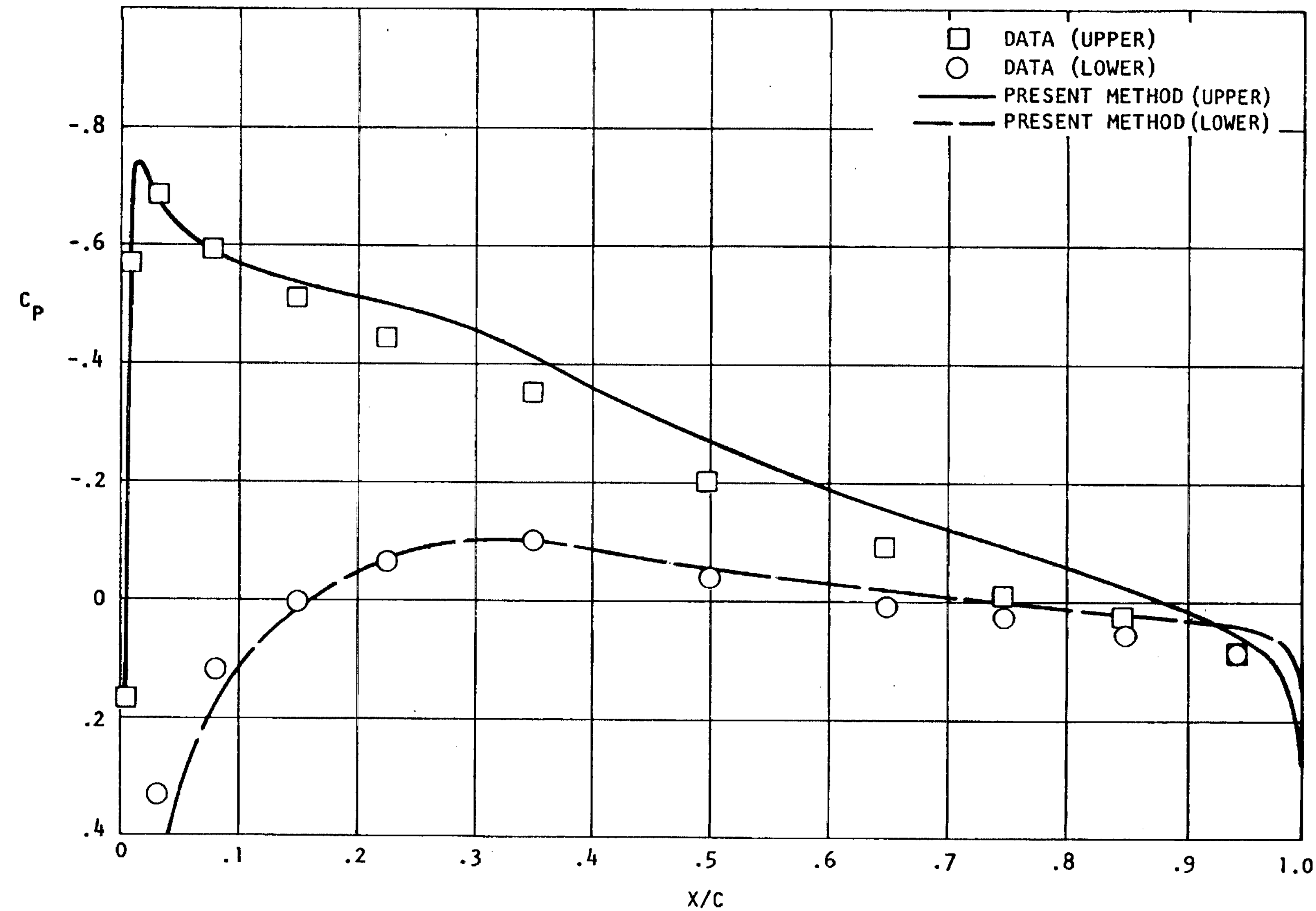


Figure 10.- Swept wing alone pressure distribution at 65.3 percent semi-span, 4.2 degrees angle of attack.

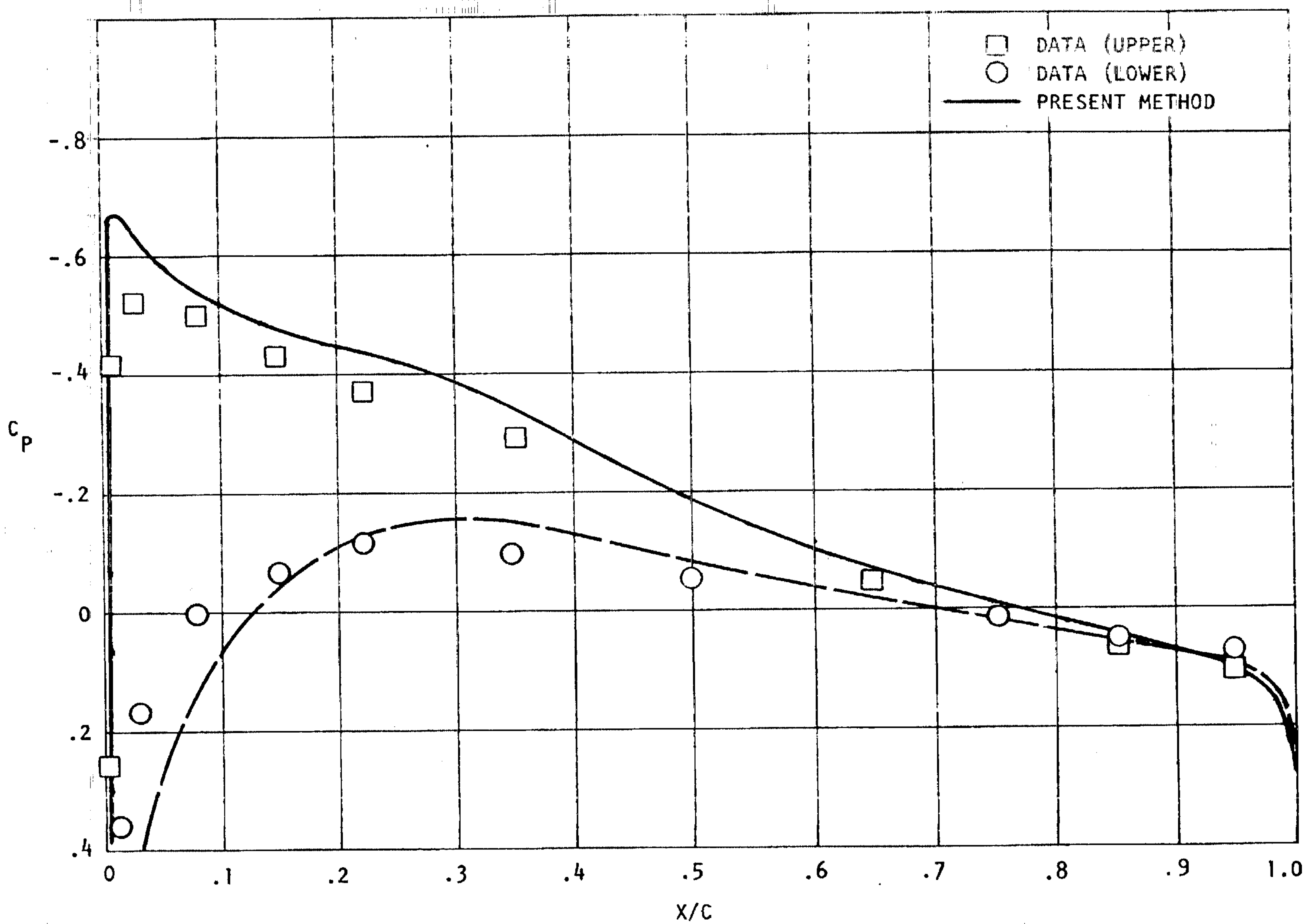


Figure 11.- Swept wing alone pressure distribution at 89.8 percent semi-span, 4.2 degrees angle of attack.

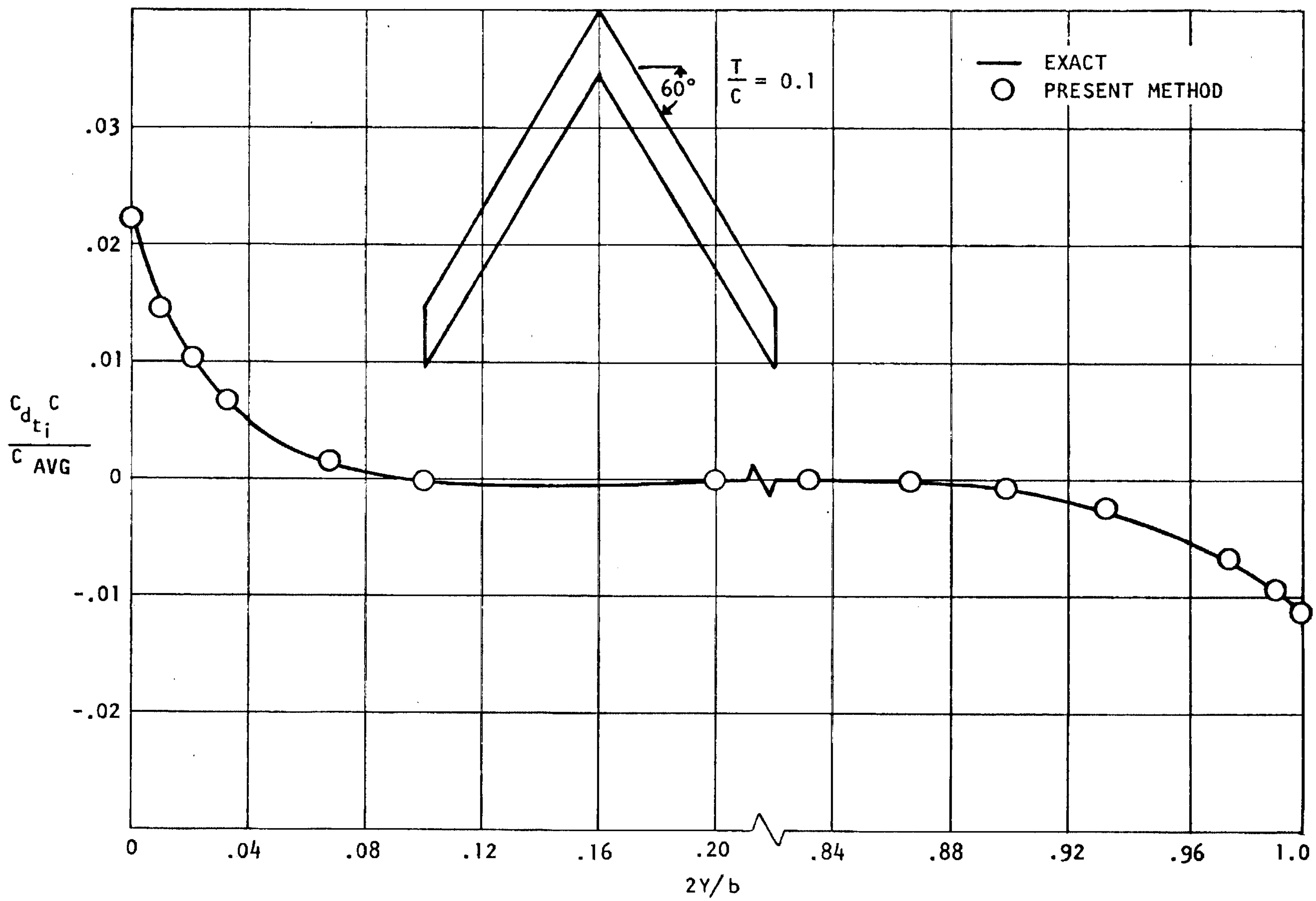


Figure 12.- Spanwise distribution of potential form drag due to thickness.

derived by Hess and Smith in reference (22). In fact Woodward's constant density source panel influence equations are identical to those derived by Hess and Smith.

As pointed out by Kuchmann and Weber the perturbation velocity due to thickness for a taper ratio one finite aspect ratio swept wing can be divided into two parts; 1) the two dimensional infinite sheared solution, and 2) the kink and tip effects. The two dimensional solution does not produce any section potential form drag provided the airfoil sections are closed. However, the kink and tip effects do produce a section drag and thrust, respectively, which when integrated across the span will give zero drag for the complete wing. Other planform effects such as taper ratio and cranks will also produce a section drag due to thickness which also integrates to zero for the complete wing.

All of the above methods will give the correct spanwise distribution of section potential form drag due to thickness. However, as also pointed out by Kuchmann and Weber the source lattice does not give the correct edge effect right at the tip, kink, or crank for a finite number of source lines in the chordwise direction. However, since this effect is only a function of the chordwise component of the thickness distribution gradient and the value of the inverse gudermannian function, with its argument being the sweep of the source line, at the point where the perturbation velocity is being computed, this effect is easily added. The region on either side of the tip, kink, or crank which is not properly handled by the source lattice is a function of the number of source lines in the chordwise direction and for practical solutions, which require about twenty source lines per chord to represent the distribution of thickness, this region is of no significance. Therefore, due to the superior numerical efficiency associated with the source lattice influence equations the source lattice appears to be the best aerodynamic finite element for predicting the perturbation velocity and potential form drag due to thickness.

Section Induced Drag Due to Lift

There are two basic approaches that have been tried in the past to solve the problem of predicting the spanwise distribution of induced drag or section potential form drag due to lift, 1) to accurately solve for the thin wing net pressure distribution, including the strength of the leading edge singularity, utilizing precise integration techniques to solve the aerodynamic influence integral equation, and 2) to utilize a vortex lattice procedure in conjunction with the Kutta-Joukowsky theorem. Both of these approaches have failed to predict a spanwise distribution of induced drag due to lift which when integrated is equal to the induced drag computed in the Trefftz plane. The reason for this is that in these attempts the assumption that the vorticity is constant in the spanwise direction along constant percent chord lines, even for only an infinitensimal distance, leads to a nonanalytic influence function for which no finite value exists for the induced velocity at span stations where the gradients of the constant percent chord lines are discontinuous.

Therefore, at span stations where the constant percent chord lines are kinked or cranked the constant vorticity distributed panel procedures, such as Woodward's, give a logarithmic singularity in the downwash. The lifting surface theories, such as Multhopp's or Wagner's, also produce a logarithmic singularity in the downwash which cannot be handled. Wagner's theory, as given in reference (50), has been investigated in great detail and is commented on in appendix E.

In the case of the skewed vortex-lattice the downwash at the control point is not singular, however, the Cauchy principal value does not exist for the downwash on the vortex line at a span station where the vortex lines have discontinuous sweeps. Therefore, the Kutta-Joukowsky theorem will give an infinite section drag at these stations.

All of these problems can be eliminated by using an unswept horseshoe vortex lattice. It is proven in appendix F that if the bound vortex lines are all parallel and the horseshoe lattice is evenly spaced in the lateral direction, the integral of the spanwise distribution of induced drag and the induced drag computed in the Trefftz plane are identical for all planform shapes. This is also true for multiple lifting surfaces, provided they are all parallel, and for lifting surfaces with jet flaps.

It is not proposed that only unswept horseshoe vortex-lattice procedures be used to compute the net pressures or loads on wings of arbitrary shape. However, this appears to be the only numerical integration procedure known at this time which will always give the same value for the induced drag in the near and far fields. During studies of the error involved in using a skewed lattice it was determined that the error in the section induced drag was limited to a vary small region on either side of the discontinuity in the sweep of the vortex lines. Also, since most of the wing is represented better by skewed vortex lines (because the lines of constant pressure do coincide with constant percent chord lines over most of the wing) a good overall answer can probably be obtained at less expense, (since fewer skewed vortices are needed in general to represent a wing than unswept vortices) if a skewed vortex lattice is used to compute the net pressures and the unswept vortex lattice is used to compute the drag once the net pressures are known.

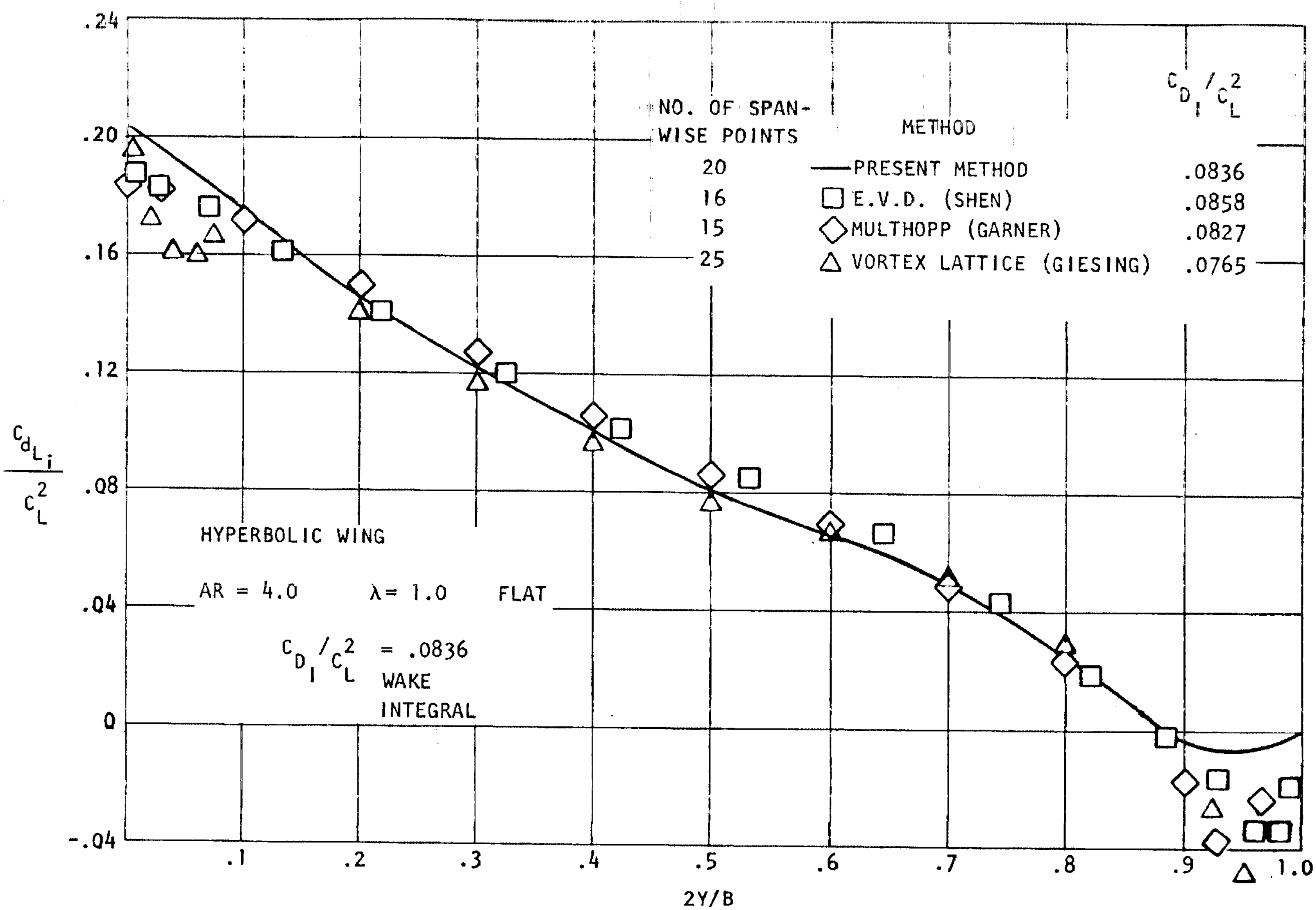


Figure 13.- Spanwise distribution of potential form drag due to lift.

A comparison of the section induced drag divided by the section lift four different procedures is shown in figure (13).

Sphere Surface Velocity

The velocity over the surface of a sphere is compared to the exact solution in figure (14).

X-15 Wing-Fuselage-Horizontal Tail-Vertical Tail

Surface velocities and pressure coefficients, section force and moment coefficients, and total configuration force and moment coefficients are computed for the X-15 wing-fuselage-horizontal tail-vertical tail shown in figure (15). The program input for this configuration plus the ventral is given in the sample input section. The program output for this configuration is given in the sample output section. In the program output the fuselage, wing, horizontal tail and vertical tail are designated as components 1, 2, 3, and 4, respectively.

Some results for this configuration are shown in figures (16), (17), (18), and (19). The data for these comparisons were obtained from References (59), (60), and (61). The force data is at .6 Mach number and the pressure coefficient data is at .2 Mach number. All of the theoretical results are for zero Mach number.

The total configuration C_{L_α} was determined experimentally to be .061. The program predicts .0617.

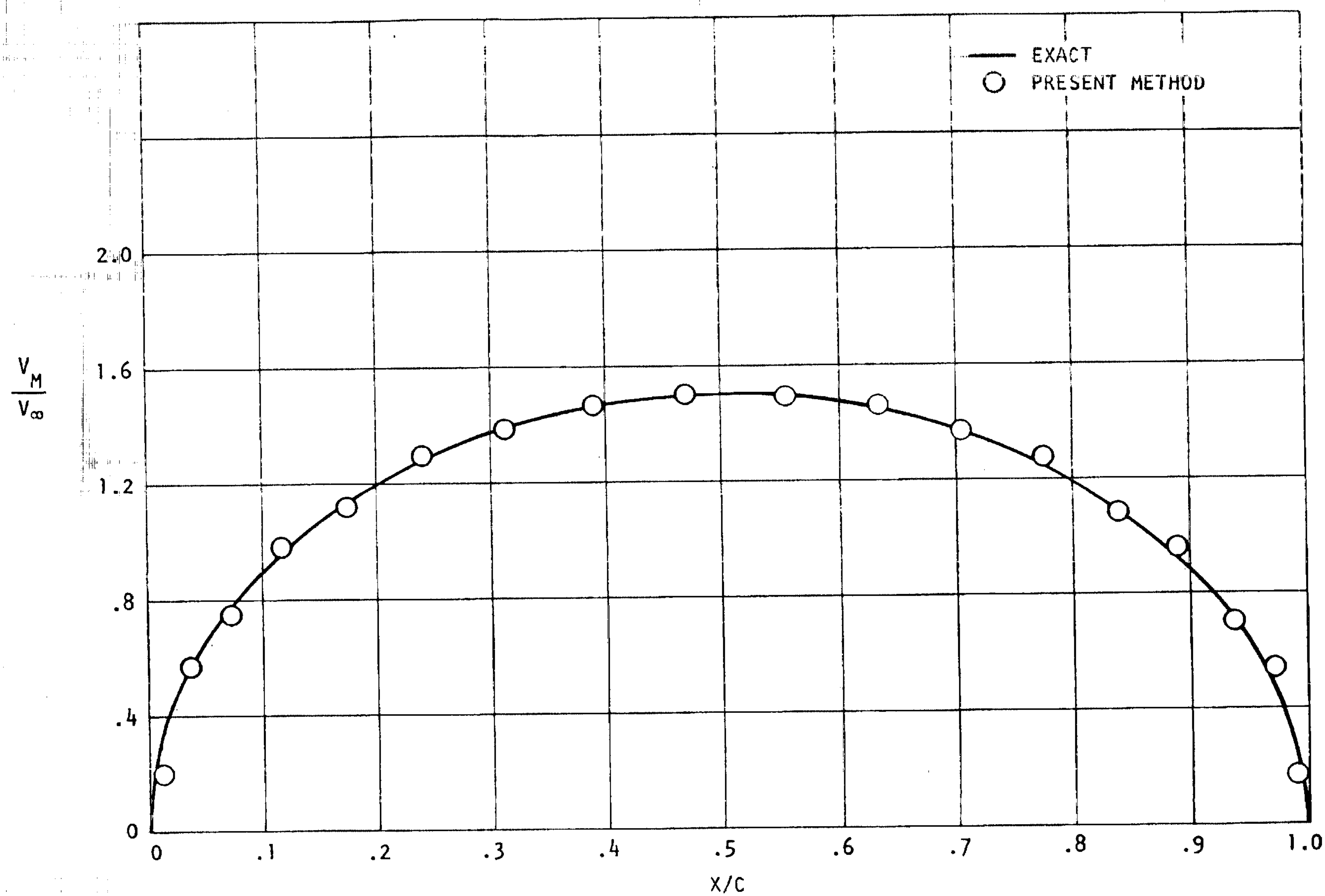


Figure 14.- Velocity ratio around a sphere.

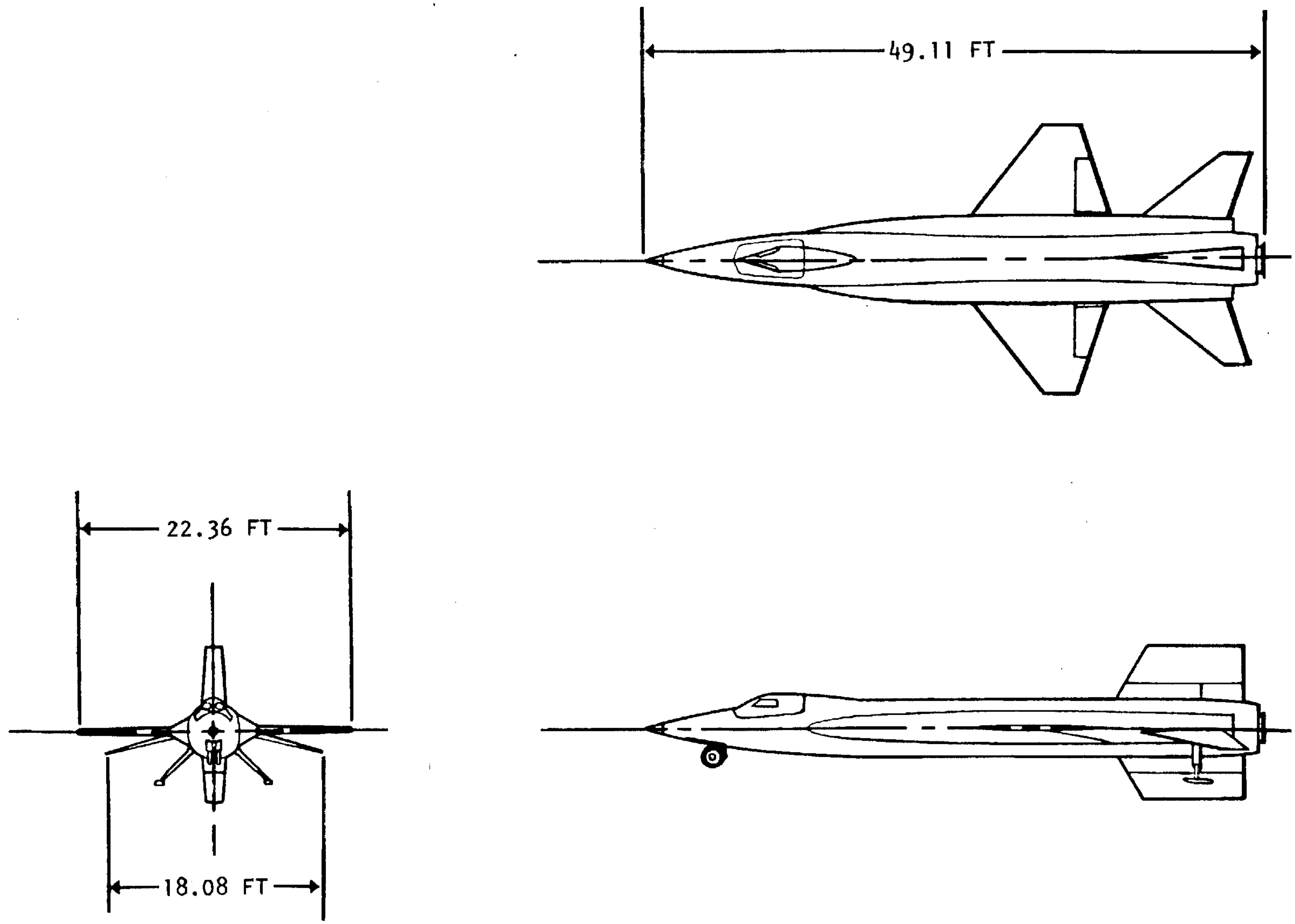


Figure 15. X-15 Configuration

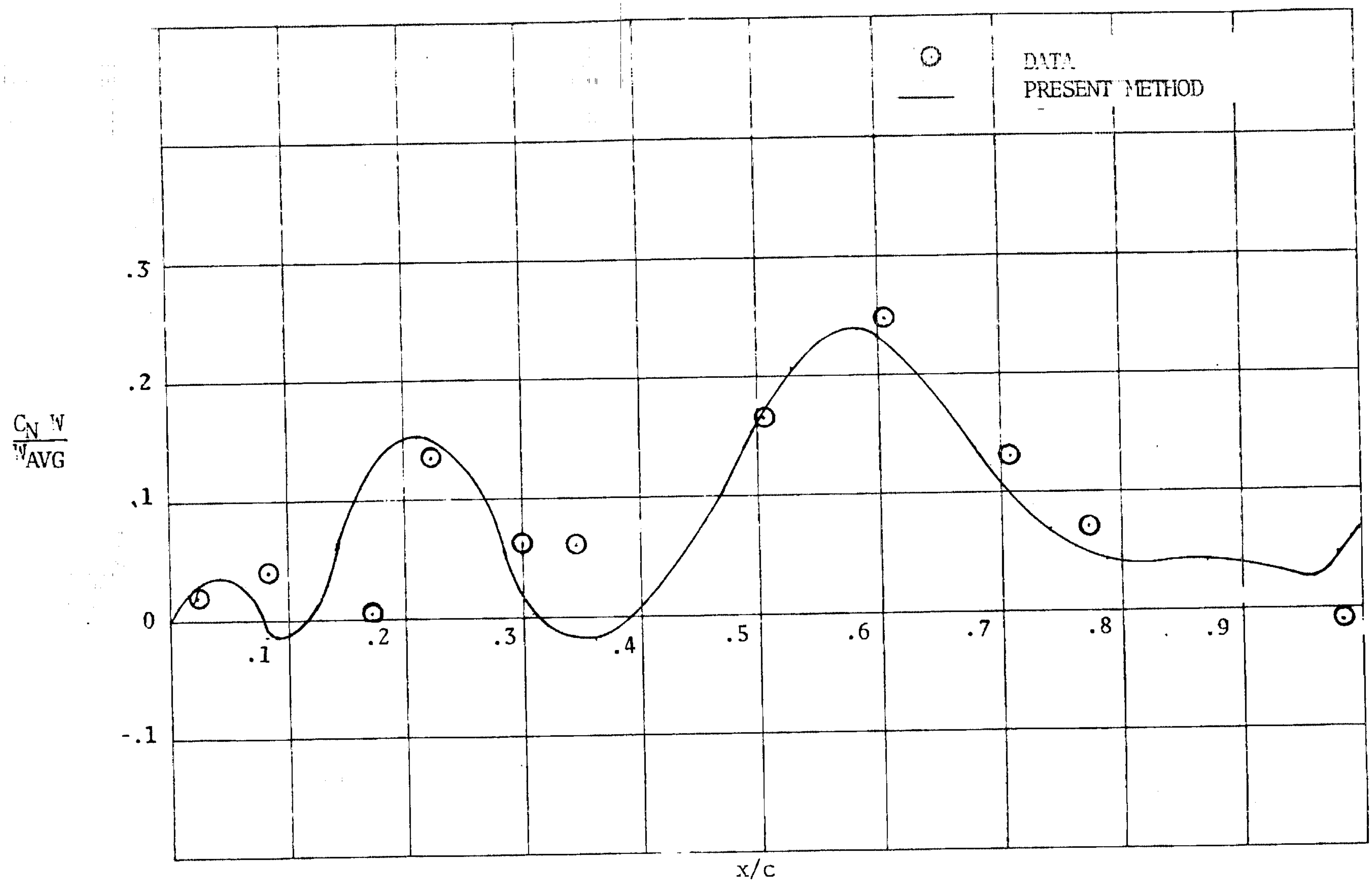


Figure 16.- Normal Load Distribution on X-15 Fuselage at Five Degrees Angle of Attack

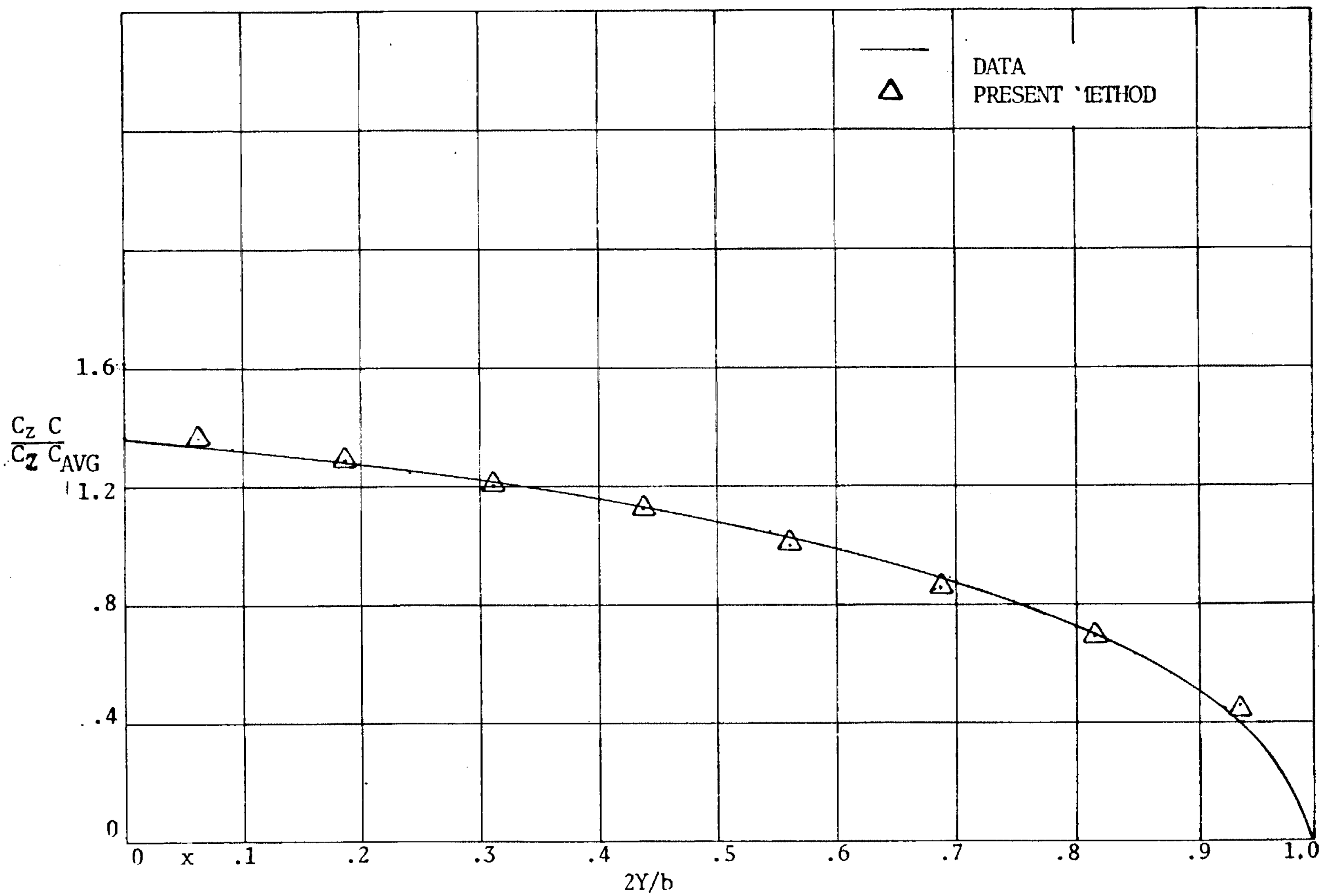


Figure 17.- Unit Span Load Distribution on X-15 Wing

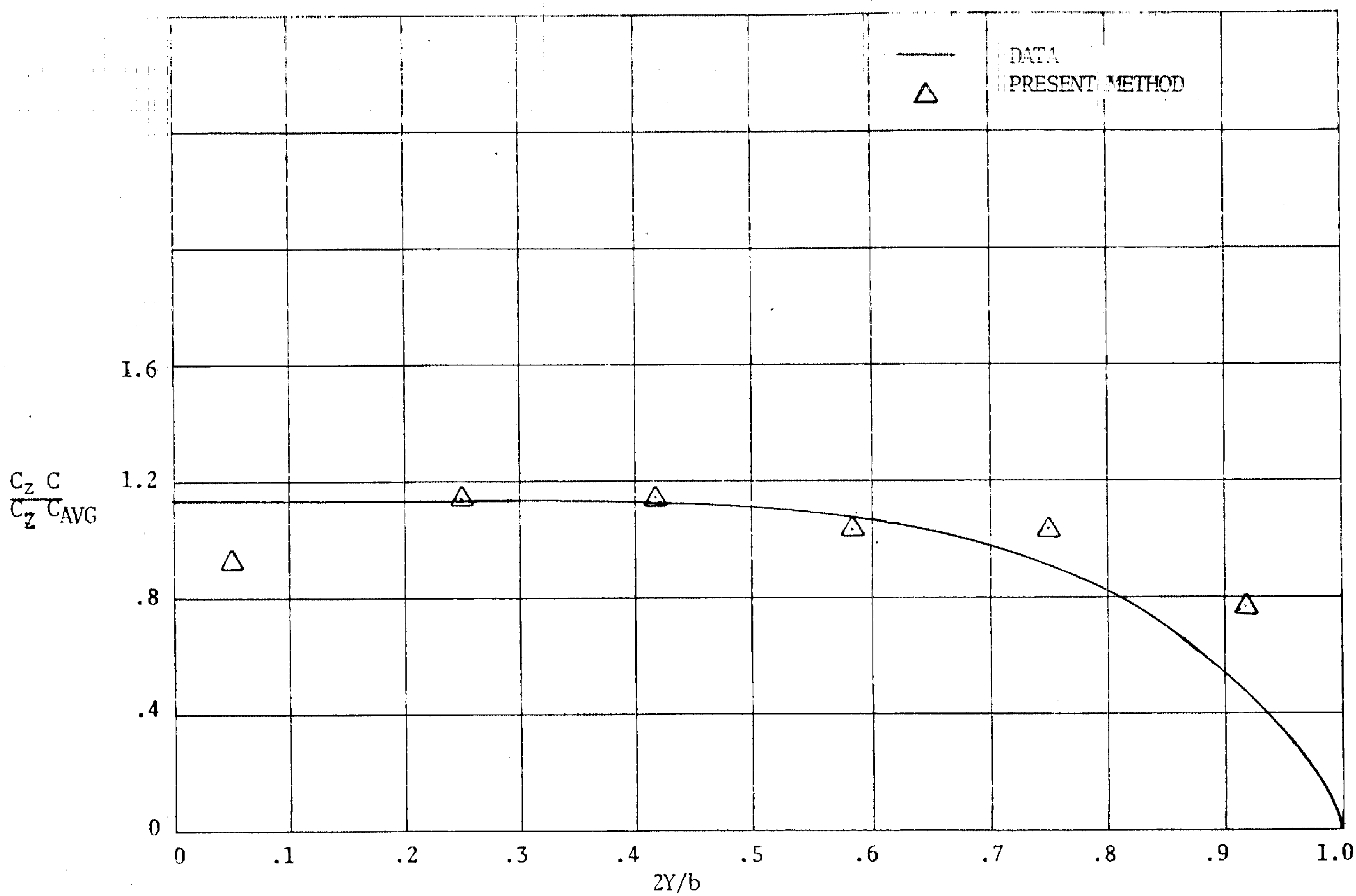


Figure 18.- Unit Span Load Distribution on X-15 Horizontal Tail

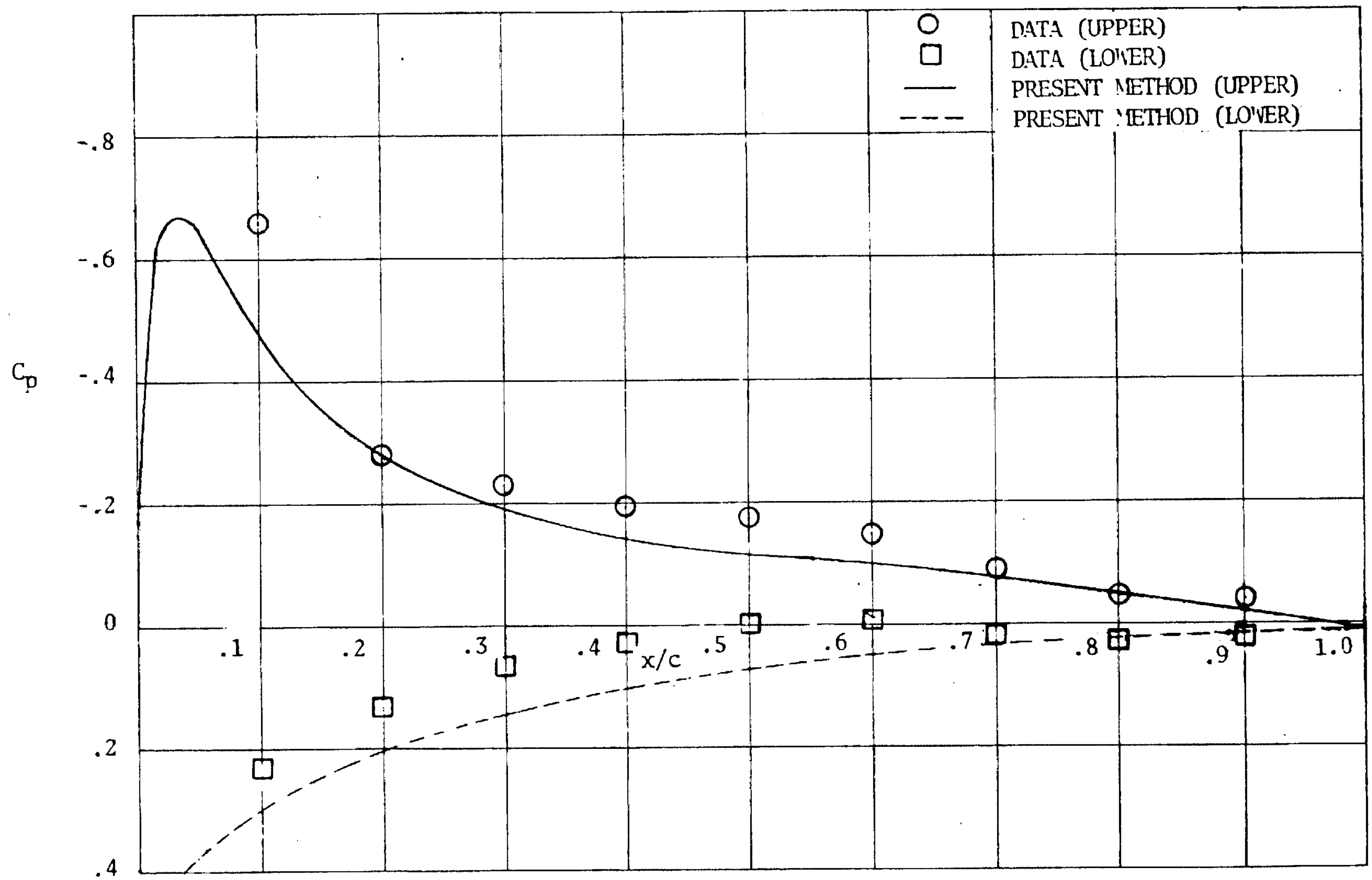


Figure 19.- Chordwise Pressure Distribution on X-15 Wing at 17.8 Percent Semispan at Five Degrees Angle of Attack

Program Setup

There are two types of data decks which can be used in this program 1) the NASA Langley input format described in reference (52) and 2) the NR input format described in the next two sections of this report. Each of these input formats requires a different program setup.

Program setup for NASA format. - In this form of data setup, routine "OUTIN" preprocesses data from Langley's format and creates a card image file for use by the rest of the subroutines. Changes or additions to the data associated with bodies and panels in the Langley form of input created by "OUTIN" are made by body and panel "INFO" decks. There should be a body and panel "INFO" deck for each body and panel in the Langley input array, respectively. These body and panel "INFO" decks utilize the NR data array format.

Additional configuration bodies and panels not described in the Langley data array are added by means of additional body and panel input decks utilizing the NR data array format.

Most of the input from the Langley data array can be directly converted to the NR data array format. The only exception is a nacelle which is converted from a body of revolution to a ring wing. This occurs if the pod data has a nonzero value at the nose of the pod. If the radius at the nose of the pod is zero, the pod is considered a solid body of revolution.

The input data in the NR data array format ahead of the body and panel descriptions pertain to the total configuration and must always be input as an "INFO" deck. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed in figure 21. Figure 20 contains the entire deck setup.

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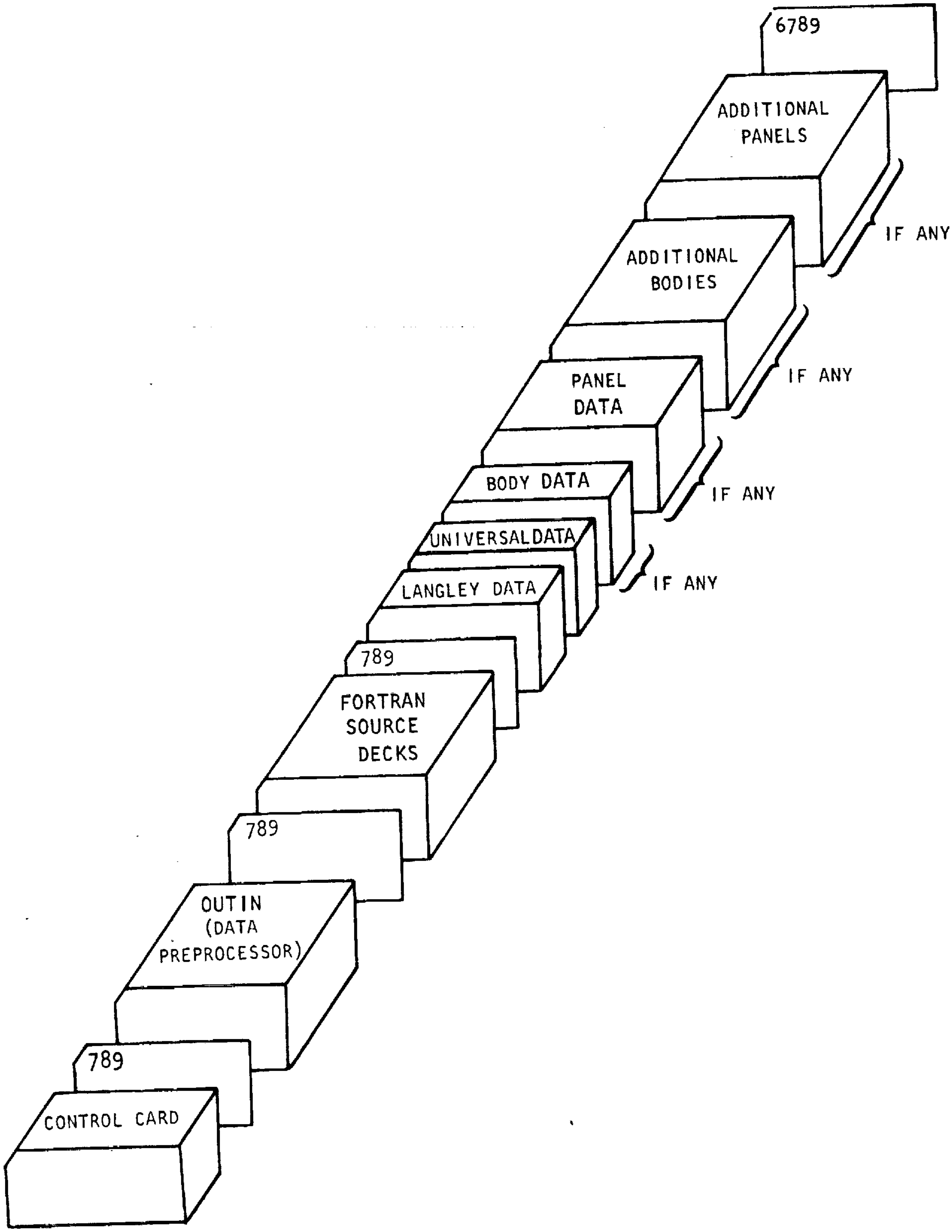

Figure 20.- Deck setup for use with NASA Langley data array format.

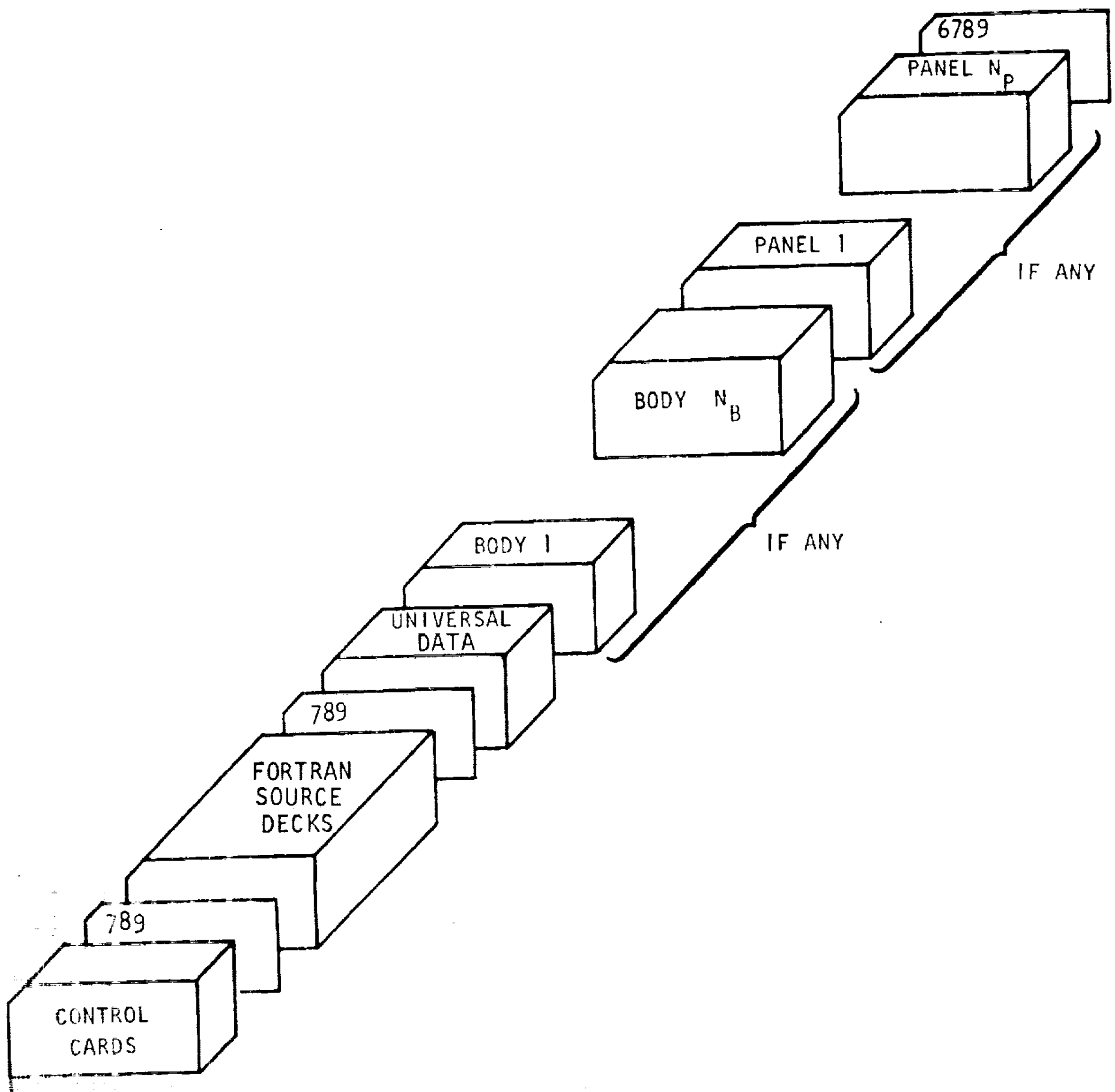

Figure 22.- Deck setup for use with NR data array format.

Program setup for NR format. - In this form of data setup all input decks utilize the NR data array format. The total configuration or universal "INFO" deck is followed by an input deck for each body and then a deck for each panel. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed for this form of data input in figure 21. Figure 22 contains the entire deck setup.

Card No.	
1.	SEQUENCE CARD
2.	CHARGE CARD
3.	PFSTAD, CM31000, T7777, P6
4.	RUN, S.
5.	SET, 0.
6.	LGO.

Figure 21.- Control card deck sequence

Input Format

The NR data array format is given in this section. Data locations of all input data with limits on size of inputs are designated. A more detailed description of the input is given in the next section.

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	1		Universal data	
13			No. of bodies	
25			No. of panels	
37			Mach No.	
49		73 … 80	A_R (reference area)	$(\text{units})^2$
61			Not used	
1	5			
13			$\bar{C}$ (reference longitudinal length)	(units)
25			b (reference lateral length)	(units)
37			$X_{C.G.}$	(units)
49		73 … 80	$Y_{C.G.}$	(units)
61			$Z_{C.G.}$	(units)
1	1 0			
13			α (angle of attack)	(deg)
25			β (angle of sideslip)	(deg)
37			$P^* = P/(2V_\infty/b)$ (nondimensional roll rate)	
49		73 … 80	$q^* = q/(2V_\infty/c)$ (nondimensional pitch rate)	
61			$r^* = r/(2V_\infty/b)$ (nondimensional yaw rate)	
1	1 5		Body Data	
13			Component No.	
25			A_{B_j} (Body reference area)	$(\text{units})^2$
37			C (chord)	(units)
49		73 … 80	Indicator for [A] matrix: (0.) compute new matrix and save on tape; (-1.) bypass matrix calculation and read from tape	
61			Symm indicator: (0.) body and vortex strengths are symm about x-z plane; (1.) if only body is symm; (-1.) no symm	

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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2.0		Body coordinate table input indicator: (0.) for (R_i vs θ_i) vs X_i; (1.) for $[(Z_{Bi}-\Delta Z_B)/Z_{BM}]$ vs $[(Y_{Bi}-\Delta Y_B)/Y_{BM}]$ vs X_i
13			Body multiplication factor indicator (0.) for $Y_{BM}=Z_{BM}$. (1.) $Y_{MB}=Z_{MB}$
25			Body camber indicator (0.) body section put ⊥ to camber line (1.) body section put ⊥ to body X axis.
37			Body longitudinal subpanel indicator: (0.) even $\Delta\phi$; (-1.) even ΔX; (1.) given x
49			Body lateral vortex indicator: (0.) even $\Delta\theta$ or $\Delta(S/S_{max})$; (1.) given θ or (S/S_{max}); (-1.) same as body definition stations
61			
1	2 5		
13			Area of influence in longitudinal direction
25			Area of influence in lateral direction
37			X_{B_O} Body origin (units)
49			Y_{B_O} Body origin (units)
61			Z_{B_O} Body origin
1	3.0		
13			No. of subpanels in longitudinal direction (max 299)
25			No. of subpanels in the lateral directions (max 40)
37			No. of longitudinal subpanel divisions (odd integer)
49			No. of lateral subpanel divisions (odd integer)
61			No. of longitudinal constraint functions (max 50)
1	3 5		
13			No. of lateral constraint functions (max 20)
25			No. of Longitudinal constraint segments (max 50)
37			No. of lateral constraint segments (max 20)
49			No. of control points in longitudinal direction (max 299)
61			No. of control points in lateral direction (max 40)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	4 0		Body coordinate table	
13			No. of X stations	(max 44)
25			$X_{B_{i_1}}$	(units)
37			$X_{B_{i_2}}$	(units)
49		73 80	⋮	
61				
1	8 5		List of X stations where θ's or $[(Y_{B_i}-\Delta Y_B)/Y_{B_M}]$'s are defined	
13			No. of X_{B_θ} stations	(max 44)
25			$X_{B_{\theta_1}}$	(units)
37			$X_{B_{\theta_2}}$ (If the list is the same as	(units)
49		73 80	⋮ in loc 41-89 it can be omitted)	
61				
1	1 3 0		Lateral coordinates for 1st X_θ station	
13			No. of θ's, $[(Y_{B_i}-\Delta Y_B)/Y_{B_M}]$'s, or $\bar{S}$'s if number in location 40 is greater than 44	(max 39)
25			θ_1	(deg)
37			θ_2	(deg)
49		73 80	⋮	
61				
1			Lateral coordinates for 2nd X_θ station	
13			No. of θ's, $[(Y_{B_i}-\Delta Y_B)/Y_{B_M}]$'s, or $\bar{S}$'s if number in location 40 is greater than 44	(max 39)
25			θ_1	(deg)
37			θ_2	(deg)
49		73 80	⋮	
61				

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	8.00		R_i or $[(Z_{B_i} - Z_B)/Z_{B_M}]$ values at X_{B_i} stations (max 800)
13			R_1 or $[(Z_{B_i} - Z_B)/Z_{B_M}]_1$
25			R_2 or $[(Z_{B_i} - Z_B)/Z_{B_M}]_2$ (Input along longitudinal direction
37			: at first lateral station, then second
49		73 80	lateral station, and etc)
61			
1	16 00		Multiplication factors and camber longitudinal stations
13			No. of X stations (max 49)
25			X_{B_1} (units)
37			X_{B_2} (units)
49		73 80	:
61			
1	1 6 5 0		Y Multiplication factors
13			$Y_{B_{M_1}}$ (units)
25			$Y_{B_{M_2}}$ (units)
37			:
49		73 80	
61			
1	1 7 0 0		Z multiplication factors
13			$Z_{B_{M_1}}$ (units)
25			$Z_{B_{M_2}}$ (units)
37			:
49			
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	1 7 5 0		ΔY displacements	
13			ΔY_1	(units)
25			ΔY_2	(units)
37			⋮	
49		73 80		
61				
1	1 8 0 0		ΔZ displacements	
13			ΔZ_1	(units)
25			ΔZ_2	(units)
37			⋮	
49		73 80		
61				
1	1 8 5 0		Longitudinal subpanel grid	(max 300)
13			X_{E1}	(units)
25			X_{E2}	(units)
37			⋮	
49		73 80		
61				
1	2 1 5 0		List of X stations where θ_E or (S/S_{max}) are defined	
13			No. of X stations	(max 49)
25			$X_{\theta E1}$	(units)
37			$X_{\theta E2}$ (omit, if same as loc 1850 - 2149)	(units)
49		73 80	⋮	
61				

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C-2

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2 2 0 0		Lateral vortex grid at first λ_{B_E} station
13			θ_{E_1} or $(S/S_{max})_1$ (deg)
25			θ_{E_2} or $(S/S_{max})_2$ (deg)
37			⋮
49			
61			
1			Lateral vortex grid at second λ_{ϕ_E} station
13			θ_{E_1} or $(S/S_{max})_1$ (deg)
25			θ_{E_2} or $(S/S_{max})_2$ (deg)
37			⋮
49			
61			
1	2 8 0 0		List of subpanels in longitudinal direction where control points are to be placed
13			J_{M_1}
25			J_{M_2}
37			⋮
49			
61			
1	3 1 0 0		List of subpanels in lateral direction where control points are to be placed
13			J_{N_1}
25			J_{N_2}
37			
49			
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. PROGRAMMER DATE PAGE of JOB NO.

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3140		Longitudinal constraint segments
13			$J_{S_{M_1}}$
25			$J_{S_{M_2}}$
37			⋮
49		73 80	
61			
1	3190		Lateral constraint segments
13			$J_{S_{N_1}}$
25			$J_{S_{N_2}}$
37			⋮
49		73 80	
61			
1	3210		Longitudinal constraint function sequence No.
13			1. 1.0
25			2. $1.0/\{1 + [Y_B(N_{X_B}/N_{Y_B}) + Z_B(N_{X_B}/N_{Z_B})]^2/(Y_B^2 + Z_B^2)\}^{1/2}$
37			3. $\cot(\phi_B/2)$
49		73 80	4. $\cot[(\pi/2) - (\phi_B/2)]$
61			5. $\sin[\pi(\phi_B - \phi_o)/(\phi_f - \phi_o)]$
1	3215		
13			6. $\cos[\pi(\phi_B - \phi_o)/(\phi_f - \phi_o)]$
25			7. $[(X_B/C_B) - (X_B/C_B)_o]/[(X_B/C_B)_f - (X_B/C_B)_o]$
37			8. $\sin[2\pi(\phi_B - \phi_o)/(\phi_f - \phi_o)]$
49		73 80	9. $\cos[2\pi(\phi_B - \phi_o)/(\phi_f - \phi_o)]$
61			10. $[(X_B/C_B) - (X_B/C_B)_o]^2/[(X_B/C_B)_f - (X_B/C_B)_o]^2$

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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3220		⋮
13			
25			3270 L_{S1} (X_B/C_B)
37			3320 L_{S2} (X_B/C_B)
49		73 80	3370 L_{S3} (X_B/C_B)
61			
1	3250		Lateral constraint function sequence No.
13			1. 1.0
25			2. $N_{Z_B}/[N_{-3}{}^2 + N_{Y_B}{}^2]^{1/2}$
37			3. $N_{Y_B}/[N_{Z_B}{}^2 + N_{Y_B}{}^2]^{1/2}$
49		73 80	4. $\sin[\pi(\phi_E - \phi_o)/(\phi_f - \phi_o)]$
61			5. $\cos[\pi(\phi_E - \phi_o)/(\phi_f - \phi_o)]$
1	3255		
13			6. $[\pi(\Theta_B - \Theta_o)/(\Theta_f - \Theta_o)]$ or $[(\eta_B - \eta_o)/(\eta_f - \eta_o)]$
25			7. $\sin[2\pi(\Theta_B - \Theta_o)/(\Theta_f - \Theta_o)]$
37			8. $\cos[2\pi(\Theta_B - \Theta_o)/(\Theta_f - \Theta_o)]$
49		73 80	9. $[\pi(\Theta_B - \Theta_o)/(\Theta_f - \Theta_o)]^2$ or $[(\eta_B - \eta_o)/(\eta_f - \eta_o)]^2$
61			⋮
1	3270		First special longitudinal constraint function
13			$(X_B/C_B)_1$
25			L_{S1} $(X_B/C_B)_1$
37			$(X_B/C_B)_2$
49		73 80	L_{S2} $(X_B/C_B)_2$
61			⋮

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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. PROGRAMMER DATE PAGE of JOB NO.

Col	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3320		Second special longitudinal constraint function
13			$(X_B/C_B)_1$
25			L_{S_2} $(X_B/C_B)_1$
37			$(X_B/C_B)_2$
49			L_{S_2} $(X_B/C_B)_2$
61			⋮
1	3370		Third special longitudinal constraint function
13			$(X_B/C_B)_1$
25			L_{S_3} $(X_B/C_B)_1$
37			$(X_B/C_B)_2$
49			L_{S_3} $(X_B/C_B)_2$
61			⋮
1	3420		Panel Data
13			Component No.
25			Reference area (units)^2
37			Reference chord (units)
49			Reference span (units)
61			Indicator to calculate near field drag; (1.) calculate; (0.) omit
1	3425		
13			Indicator to save [A] matrix (0.) compute new matrix (-1.) bypass matrix calculation
25			Symm indicator (0.) if panel and unknowns have an image (1.) if just panel (-1.) no image (2.) antisymmetric
37			Panel inboard trailing vortex is attached to panel (0.) positive normal side (1.) negative normal side
49			Trailing vortex indicator (0.) straight (1.) force free
61			Panel contour indicator (0.) local angles of attack (1.) percent deflection relative to chord line

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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

Column	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3430		
13			Panel longitudinal vortex indicator (0.) even $\Delta\phi$ (-1.) even $\Delta(X/C)$ (1.) given (X/C)
25			Panel lateral vortex indicator (0.) even $\Delta\eta$ (-1.) even $\Delta\theta$ (1.) given η
37			X_{P_o} Panel Origin (units)
49		73 80	Y_{P_o} Panel Origin (units)
61			Z_{P_o} Panel Origin (units)
1	3435		
13			Area of influence in longitudinal direction
25			Area of influence in lateral direction
37			No. of longitudinal subpanels (max 40)
49		73 80	No. of lateral subpanels (max 40)
61			No. of longitudinal constraint functions (max 15)
1	3440		
13			No. of lateral constraint functions (max 20)
25			No. of longitudinal control points (max 40)
37			No. of lateral control points (max 40)
49		73 80	No. of points defining panel perimeter (max 50)
61			No. of body or panel to which inboard trailing vortex is attached
1	3445		
13			No. of trailing vortex to which panel inboard trailing vortex is attached
25			No. of panel to which outboard trailing vortex is attached
37			No. of trailing vortex to which panel outboard trailing vortex is attached
49		73 80	No. of leading edge control surfaces
61			No. of trailing edge control surfaces

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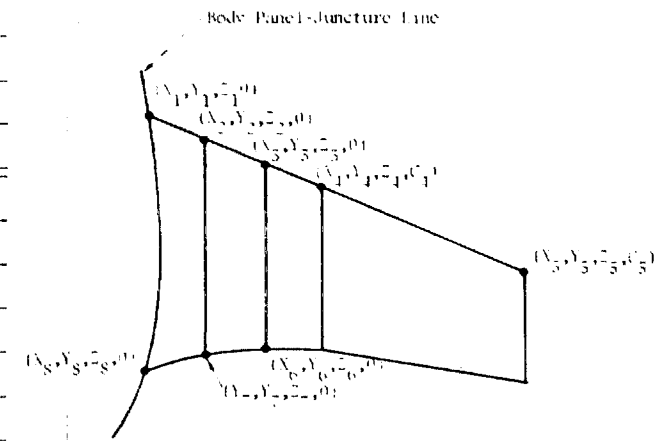
FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ______ OF ______ JOB NO. ______

Column	NUMBER	IDENTIFICATION (73–80)	DESCRIPTION DO NOT KEY PUNCH	
1	3 4 5 0		Panel perimeter description	
13			X_1 (Note: The chord is not given in the root section)	(units)
25			Y_1	(units)
37			Z_1	(units)
49			C_1	(units)
61			X_2	(units)
1	3 4 5 5			
13			Y_2	(units)
25			Z_2	(units)
37			C_2	(units)
49				
61				
1	3 6 0 0		Longitudinal stations where camber is defined	
13			No. of percent chord stations	(max 30)
25			$(X/C)_1$	
37			$(X/C)_2$	
49				
61				
1	3 6 3 0		Lateral stations where camber and twist are defined	
13			No. of percent semi-span stations	(max 30)
25			η_1	
37			η_2	
49				
61				



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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	3 6 6 0	73–80	Mean camber surface definition	
13			$\alpha[(x/c)_1, \eta_1]$ on $Z_c/c\ [(x/c)_1, \eta_1]$	(Rad)
25			$\alpha[(x/c)_2, \eta_1]$ or $Z_c/c\ [(x/c)_2, \eta_1]$	(Rad)
37			⋮ ⋮	
49			$\alpha[(x/c)_1, \eta_2]$ or $Zc/c\ [(x/c)_1, \eta_2]$	(Rad)
61			⋮ ⋮	
1	4 1 0 0	73–80	Twist definition	
13			$\epsilon(\eta_1)$	(Rad)
25			$\epsilon(\eta_2)$	(Rad)
37			⋮	
49				
61				
1	4 1 3 0	73–80	Longitudinal stations where thickness is defined	
13			No. of percent chord stations.	(30)
25			$(x/c)_1$	
37			$(x/c)_2$	
49			⋮	
61				
1	4 1 6 0	73–80	Lateral stations where thickness is defined	
13			No. of percent semi-span stations	(30)
25			η_1	
37			η_2	
49			⋮	
61				

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.____ PROGRAMMER____ DATE____ PAGE____ of____ JOB NO.____

Column	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	4190		Thickness surface definition
13			$Z_t/c\ [(x/c)_1, \eta_1]$
25			$Z_t/c\ [(x/c)_2, \eta_1]$
37			⋮
49		73 80	$Z_t/c\ [(x/c)_1, \eta_2]$
61			⋮
1	4600		Longitudinal subpanel stations
13			$(x/c)_1$
25			$(x/c)_2$
37			⋮
49		73 80	
61			
1	4640		Lateral subpanel stations
13			η_1
25			η_2
37			⋮
49		73 80	
61			
1	4680		Longitudinal control points
13			J_{M1}
25			J_{M2}
37			⋮
49		73 80	
61			

FORM 114-C-17 REV. 7-66

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO._____ PROGRAMMER_____ DATE_____ PAGE___ of___ JOB NO._____

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	4720		Lateral Control Points
13			J_{N_1}
25			J_{N_2}
37			⋮
49		73 80	
61			
1	4760		List of lateral constraint functions
13			W_1 (Note: Standard functions are given by $\eta^w\sqrt{1-\eta^2}$ and
25			W_2 special functions are defined at given input data
37			⋮ 4780 array locations)
49		73 80	4785
61			⋮
1	4780		Special lateral constraint functions
13			Type Indicator "P" Function 0. L.E. Control Surface 1. T.E. Control Surface -1.
25			η_b
37			$\Delta\eta_i$ η_{k_i} η_{f_i}
49		73 80	$\Delta\eta_o$ η_{k_o} η_{f_o}
61			(Note: Use same format for additional special functions)
1	4880		Special longitudinal constraint functions
13			Type Indicator L.E. Control Surface 1. T.E. Control Surface -1.
25			δ (Control Surface Angle) (Deg.)
37			η_{f_i} (Inboard Control Surface Edge)
49		73 80	η_{f_o} (Outboard Control Surface Edge
61			x_i (Inboard L.E. or T.E. Corner of Control Surface) (Units)

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FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

	NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	-	4 8 8 5		Special Longitudinal Constraint Functions Continued
13				x_o (Outboard L.E. or T.E. Corner of Control Surface) (Units)
25				$x_{\eta i}$ (Inboard Edge - Hinge Line Intersection) (Units)
37				$x_{\eta o}$ (Outboard Edge - Hinge Line Intersection) (Units)
49			73 80	Note: Use same format for special longitudinal constraint functions
61				due to additional control surfaces. Start data sets at 4890, 4900, and etc.)
1				
13				
25				
37				
49			73 80	
61				
1				
13				
25				
37				
49			73 80	
61				
1				
13				
25				
37				
49			73 80	
61				

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Input Description

Location	Description
1	Total number of bodies used to represent the configuration
2	Total number of panels used to represent the configuration. This number does not include those due to images when location 3426 has a zero in it.
3	Mach number
* 4	Total configuration reference area.
5	Total configuration longitudinal reference length.
6	Total configuration lateral reference length.
7	X component of center of gravity position vector.
8	Y component of center of gravity position vector.
9	Z component of center of gravity position vector.
10	Angle of attack.
11	Angle of sideslip.
12	Nondimensional roll rate $P/(2V_\infty/b)$.
13	Nondimensional pitch rate $q/(2V_\infty/\bar{c})$.
14	Nondimensional yaw rate $\gamma/(2V_\infty/b)$.
15	The number of the component. The bodies are numbered first and then the panels.
* 16	The body reference area.
17	The body chord.

* Items not needed if data format from reference (52) is used.

Location	Description
18	This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this body. Input a zero to compute new matrices and a minus one if the calculation of the matrices is to be bypassed.
*19	This is a symmetry indicator used to take advantage of both symmetrical geometries and loadings. If both the body geometry and the vortex strength are symmetrical about the X-Z plane input a zero, if only the body geometry is symmetrical and not the vortex strengths input a one, and if neither symmetry exists input a minus one.
*20	This is an indicator used to signify whether cartesian or polar coordinates are used to input the body cross-sections. If polar coordinates are used input a zero and if cartesian coordinates are used input a one.
21	This is an indicator used to signify whether the lateral scaling factors are equal in the Y and Z directions. Input a zero if they are equal and a one if they are not equal. The Z_{B_M} array is not input if $Y_{B_M} = Z_{B_M}$.
*22	This indicator is used to signify whether the body cross-sections are placed perpendicular to the mean camber line or the X axis. Input a zero if the cross-section is to be placed perpendicular to the mean camber line or a one if it is to be placed perpendicular to the X axis.
23	This indicator signifies the type of subpanel spacing used on the body in the longitudinal direction. Input a zero if the subpanels are to be spaced at even increments of ϕ, where $\phi = \cos^{-1}[1-2(x/c)]$, input a minus one if the subpanels are to be spaced at even increments of X, and input a one if the spacing is specified at a given set of X stations. The X stations are to be given in locations 1850 - 2149.
24	This indicator signifies the type of subpanel spacing used on the body in the lateral direction. Input a zero if the subpanels are to be spaced at even increments of θ or $\eta_B = (S/S_{max})$, a one if the subpanel side edges are given at a set of θ or η_B stations, and a minus one if the subpanel edges are at the same lateral stations as the body geometry is defined. The subpanel side edges are input in locations 2200 - 2799.

* Items not needed if data format from reference (52) is used.

Location	Description
25	This is the longitudinal distance, in terms of the length of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a longitudinal distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.
26	This is the lateral distance, in terms of the width of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a lateral distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.
*27	This is the X component of the origin of the body coordinate frame.
*28	This is the Y component of the origin of the body coordinate frame.
*29	This is the Z component of the origin of the body coordinate frame.
30	This is the number of subpanels in the longitudinal direction on the body.
31	This is the number of subpanels in the lateral direction on the body.
32	This is the number of equally spaced divisions the body subpanels are divided into in the longitudinal direction. These divisions are used to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.

* Items not needed if data format from reference (52) is used.

Location	Description
33	This is the number of equally spaced divisions the body subpanels are divided into in the lateral direction. These divisions are used to map the vortex grid closer to the actual body surface and to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.
34	This is the number of functions used to constrain the body surface vorticity in the longitudinal direction. The list of functions used is given in locations 3210-3249. These same functions are used over each longitudinal constraint function segment. The list defining these segments is given in locations 3140-3189. The maximum number of longitudinal constraint functions is 50.
35	This is the number of functions used to constrain the body surface vorticity in the lateral direction. The list of functions used is given in locations 3250-3269. These same functions are used over each lateral constraint function segment. The list defining these segments is given in locations 3190-3209. The maximum number of lateral constraint functions is 20.
36	This is the number of segments in the longitudinal direction over which the body longitudinal constraint functions are defined. The list of subpanels defining the segments is given in locations 3140-3189. The maximum number of segments is 50.
37	This is the number of segments around the circumference of the body over which the lateral constraint functions are defined. The list of subpanels defining the segments is given in locations 3190-3209. The maximum number of segments is 20.
38	This is the number of control points in the longitudinal direction on the body. The maximum number of points in the longitudinal direction is 299. The list of body control points in the longitudinal direction is given in location 2800 - 3099.

* Items not needed if data format from reference (52) is used.

Location	Description
39	This is the number of control points in the lateral direction on the body. The maximum number of points in the lateral direction is 40. This list of control points is given in locations 3100 - 3139.
*40	This is the number of longitudinal stations where the body cross-sections are defined. The maximum number is 44.
*41-89	This is the list of longitudinal stations where the body cross-sections are defined. This list starts at the nose and ends at the aft end if the body is solid. For a flow through body this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.
*85	This is the number of longitudinal stations where the body cross-section lateral stations are defined. The maximum number of these longitudinal stations is 44.
*86-129	This is the list of longitudinal stations where the body cross-section lateral stations are defined. The body cross-section lateral stations are constant between each longitudinal station in this list. If this list is the same as that in locations 41-89, it can be omitted. If the body is solid, this list starts at the nose and ends at the aft end. If the body is a flow through type, this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.
*130	This is the number of lateral stations at the first longitudinal station in the list at locations 91-129. The maximum number of these lateral stations is 39.
*131	This is the list of lateral stations at the first longitudinal station in the list at locations 91-129. These lateral stations where the body is defined can be given by the angle θ, the lateral fraction $[(Y_{B_i} - \Delta Y_B)/Y_{B_M}]$, or fraction of lateral circumferential length $\bar{S}$.
	Note: Lists of lateral stations at the other longitudinal stations, given in the list in locations 91-129, follow this data as shown in the input format.

* Items not needed if data format from reference (52) is used.

Location	Description
*800-1599	In these locations the body cross-section radii or fractional distances $[(Z_{B_i}-\Delta Z_B)/Z_{B_M}]$, for each of the X_{B_i} longitudinal stations and θ_i or $[(Y_{B_i}-\Delta Z_B)/Y_{B_M}]$ lateral stations, are input.
*1600	This is the number of longitudinal stations where the multiplication factors Y_{B_M} and Z_{B_M} and the translation increments ΔY and ΔZ are given. The maximum number of stations that can be used here is 49.
1601-1649	This is the list of longitudinal stations where the multiplication factors Y_{B_M} and Z_{B_M} and the translation increments ΔY_B and ΔZ_B are given.
1650-1699	This is the list of multiplication factors Z_{B_M} at the longitudinal stations given in locations 1601-1649.
1750-1799	This is the list of translation increments ΔY_B at the longitudinal stations given in locations 1601-1649.
*1800-1849	This is the list of translation increments ΔZ_B at the longitudinal stations given in locations 1601-1649.
*1850-2149	This is the list of longitudinal stations where the body subpanel edges are defined.
2150	This is the number of longitudinal stations where the body lateral subpanel edges are defined. The maximum number of these longitudinal stations is 49.
2151-2199	This is the list of longitudinal stations where the body lateral subpanel edges are defined. This list can be omitted if it is the same as that in locations 1850-2149.
2200	This is the list of lateral subpanel edges at the first longitudinal station given in the list at locations 2151-2199. These stations can be defined by θ's or fractions of body circumference $\eta_b = (S/S_{max})$.
	Note; Lists of lateral subpanel edges at the other longitudinal stations, given in the list at locations 2151-2199, follow this input as shown in the input format.

* Items not needed if data format from reference (52) is used.

Location	Description
2800-3099	This is the list of the number of the subpanels where the control points are located in the longitudinal direction on the body. The integer in this list indicates the number of the subpanel, aft of the nose of a solid body or aft of the tail end on the inner surface and around to the outer surface of a flow through body, at which a control point is placed. If no longitudinal constraint functions are used, this input can be omitted.
3100-3139	This is the list of control point locations in the lateral direction on the body. The integer in this list indicates the number of the subpanel in the lateral direction from the top of the body at which a control point is placed. If no lateral constraint functions are used, this input can be omitted.
3140-3189	This is the list of longitudinal constraint segment boundaries. The integers in this list indicate the subpanels between which the longitudinal constraint functions are applied. For example, if this list included the longitudinal number of every tenth subpanel, the longitudinal constraint functions would be applied over a range of ten subpanels and repeated over the segments defined by every tenth subpanel.
3190-3209	This is the list of lateral constraint segment boundaries. This list is utilized in the same manner as the longitudinal constraint segment boundaries.
3210-3249	This is the list of longitudinal constraint functions used on the body. The functions to be used are signified by entering here the sequence number of the function as given in the input format. If special functions are to be used the data location where the function is described is input here.
3250-3269	This is the list of lateral constraint functions used on the body. The same procedure is used here to signify the desired functions as is used in the definition or the longitudinal constraint functions.
3270-3419	In these locations the special longitudinal constraint functions are described as shown in the input format. These are described in tabular form.

* Items not needed if data format from reference (52) is used.

Location	Description
3420	This is the component number for the panels.
*3421	This is the panel reference area.
3422	This is the panel reference chord.
3423	This is the panel lateral reference length.
3425	This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this panel. Input a zero if a new matrix is computed and a minus one if the calculation of the matrix is to be bypassed.
*3426	This is a symmetry indicator used to take advantage of a configuration's symmetry to save input effort and computer time. Input a zero if both the panel geometry and the vortex strengths have images on the port side of the X-Z plane, a one if just the panel geometry has an image, a minus one if neither have an image, and a two if the vortex strengths on the panel and its image are antisymmetric.
3427	This indicator is used to signify whether the panel is attached to the positive normal or negative normal side of another panel. Input a zero if it is attached to the positive normal side and a one if it is attached to the negative normal side.
3428	This indicator is used to signify whether the wake is force free or fixed. Input a zero if the wake is fixed and a one if it is force free. If any panel has a force free wake, the wake or a body it is attached to is also assumed to be force free.
*3429	This indicator signifies whether the mean camber surface of the panel is described in terms or local angles or attack or as fractions of chord. Input a zero if local angles of attack are input and a one if fractions of chord are input.

* Items not needed if data format from reference (52) is used.

Location	Description
3430	This input indicates the type of longitudinal subpanel spacing that is used. If the subpanels are spaced at even increments of ϕ input a zero, if they are at equal increments of percent chord input a minus one, and if they are input as a given set of percent chord use a one.
3431	This input indicates the type of lateral subpanel spacing that is used on a panel. Input a zero if the spacing is at equal increments of η, a minus one if it is at equal increments of Θ, and a one if the spacing is specified at a given set of η stations.
*3432	This is the X component of the panel origin point.
*3433	This is the Y component of the panel origin point.
*3434	This is the Z component of the panel origin point.
3435	This is the area of influence of a quadrilateral vortex on a panel in the longitudinal direction. This input is in terms of fraction of quadrilateral vortex length.
3436	This is the area of influence of a vortex on a panel in the lateral direction. This input is in terms of fraction of vortex width.
3437	This is the number of subpanels in the longitudinal direction on the panel. The maximum number of longitudinal subpanels is 40.
3438	This is the number of subpanels in the lateral direction on the panel. The maximum number of lateral subpanels is 40.
3439	This is the number of longitudinal constraint functions on the panel. This number does not include special functions due to control surfaces. The maximum number is 15.
3440	This is the number of lateral constraint functions on the panel. This number does include special functions due to control surfaces. The maximum number is 20.

* Items not needed if data format from reference (52) is used.

Location	Description
3441	This is the number of control points in the longitudinal direction on the panel. The maximum number is 40.
3442	This is the number of control points in the lateral direction on the panel. The maximum number is 40.
*3443	This is the number of points used to define the panel perimeter. In the root section the perimeter is defined by X, Y, and Z components of points along the leading and trailing edges. The outboard section is defined by X, Y, and Z components of points along the leading edge and the local chord. The root section leading edge is input first, then the outboard section, and then the root section trailing edge. The maximum number of points is 50.
3444	This input is the number of the body or panel to which this panel's inboard trailing vortex is attached.
3445	This is the number of the trailing vortex leg or subpanel side edge to which this panel's inboard trailing vortex is attached. The subpanel side edges are numbered consecutively starting at the top of a body or at the inboard edge of a panel and going in the clockwise direction when an observer is looking in the negative X direction.
3446	This is the number of the panel to which the outboard trailing vortex of this panel is attached.
3447	This is the number of the trailing vortex leg or subpanel side edge to which this panel's outboard trailing vortex is attached.
3448	This is the number of leading edge control surfaces on the panel.
3449	This is the number of trailing edge control surfaces on the panel.
*3450-3599	In these locations the panel perimeter is described.

* Items not needed if data format from reference (52) is used.

Location	Description
* 3600	This is the number of longitudinal stations where the panel camber is described. The maximum number is 30.
*3601-3629	This is the list of chord fractions which define the chordwise locations where the panel camber is defined.
* 3630	This is the number of lateral stations where the panel camber and twist is defined. The maximum number is 30.
*3631-3659	This is the list of lateral surface length fraction η where the panel camber and twist is defined.
*3660-4099	This is the table of local angle of attack or deflection, in terms of fraction of chord, of the panel mean camber surface. The table is input as shown in the input format.
4100-4129	This is the table of panel twist.
* 4130	This is the number of longitudinal stations where the panel thickness is described. The maximum number is 30.
*4131-4159	This is the list of chord fractions which define the chordwise locations where the panel thickness is defined.
* 4160	This is the number of lateral stations where the panel thickness is defined. The maximum number is 30.
*4161-4189	This is the list of lateral surface length fraction η where the panel thickness is defined.
*4190-4599	This is the table of panel thickness in terms of fraction of chord. The table is input as shown in the input format.
4600-4639	This is the list of longitudinal subpanel edge locations in terms of fraction of chord.
4640-4679	This is the list of lateral subpanel side edge locations in terms of η.
4680-4719	This is the list of panel control points in the longitudinal direction.

* Items not needed if data format from reference (52) is used.

Location	Description
4720-4759	This is the list of panel control points in the lateral direction.
4760-4779	This is the list of exponents W for the standard lateral constraint functions or data locations of the special lateral constraint functions.
4780-4879	In these locations the special panel lateral constraint functions are described. The input is as shown in the input format.
4880 - ...	In these locations the special panel longitudinal constraint functions are described. The input is as shown in the input format.

* Items not needed if data format from reference (52) is used.

SAMPLE INPUT

1 1.0	4.0	0.0	28800.0	
5 123.23	268.32	339.1675	0.0	0.0
10 5.0				
151.	28800.0	580.0	0.0	-1.0
200.0	1.0	1.0		0.0
25		0.0	0.0	0.0
3028.0	4.0	1.0	7.0	
350.0			28.0	4.0
402.0	0.0	580.0		
1302.0	0.0	360.0		
1332.0	0.0	360.0		
8001.	1.	1.	1.	
160038.0	0.0	3.0	6.0	10.0
160515.0	20.0	30.0	40.0	50.0
161060.0	70.0	80.0	90.0	100.0
1615110.0	120.0	130.0	140.0	150.0
1620160.0	170.0	180.0	190.0	200.0
1625210.0	220.0	230.0	240.0	250.0
1630260.0	270.0	280.0	290.0	300.0
1635310.0	320.0	400.0	580.0	
16500.0	2.5	4.0	5.5	7.0
16558.25	10.75	12.75	14.50	16.25
166018.0	19.50	21.0	22.25	23.50
166524.50	25.75	26.75	28.0	29.50
167031.0	32.5	34.0	35.0	36.5
167537.5	38.75	39.75	40.5	41.5
168042.0	43.0	43.5	43.75	44.0
168544.0	44.0	44.0		
17000.0	2.5	4.0	5.0	6.5
17058.0	10.25	12.5	14.5	16.25
171018.5	21.0	24.5	26.75	28.0
171529.0	29.5	29.75	30.0	29.75
172029.25	28.75	28.25	27.75	27.50
172527.50	27.50	27.50	27.50	27.50
173027.50	27.50	27.50	27.50	27.50
173527.50	27.50	27.50		
1810 0.5	1.75	3.4	4.5	4.875
1815 4.675	4.55	4.25	3.75	3.15
1820 2.3	1.55	0.65	0.2	

```
1850 0.0         2.6          11.54        27.25        49.0
1855 77.0        110.0        148.0        189.0        233.0
1860 270.0       295.0        308.42       317.761      341.41
1865 374.4       407.39       431.53868    440.38       450.0
1870 460.        471.61       477.28644    492.79075    513.9715
1875 535.1572    550.6566     565.0        580.0
5000  -1.
3420 1.0         7566.58366   123.23       90.218
3435                          6.0          8.0
3440             6.0          8.0          8.0          1.0
3445 2.0
3450 308.42      43.5                                   316.9416
3455 54.9246                               325.4632     66.3492
3460                          333.9848     77.7738
3465 95.6841     376.04       134.16                    35.78
3470 433.23925   66.4965                                436.8096
3475 55.21925                              440.38       43.942
413028.0        .0           0.001        0.0025       0.005
41350.0075      0.0125       0.025        0.050        0.075
41400.10        0.15         0.20         0.25         0.30
41450.35        0.40         0.45         0.50         0.55
41500.60        0.65         0.67         0.70         0.75
4155 0.8         0.85         0.90         1.0
41602.0         0.0          1.0
4190.0          0.00402      0.00617      0.00824      0.00947
41950.01048     0.01124      0.01263      0.01395      0.01523
42000.01769     0.02001      0.02183      0.02317      0.02415
42050.02476     0.02499      0.02487      0.02437      0.02345
42100.02174     0.02085      0.019409     0.017008     0.014606
4215 0.012205    0.009803     0.005        0.0          0.00402
42200.00617     0.00824      0.00947      0.01048      0.01124
42250.01263     0.01395      0.01523      0.01769      0.02001
42300.02183     0.02317      0.02415      0.02476      0.02499
42350.02487     0.02437      0.02345      0.02174      0.02085
42400.019409    0.017008     0.014606     0.012205     0.009803
42450.005
5000  -1.
3420 2.0         3722.7580    123.23       67.662
3435                          6.0          6.0
```

```
3440              6.0         6.0         8.0          1.0
3445 2.0
3450 471.61       43.942                               484.8699
3455 54.834      -2.918                   498.1298     65.726
3460-5.836                   511.3898     76.618      -8.754
3465 56.4264      553.9793    109.294    -17.51        25.28
3470 564.0019     65.726     -5.836                    560.1875
3475 54.834      -2.918                   556.3731     43.942
413023.0         0.0         0.001       0.0025       0.0050
41350.0075       0.0125      0.025       0.050        0.075
41400.10         0.15        0.20        0.250        0.30
41450.35         0.40        0.45        0.50         0.55
41500.60         0.75        0.90        1.00
41602.0          0.0         1.0
41900.0          0.00348     0.00538     0.00728      0.00847
41950.00969      0.01052     0.01206     0.01353      0.01495
42000.01768      0.02002     0.02184     0.02318      0.02417
42050.02476      0.02499     0.02492     0.02460      0.02346
42100.01653      0.00961     0.00498     0.0          0.00348
42150.00538      0.00728     0.00847     0.00969      0.01052
42200.01206      0.01353     0.01495     0.01768      0.02002
42250.02184      0.02318     0.02417     0.02476      0.02499
42300.02492      0.02460     0.02346     0.01653      0.00961
42350.00498
5000  -1.
3420 3.0          5864.375    123.23      55.0
3425             -1.0
3435                          6.0         5.0
3440              6.0         5.0         8.0          1.0
3445 1.0
3450 451.5                    27.5                     457.8503
3455              38.5                    464.2006
3460 49.5                     470.5509                 60.5
3465 103.4491     483.25                  82.5         90.75
3470 574.0                    49.5                     574.0
3475              38.5                    574.0
3480 27.5
4130 5.0          0.0         0.25        0.5          0.75
4135 1.0
```

```
4160 2.0           0.0          1.0
4190 0.00408       0.0259575    0.047835     0.0697125    0.09159
4195 0.00551       0.0273825    0.049255     0.0711275    0.093
5000  -1.
3420 4.0           5024.12      123.23       46.0
3425              -1.0
3435                            6.0          5.0
3440               6.0          5.0          8.0          1.0
3445 3.0
3450 451.5                     -27.5                      456.8121
3455              -36.7                      462.1242
3460-45.9                       467.4362                 -55.1
3465 106.5638      478.06                   -73.5         95.94
3470 462.1242                  -45.9                      456.8121
3475              -36.7                      451.5
3480-27.5
4130 5.0           0.0          0.25         0.5          0.75
4135 1.0
4160 2.0           0.0          1.0
4190 0.00408       0.0259575    0.047835     0.0697125    0.09159
4195 0.00521       0.0270825    0.048955     0.0708275    0.0927
5000  -1.
```

SAMPLE OUTPUT

BODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 1

LATERAL STATION 1

NO	XQ	YQ	ZQ	VM/V	VT/V	CP
1	1.367594	.885863	.887939	.590518	-.068040	.646659
2	5.909044	2.742717	2.642261	1.036237	-.096102	-.083023
3	14.023102	4.689037	4.410988	1.059708	-.100525	-.133086
4	25.607729	6.823918	6.481009	1.070975	-.099997	-.156986
5	40.517242	8.986607	8.847891	1.064636	-.091717	-.141862
6	58.564146	11.263331	11.314363	1.025605	-.071717	-.057008
7	79.521490	13.655558	16.967221	1.084864	-.049334	-.179363
8	103.125723	15.920157	23.280504	1.162270	-.082142	-.357618
9	129.080009	17.959463	25.290380	1.134511	-.138510	-.306301
10	157.057956	20.478761	24.134684	1.099374	-.170262	-.237612
11	186.707727	23.454137	20.871473	1.047057	-.177496	-.127833
12	217.656459	26.221138	19.432546	1.052650	-.146017	-.129392
13	249.514952	28.514243	19.323167	1.067546	-.135301	-.157960
14	281.882568	30.266977	19.323167	1.075833	-.134304	-.175454
15	314.352265	30.869521	19.323167	1.082484	-.113127	-.184570
16	346.515719	30.917069	19.323167	1.068542	-.049121	-.144195
17	377.968455	30.917069	19.323167	1.046688	-.034994	-.096781
18	408.314936	30.917069	19.323167	1.026233	-.029783	-.054041
19	437.173539	30.917069	19.323167	1.010593	-.026822	-.022018
20	464.181349	30.917069	19.323167	1.002017	-.021138	-.004485
21	488.998727	30.917069	19.323167	1.000092	-.011672	-.000321
22	511.313581	30.917069	19.323167	.999202	-.006057	.001558
23	530.845289	30.917069	19.323167	.997655	-.004160	.004667
24	547.348227	30.917069	19.323167	.996394	-.005571	.007169
25	560.614862	30.917069	19.323167	.997776	-.010173	.004339
26	570.478359	30.917069	19.323167	1.002328	-.013911	-.004855
27	576.814677	30.917069	19.323167	1.002574	-.013173	-.005329
28	579.544135	30.917069	19.323167	.998205	-.011736	.003450

LATERAL STATION 2

NO	XQ	YQ	ZQ	VM/V	VT/V	CP
1	1.367594	.885863	-.887939	.470534	-.063820	.774525
2	5.909044	2.742717	-2.642261	.912473	-.090469	.159209
3	14.023102	4.689037	-4.410988	.983299	-.101659	.022790
4	25.607729	6.823918	-6.481009	1.013680	-.109462	-.039529
5	40.517242	8.986606	-8.859000	1.034137	-.103806	-.080215
6	58.564146	11.263330	-11.284960	1.058615	-.091148	-.128974
7	79.521490	13.655557	-13.047635	1.065652	-.087291	-.143233
8	103.125723	15.920156	-14.403721	1.057658	-.093260	-.127339
9	129.080009	17.959462	-16.019709	1.049375	-.103364	-.111872
10	157.057956	20.478760	-17.601603	1.053666	-.104389	-.121109
11	186.707727	23.454135	-19.014641	1.071217	-.100577	-.157622
12	217.656459	26.221136	-19.318038	1.063202	-.092699	-.138991
13	249.514952	28.514240	-19.323169	1.050094	-.095140	-.111748
14	281.882568	30.266975	-19.323169	1.037424	-.103187	-.086895
15	314.352265	30.869518	-19.323169	1.004328	-.091440	-.017037
16	346.515719	30.917066	-19.323169	.988452	-.029404	.022097
17	377.968455	30.917066	-19.323169	.988548	-.016340	.022507
18	408.314936	30.917066	-19.323169	.990491	-.013453	.018747
19	437.173539	30.917066	-19.323169	.989040	-.016152	.021539
20	464.181349	30.917066	-19.323169	.985564	-.020544	.028242
21	488.998727	30.917066	-19.323169	.982855	-.018533	.033652
22	511.313581	30.917066	-19.323169	.982864	-.015374	.033742
23	530.845289	30.917066	-19.323169	.985097	-.013962	.029390
24	547.348227	30.917066	-19.323169	.987694	-.014689	.024245
25	560.614862	30.917066	-19.323169	.987333	-.017740	.024856
26	570.478359	30.917066	-19.323169	.982742	-.019734	.033828
27	576.814677	30.917066	-19.323169	.980573	-.017752	.038162
28	579.544135	30.917066	-19.323169	.985343	-.015788	.028851

LATERAL STATION . 3

NO	XQ	YQ	ZQ	VM/V	VT/V	CP
1	1.367594	-.885863	-.887939	.470531	.054860	.775591
2	5.909044	-2.742718	-2.642261	.912469	.106063	.156151
3	14.023102	-4.689037	-4.410987	.983294	.108061	.021455
4	25.607729	-6.823919	-6.481008	1.013675	.110225	-.039686
5	40.517242	-8.986607	-8.858998	1.034131	.106109	-.080686
6	58.564146	-11.263332	-11.284958	1.058608	.092209	-.129153
7	79.521490	-13.655559	-13.047633	1.065645	.081929	-.142311
8	103.125723	-15.920158	-14.403718	1.057653	.092739	-.127230
9	129.080009	-17.959465	-16.019706	1.049371	.103859	-.111966
10	157.057956	-20.478763	-17.601599	1.053663	.104291	-.121082
11	186.707727	-23.454139	-19.014638	1.071213	.100677	-.157633
12	217.656459	-26.221140	-19.318035	1.063194	.092515	-.138941
13	249.514952	-28.514245	-19.323166	1.050077	.095113	-.111707
14	281.882568	-30.266980	-19.323166	1.037381	.103051	-.086779
15	314.352265	-30.869523	-19.323166	1.004228	.104047	-.019299
16	346.515719	-30.917072	-19.323166	.988233	.089126	.015453
17	377.968455	-30.917072	-19.323166	.988038	.088269	.015989
18	408.314936	-30.917072	-19.323166	.989610	.091745	.012256
19	437.173539	-30.917072	-19.323166	.989461	.096468	.011660
20	464.181349	-30.917072	-19.323166	.987045	.101795	.015381
21	488.998727	-30.917072	-19.323166	.983701	.103668	.021585
22	511.313581	-30.917072	-19.323166	.983235	.103617	.022512
23	530.845289	-30.917072	-19.323166	.985181	.104204	.018560
24	547.348227	-30.917072	-19.323166	.987455	.105814	.013735
25	560.614862	-30.917072	-19.323166	.986829	.108429	.014412
26	570.478359	-30.917072	-19.323166	.982309	.110354	.022891
27	576.814677	-30.917072	-19.323166	.980352	.108860	.027059
28	579.544135	-30.917072	-19.323166	.985285	.106955	.017773

LATERAL STATION 4

NO	XO	YO	ZO	VM/V	VT/V	CP
1	1.367594	-.885863	.887940	.590515	.059083	.647801
2	5.909044	-2.742717	2.642261	1.036232	.111706	-.086256
3	14.023102	-4.689036	4.410989	1.059701	.106943	-.134403
4	25.607729	-6.823917	6.481010	1.070963	.100787	-.157120
5	40.517242	-8.986605	8.847892	1.064618	.094061	-.142259
6	58.564146	-11.263329	11.314365	1.025575	.072831	-.057108
7	79.521490	-13.655555	16.967224	1.084819	.044029	-.178770
8	103.125723	-15.920154	23.280508	1.162234	.081644	-.357453
9	129.080009	-17.959460	25.290384	1.134504	.139019	-.306426
10	157.057956	-20.478758	24.134688	1.099378	.170194	-.237598
11	186.707727	-23.454133	20.871477	1.047062	.177646	-.127898
12	217.656459	-26.221134	19.432549	1.052651	.145892	-.129359
13	249.514952	-28.514238	19.323171	1.067553	.135336	-.157985
14	281.882568	-30.266972	19.323171	1.075863	.134234	-.175501
15	314.352265	-30.869515	19.323171	1.082570	.125806	-.187786
16	346.515719	-30.917064	19.323171	1.068742	.108925	-.154075
17	377.968455	-30.917064	19.323171	1.047171	.107010	-.108019
18	408.314936	-30.917064	19.323171	1.027078	.108169	-.066589
19	437.173539	-30.917064	19.323171	1.010119	.107428	-.031881
20	464.181349	-30.917064	19.323171	1.000460	.102497	-.011426
21	488.998727	-30.917064	19.323171	.999125	.096834	-.007628
22	511.313581	-30.917064	19.323171	.998725	.094212	-.006328
23	530.845289	-30.917064	19.323171	.997547	.094273	-.003987
24	547.348227	-30.917064	19.323171	.996646	.096565	-.002628
25	560.614862	-30.917064	19.323171	.998281	.100756	-.006717
26	570.478359	-30.917064	19.323171	1.002773	.104440	-.016461
27	576.814677	-30.917064	19.323171	1.002819	.104177	-.016499
28	579.544135	-30.917064	19.323171	.998267	.102788	-.007102

BODY SECTIONAL LOADS FOR COMPONENT 1

X/C	CXW/WAVG	CYH/HAVG	CZW/WAVG
.001572	.050144	.000047	.003062
.007840	.029743	-.000190	.019492
.020297	-.006168	-.000362	.032213
.038787	-.017519	-.000101	.035236
.063077	-.024219	-.000166	.029833
.092861	-.022710	-.000183	-.020046
.127765	-.050716	.000482	.000689
.167351	-.090340	.000218	.120171
.211120	-.043493	-.000174	.151413
.258521	-.025014	-.000026	.118753
.308959	-.016557	-.000035	.001799
.361800	-.016400	.000035	-.020793
.416379	-.015693	.000014	.040604
.472009	-.011005	.000174	.103462
.527991	-.002040	-.001718	.207591
.583621	.000000	-.007340	.238464
.638200	-0.000000	-.008775	.185028
.691041	-.000000	-.009435	.120487
.741479	.000000	-.009973	.069956
.788880	.000000	-.010114	.043494
.832649	-.000000	-.009853	.042609
.872235	.000000	-.009701	.043201
.907139	-.000000	-.009795	.035241
.936923	-0.000000	-.010084	.025196
.961213	-.000000	-.010683	.025866
.979703	-0.000000	-.011288	.049624
.992160	.000000	-.011363	.061495
.998428	-.000000	-.010976	.039674

TOTAL BODY LOADS

CX	CY	CZ	CMX	CMY	CMZ	CDI
-.019493	-.004430	.114347	-.000000	.000315	-.000030	.019155

X/C CP	Y/C CP	Z/C CP	SX	SY	SZ
.582145	-.000046	-.000455	2086.334115	29548.720723	41111.352224

PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 2

ETA= .062500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.181120	.656489	-.311229	.406818	-.491906	.403521	.895427
.204247	1.081112	.866413	-.296867	-.036583	-.256934	.247990	.504924
.437500	1.069295	.979515	-.220368	-.103482	-.191954	.029841	.221795
.687500	1.025080	.978685	-.168749	-.090304	-.079265	.034021	.113286
.887260	1.087280	1.067276	-.101465	-.031939	-.192473	-.140099	.052374
.983253	.952089	.946204	-.077220	.002996	.087564	.104688	.017125

ETA= .187500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.263527	.773080	-.222740	.195780	-.646114	.364017	1.010131
.204247	1.118429	.874160	-.144025	.082749	-.271627	.228997	.500624
.437500	1.054999	.946643	-.089491	.020618	-.121032	.103443	.224474
.687500	1.027543	.973618	-.066897	.006097	-.060319	.052031	.112351
.887260	1.011453	.988791	-.050859	.010839	-.025625	.022175	.047799
.983253	.995865	.989405	-.055099	.006077	.005217	.021041	.015824

ETA= .312500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.260104	.754962	-.217921	.179592	-.635353	.397779	1.033131
.204247	1.136498	.870238	-.133686	.096616	-.309499	.233351	.542850
.437500	1.061764	.939789	-.083033	.043308	-.134238	.114921	.249159
.687500	1.028805	.968763	-.064228	.026749	-.062564	.060783	.123348
.887260	1.009212	.984526	-.057023	.024083	-.021760	.030130	.051890
.983253	.999502	.992695	-.056792	.024304	-.002230	.013966	.016196

ETA= .437500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.269664	.739954	-.223053	.193428	-.661800	.415054	1.076854
.204247	1.142170	.864343	-.141068	.111018	-.324452	.240586	.565037
.437500	1.065755	.936223	-.098340	.067994	-.145504	.118863	.264367
.687500	1.031067	.967045	-.084377	.055413	-.070219	.061753	.131972
.887260	1.010733	.984547	-.080200	.053367	-.028013	.027820	.055833
.983253	1.000695	.993544	-.080154	.054449	-.007815	.009905	.017720

ETA= .562500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.272811	.734412	-.228597	.204819	-.672303	.418688	1.090992
.204247	1.143606	.861439	-.147200	.122977	-.329502	.242800	.572302
.437500	1.066636	.935267	-.106949	.082802	-.149150	.118420	.267570
.687500	1.031851	.967130	-.095781	.072593	-.073891	.059390	.133281
.887260	1.011707	.985262	-.093491	.071644	-.032291	.024125	.056416
.983253	1.001689	.994466	-.094099	.073051	-.012235	.005700	.017936

ETA= .687500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.279087	.726329	-.231136	.211646	-.689488	.427652	1.117140
.204247	1.145863	.858036	-.147972	.128280	-.334899	.247319	.582217
.437500	1.066623	.935111	-.109870	.090311	-.149757	.117412	.267168
.687500	1.031729	.967925	-.101516	.082613	-.074769	.056296	.131066
.887260	1.012103	.986104	-.100526	.082487	-.034457	.020795	.055252
.983253	1.002354	.995242	-.101382	.083853	-.014992	.002463	.017455

ETA= .812500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.283194	.720877	-.233841	.217733	-.701268	.432929	1.134197
.204247	1.144662	.858396	-.154770	.138599	-.334204	.243948	.578152
.437500	1.062995	.938607	-.125686	.109654	-.145754	.106993	.252747
.687500	1.029330	.970847	-.123072	.107472	-.074668	.045906	.120574
.887260	1.011507	.987656	-.124080	.109013	-.038542	.012653	.051195
.983253	1.002645	.996076	-.125143	.110385	-.020957	-.004353	.016604

ETA= .937500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.268775	.734306	-.238602	.225210	-.666721	.410076	1.076796
.204247	1.126214	.876222	-.192559	.179171	-.305438	.200132	.505570
.437500	1.048174	.953330	-.196325	.183057	-.137213	.057653	.194866
.687500	1.021807	.978769	-.205868	.192867	-.086472	.004814	.091286
.887260	1.008961	.990936	-.212099	.199405	-.062989	-.021716	.041273
.983253	1.002336	.997257	-.214699	.202180	-.050773	-.035399	.015375

PANFL SECTIONAL LOADS FOR COMPONENT 2

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.062500	0.000000	-.000000	.443564	.443564	.001636	-.114514	-.000000
.187500	0.000000	-.000000	.416712	.416712	.001943	-.103769	-.000000
.312500	0.000000	-.000000	.384872	.384872	.001593	-.098811	-.000000
.437500	0.000000	-0.000000	.365286	.365286	0.000000	-.093836	0.000000
.562500	0.000000	-0.000000	.320442	.320442	0.000000	-.082251	0.000000
.687500	0.000000	-0.000000	.274300	.274300	.000000	-.069529	0.000000
.812500	0.000000	-0.000000	.220765	.220765	0.000000	-.054293	0.000000
.937500	0.000000	-0.000000	.149743	.149743	0.000000	-.034260	0.000000

ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.062500	.258168	.003689	.000000	0.000000	0.000000	0.000000	0.000000
.187500	.249019	.004662	.000000	0.000000	0.000000	0.000000	0.000000
.312500	.256738	.004138	.000000	0.000000	0.000000	0.000000	0.000000
.437500	.256883	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.562500	.256680	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.687500	.253479	.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.812500	.245932	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.937500	.228791	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TOTAL PANEL LOADS

CX	CY	CZ	CMX	CMY	CMZ
0.000000	-.000000	.321910	.212417	-.054188	-.000000

CDT=0	CDLI	CDI	CT	CDTI
0.000000	0.000000	.013985	0.000000	0.000000

X/C CP	Y/C CP	Z/C CP
2.920645	.659864	.000000

PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 3

ETA= .083333

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.025054	.956949	-.095312	.022608	-.059820	.083737	.143557
.204247	1.006177	.959123	-.023159	.057871	-.012929	.076734	.089663
.437500	.995129	.966151	-.016503	.026161	.009447	.065867	.056420
.687500	.996815	.977835	-.001801	.014707	.006358	.043623	.037265
.887260	.995186	.986316	.006937	.004910	.009558	.027157	.017599
.983253	.995323	.992755	.007548	.001089	.009275	.014437	.005162

ETA= .250000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.041796	.941670	-.010696	.095229	-.085454	.104190	.189644
.204247	1.025326	.959699	.022339	.067095	-.051793	.074477	.126270
.437500	1.013987	.972254	.050509	.045808	-.030721	.052625	.083345
.687500	1.006866	.980639	.065026	.032162	-.018008	.037313	.055321
.887260	1.000507	.987937	.073115	.023021	-.006361	.023451	.029812
.983253	.996244	.992552	.076488	.019051	.001648	.014477	.012829

ETA= .416667

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.079304	.908199	.110317	.294157	-.177068	.088647	.265715
.204247	1.042394	.945292	.166965	.250391	-.114462	.043727	.158190
.437500	1.021030	.967689	.201900	.222396	-.083266	.014118	.097384
.687500	1.010246	.979548	.215199	.204890	-.066908	-.001493	.065414
.887260	1.002111	.988479	.219766	.192880	-.052524	-.014293	.038231
.983253	.997383	.993598	.220956	.187760	-.043595	-.022490	.021105

ETA= .583333

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.106353	.883410	.076604	.347538	-.229886	.098803	.328689
.204247	1.055318	.934881	.141176	.284012	-.133627	.045334	.178961
.437500	1.025908	.964988	.175525	.251135	-.083297	.005729	.089026
.687500	1.012533	.979137	.189930	.238172	-.061296	-.015435	.045861
.887260	1.003453	.988858	.197049	.232128	-.045746	-.031723	.014023
.983253	.998361	.994273	.199961	.229706	-.036709	-.041344	-.004634

ETA= .750000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.109203	.882289	-.089254	.205859	-.238297	.179188	.417485
.204247	1.059027	.932806	-.026707	.143932	-.122252	.109158	.231410
.437500	1.028351	.964010	.007470	.110636	-.057562	.058445	.116008
.687500	1.013245	.979697	.019760	.099216	-.027055	.030350	.057405
.887260	1.003666	.989754	.025191	.094422	-.007980	.011472	.019452
.983253	.998732	.994922	.027556	.092344	.001775	.001603	-.000172

ETA= .916667

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.130268	.862901	-.095507	.220482	-.286628	.206789	.493417
.204247	1.065859	.927537	-.032051	.157370	-.137082	.114911	.251993
.437500	1.026608	.967134	-.014929	.140738	-.054147	.044844	.098991
.687500	1.010001	.984119	-.010977	.137271	-.020222	.012666	.032888
.887260	1.002144	.992282	-.009366	.136017	-.004381	-.003125	.001256
.983253	.998607	.995968	-.008639	.135450	.002710	-.010298	-.013008

PANEL SECTIONAL LOADS FOR COMPONENT 3

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.083333	0.000000	.023200	.086572	.089626	.000203	-.027748	.007436
.250000	0.000000	.029254	.109203	.113054	.000714	-.036636	.009814
.416667	0.000000	.029709	.110471	.114396	.000519	-.035766	.009619
.583333	0.000000	.026035	.097158	.100585	0.000000	-.025209	.006755
.750000	0.000000	.026722	.099721	.103240	0.000000	-.026123	.007000
.916667	0.000000	.019853	.074087	.076701	0.000000	-.015315	.004104

ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.083333	.320525	.002183	-.000585	0.000000	0.000000	0.000000	0.000000
.250000	.335482	.006098	-.001633	0.000000	0.000000	0.000000	0.000000
.416667	.323759	.004378	-.001177	0.000000	0.000000	0.000000	0.000000
.583333	.259464	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.750000	.261961	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.916667	.206721	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TOTAL PANEL LOADS

CX	CY	CZ	CMX	CMY	CMZ
0.000000	.025550	.095285	.060188	-.145875	.039115

CDT=0	CDLI	CDI	CT	CDTI
0.000000	0.000000	.004900	0.000000	0.000000

X/C CP	Y/C CP	Z/C CP
4.283246	.589289	-.158016

PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 4

ETA= .100000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	.932949	.932564	-.077877	-.076001	.123541	.124549	.001008
.204247	1.024072	1.024938	-.091636	-.092151	-.057121	-.058990	-.001869
.437500	.985155	.985719	-.086986	-.088775	.021903	.020477	-.001426
.687500	.998520	.998490	-.055047	-.056996	-.000073	-.000231	-.000158
.887260	1.012247	1.011969	-.042618	-.044008	-.026460	-.026018	.000442
.983253	1.057625	1.057486	-.021044	-.021831	-.119014	-.118753	.000261

ETA= .300000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	.953746	.953741	.002525	.002958	.090363	.090370	.000007
.204247	.984768	.985292	-.016717	-.016948	.029953	.028912	-.001041
.437500	.996926	.997314	-.017246	-.017980	.005841	.005042	-.000798
.687500	1.004757	1.004793	-.011580	-.012328	-.009672	-.009761	-.000090
.887260	1.023596	1.023507	-.001439	-.001902	-.047750	-.047571	.000180
.983253	1.072462	1.072425	.006250	.005918	-.150213	-.150131	.000082

ETA= .500000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	.954350	.954499	.029335	.029413	.088356	.088067	-.000288
.204247	.985299	.985656	.015500	.015293	.028946	.028249	-.000697
.437500	.999663	.999929	.014749	.014223	.000456	-.000061	-.000517
.687500	1.008530	1.008589	.017911	.017342	-.017453	-.017553	-.000099
.887260	1.029149	1.029124	.022322	.021885	-.059645	-.059576	.000070
.983253	1.078532	1.078520	.024845	.024437	-.163848	-.163803	.000045

ETA= .700000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	.956354	.956544	.051852	.051796	.082698	.082341	-.000357
.204247	.988521	.988770	.043495	.043272	.020934	.020461	-.000473
.437500	1.001854	1.002031	.044823	.044373	-.005720	-.006036	-.000316
.687500	1.010658	1.010716	.046103	.045593	-.023555	-.023625	-.000070
.887260	1.030562	1.030563	.045263	.044805	-.064108	-.064067	.000041
.983253	1.079426	1.079422	.043048	.042599	-.167013	-.166968	.000045

ETA= .900000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	.968136	.968297	.081258	.081133	.056111	.055819	-.000292
.204247	.994229	.994378	.086369	.086080	.004049	.003802	-.000246
.437500	1.005005	1.005106	.089418	.088908	-.018030	-.018143	-.000112
.687500	1.012164	1.012205	.089343	.088735	-.032458	-.032432	.000025
.887260	1.024652	1.024664	.082011	.081632	-.056638	-.056601	.000038
.983253	1.069703	1.069703	.069102	.068775	-.149039	-.148995	.000044

PANEL SECTIONAL LOADS FOR COMPONENT 4

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.100000	0.000000	.000639	0.000000	-.000639	-.000000	0.000000	.000198
.300000	0.000000	.000420	0.000000	-.000420	-.000000	0.000000	.000120
.500000	0.000000	.000308	0.000000	-.000308	-.000000	0.000000	.000088
.700000	0.000000	.000207	0.000000	-.000207	0.000000	0.000000	.000055
.900000	-.001351	.000086	0.000000	-.000086	0.000000	0.000000	.000011

ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.100000	.310517	0.000000	.000306	0.000000	0.000000	0.000000	0.000000
.300000	.284980	0.000000	.000825	0.000000	0.000000	0.000000	0.000000
.500000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.700000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.900000	.000527	.008250	0.000000	0.000000	0.000000	0.000000	0.000000

TOTAL PANEL LOADS

CX	CY	CZ	CMX	CMY	CMZ
-.000290	.000334	0.000000	-.000125	-.000181	.000424

CDT=0	CDLI	CDI	CT	CDTI
0.000000	0.000000	-.000008	0.000000	0.000000

X/C CP	Y/C CP	Z/C CP
3.476786	.628013	.481290

TOTAL CONFIGURATION LOADS

CX	CY	CZ	CMX	CMY	CMZ
-.0195524	-.0043623	.3081308	-.0000265	-.0647415	-.0000556

(X/CR) CP	(Y/CR) CP	(Z/CR) CP
339.3767286	-.0000970	.0132752

CONCLUSIONS

The theory and program described herein are capable of predicting surface velocities and pressure coefficients and section, control surface, and total configuration loads and moments for a diverse class of airplanes. In particular the source - vortex lattice with second order corrections was shown to agree well with the exact solution for a Karman-Trefftz airfoil section. Also, the quadrilateral vortex element predicted surface velocities which agreed well with the exact solution for a sphere.

Because of the options built into the program simple configurations can be input with a minimum of effort while the capability exists to input complicated configurations in significant detail. The program is general enough to analyze complete configurations at angle of attack and in side slip, pitching motion, rolling motion, yawing motion, and with control surfaces deflected. The configuration can be run in each of these modes to obtain static and rotary stability derivatives or in a combination of the modes to predict the loads and moments on the configuration while in a quasi-steady maneuver. The program also has the capability to account for a free wake while operating in any of the above modes.

Within the scope of this study a significant effort was devoted toward understanding the limitations of and the relationships between the different types of aerodynamic finite elements and lifting surface theories available at present. It has been concluded that those used in this analysis are presently the most efficient and generally available. It was also shown that the spanwise variation of potential form drag due to both thickness and lift are correctly computed by the program described in this report.

Appendix A

NUMERICAL PROCEDURES

Discussions of the prime numerical procedures used within the program are given in this appendix. There are essentially three such procedures; (1) straight line interpolation and extrapolation, (2) controlled deviation interpolation, and (3) Householder's simultaneous equation solution.

For straight line interpolation and extrapolation about two given points (X_1, Y_1) and (X_2, Y_2);

$$Y = \alpha Y_2 + (1-\alpha) Y_1 \tag{1}$$

where

$$\alpha = \frac{X-X_1}{X_2-X_1} \tag{2}$$

The slope $\frac{dY}{dX}$ for this case is given by;

$$\frac{dY}{dX} = \alpha' (Y_2-Y_1) \tag{3}$$

where

$$\alpha' = 1/(X_2-X_1) \tag{4}$$

In the case of the controlled deviation interpolation method (CODIM) parabolae are used to curve fit a set of four given points (X_N-1, Y_N-1), (X_N, Y_N), (X_N+1, Y_N+1), and (X_N+2, Y_N+2) to obtain interpolated Y and dY/dX values for $X_N \leq x \leq X_N+1$. Only that information, relative to this method, which is necessary to judiciously pick input points will be discussed here. A complete derivation is given in reference (20).

One parabola P_1 is fit through (X_{N-1}, Y_{N-1}), (X_N, Y_N), and (X_{N+1}, Y_{N+1}). The other parabola P_2 is fit through (X_N, Y_N), (X_{N+1}, Y_{N+1}), and (X_{N+2}, Y_{N+2}). This curve fitting process involves the solution of two sets of simultaneous equations. If,

$$P_1 = A_1X^2 + B_1X + C_1 \tag{5}$$

and

$$P_2 = A_2X^2 + B_2X + C_2 \tag{6}$$

Then

$$\begin{Bmatrix} A_1 \\ B_1 \\ C_1 \end{Bmatrix} = \begin{bmatrix} X^2_{N-1} & X_{N-1} & 1 \\ X^2_N & X_N & 1 \\ X^2_{N+1} & X_{N+1} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} Y_{N-1} \\ Y_N \\ Y_{N+1} \end{Bmatrix} \tag{7}$$

and

$$\begin{Bmatrix} A_2 \\ B_2 \\ C_2 \end{Bmatrix} = \begin{bmatrix} X^2_N & X_N & 1 \\ X^2_{N+1} & X_{N+1} & 1 \\ X^2_{N+2} & X_{N+2} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} Y_N \\ Y_{N+1} \\ Y_{N+2} \end{Bmatrix} \tag{8}$$

The interpolated values of Y and dY/dX between X_N and X_{N+1} are defined by either P_1, P_2, or a linear combination of P_1 and P_2. The amount of P_1 and P_2 used in the linear combination is determined by comparing both of the parabolae to the straight line.

$$S = \alpha Y_{N+1} + (1-\alpha) Y_N \tag{9}$$

Therefore;

$$\frac{dD}{dX} = \frac{d\alpha}{dX} E_1 + \alpha \frac{dE_1}{dX} - \frac{d\alpha}{dX} E_2 + (1-\alpha) \frac{dE_2}{dX} \tag{18}$$

$$Y = \begin{cases} P_1 & \text{for } X = X_N \\ \dfrac{\alpha E_1 P_2 + (1-\alpha) E_2 P_1}{\alpha E_1 + (1-\alpha) E_2} & \text{for } X_N \quad X \quad X_{N+1} \\ P_2 & \text{for } X = X_{N+1} \end{cases} \tag{19}$$

and

$$\frac{dY}{dX} = \begin{cases} \dfrac{dP_1}{dX} & \text{for } X = X_N \\ \dfrac{dN}{dX} / D - N \dfrac{dD}{dX} / D^2 & \text{for } X_N \quad X \quad X_{N+1} \\ \dfrac{dP_2}{dX} & \text{for } X = X_{N+1} \end{cases} \tag{20}$$

In the case of an end interval $X_{N-1} \leq x \leq X_N$; P_1 is set equal to;

$$P_1 = S + K(P_2 - S) \tag{21}$$

where

$$K = 1 - \frac{\left| |M_1| - |M_2| \right|}{|M_1| + |M_2|} \tag{22}$$

where

$$\alpha = \frac{Y_{N+1} - Y_N}{X_{N+1} - X_N} \tag{10}$$

The parabola which has the least deviation from the straight line is given the greatest weight. The weighting factors E_1 and E_2 are determined as follows:

$$E_1 = \left| P_1 - S \right| \tag{11}$$

$$E_2 = \left| P_2 - S \right| \tag{12}$$

The weighted expression for Y in the range $X_N \leq x \leq X_{N+1}$ is then;

$$Y = \frac{\alpha E_1 P_2 + (1-\alpha) E_2 P_1}{E_1 + (1-\alpha) E_2} \tag{13}$$

The derivative dY/dX in the range $X_N \leq x \leq X_{N+1}$ is then;

$$\frac{dY}{dX} = \frac{dN}{dX} \Big/ D - N\frac{dD}{dX} \Big/ D^2 \quad \text{for } D \neq 0 \tag{14}$$

where

$$N = \alpha E_1 P_2 + (1-\alpha) E_2 P_1 \tag{15}$$

$$D = \alpha E_1 + (1-\alpha) E_2 \tag{16}$$

And then

$$\frac{dN}{dX} = \frac{d\alpha}{dX} E_1 P_2 + \alpha \frac{dE_1}{dX} P_2 + \alpha E_1 \frac{dP_2}{dX} - \frac{d\alpha}{dX} E_2 P_1 + (1-\alpha) \frac{dE_2}{dX} P_1 + E_2 \frac{dP_1}{dX} \tag{17}$$

and

$$M_1 = \frac{Y_N - Y_{N-1}}{X_N - X_{N-1}} \tag{23}$$

$$M_2 = \frac{Y_N - Y_{N+1}}{X_N - X_{N+1}} \tag{24}$$

A similar procedure is followed for the other end interval $X_{N+1} \leq x \leq X_{N+2}$.

Householder's method for solving simultaneous equations is used in the solution of the aerodynamic influence equations. The method is applicable to both square and rectangular influence matrices. In the case of rectangular matrices it is not necessary to least square the equations first, since Householder's procedure least squares and triangularizes simultaneously. Also, the influence matrix is triangularized by means of orthogonal transformation matrices, which preserve the conditioning of the matrix. The combination of these two advantages, along with a reduction in the number of required computer operations, greatly improves the numerical accuracy and stability of the solution over that of the standard Gaussian reduction method.

A complete, but rather abstract, derivation of the method is given in reference (21). The method in the subroutine has been altered from the original to allow the operation on a single row of the matrix at a time. This reduces the required core allocation necessary to triangularize the matrix.

A derivation of the method, developed by the writer, will be given here in order to describe the basic philosophy of the method.

If [A] is the rectangular influence matrix, the upper triangle is given by;

$$[R] = [W]\,[A] \tag{25}$$

where [W] is the combined orthogonal transformation matrix used by Householder to triangularize [A].

The relationship between Householder's triangularized matrix [R] and that obtained by Gaussian elimination of the least squared influence matrix $[A]^T[A]$, is

$$[G] = [T][A]^T[A] = [T][R]^T[R] = [D][R] \tag{26}$$

where [G] is the triangular matrix obtained by Gaussian elimination of $[A]^T[A]$. The matrix [T] is the Gaussian transformation matrix used to triangularize $[A]^T[A]$. And, [D] is a diagonal matrix with the same diagonal as [R]. It can be seen from equation (26) that a nonsquare matrix must be least squared first, before applying the Gaussian transformation [T]. Whereas, the Householder transformation matrix [W] can be applied directly. The least squared matrix $[A]^T[A]$ is usually more illconditioned than [A], and therefore, less accurate results are obtained.

In the Householder method [W] is equal to the product of N+1 individual orthogonal transformation matrices, where n equals the number of unknowns. There are N+1 transformations because the augmented influence matrix, made up of the influence matrix itself plus the boundary condition matrix, added on as the last column, has N+1 columns. Each transformation results in reducing all elements below the diagonal to zero for one column. The columns are reduced from left to right.

The individual transformation matrices $[W]_m$ are defined by;

$$[W]_m = ([I] - 2\ \{u\}_m\ \{u\}^T_m) \tag{27}$$

where [I] is a unit diagonal matrix and $[u]_m$ is a column matrix defined by the unit vector $\vec{u}_m = (\vec{a}_m - \alpha_m \vec{v}_m)/\ \mu_m$. The vector $\vec{a}_m$ is defined by the mth column of [A] where the elements on rows less than m are replaced by zeros. The unit vector $\vec{v}_m$ is defined by a column matrix $[v]_m$ with all zeros except for the mth. row, which is equal to one. The constants α_m and μ_m are defined as,

$$\alpha_m = |\ \{a_m\}\ | \tag{28}$$

$$\mu_m = \sqrt{2\,\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)} \tag{29}$$

It can be shown that $[a_m]$ is reduced to $|\{a_m\}|\{v_m\}$ if $\{a_m\}$ is premultiplied by $([I] - 2\{u_m\}\{u_m\}^T)$. Also, that the first m-1 rows of

$$[W_{m-1}][W_{m-2}] \ \ldots \ [W_1][A] \tag{30}$$

remain unchanged by the mth transformation. The result after m transformations is then zeros below the diagonal for the first m columns and $|\{a_1\}|, |\{a\}_2|, \ldots, |\{a\}_{m-2}|, |\{a\}_{m-1}|, \{a\}_m|$ on the diagonal. The elements above the diagonal have been defined by the m preceding transformations and will remain unchanged for the N+1-m remaining transformations.

$$([I] - 2\{u_m\}\{a_m\}^T)\ \{a_m\} = |\{a_m\}|\{v_m\} \tag{31}$$

$$\vec{u}_m = (\vec{a}_m - \alpha_m \vec{v}_m)/\mu_m \quad \text{or} \quad \{u_m\} = (\{a_m\} - \alpha_m\{v_m\})/\mu_m \tag{32}$$

$$\alpha_m = |\{a_m\}| \tag{33}$$

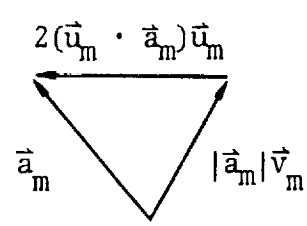
and

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)} \quad \text{or} \quad \mu_m = \sqrt{2\alpha_m(\alpha_m - \{a_m\}^T\{v_m\})} \tag{34}$$

remain to be proved. It is helpful in the derivation of equation (31) if the vector identity

$$|\vec{a}_m|\vec{v} + 2(\vec{u}_m \cdot \vec{a}_m)\ \vec{u}_m = \vec{a}_m \tag{35}$$

is observed from the following vector diagram.



Then from equation (35);

$$|\vec{a}_m|\vec{v}_m + 2\,\vec{u}_m(\vec{u}_m \cdot \vec{a}_m) = \vec{a}_m \tag{36}$$

$$|\vec{a}_m|\vec{v}_m + 2\,\vec{u}_m\vec{u}_m \cdot \vec{a}_m = \vec{a}_m \tag{37}$$

Therefore;

$$|\vec{a}_m|\vec{v}_m = (\hat{i}_m\hat{i}_m - 2\,\vec{u}_m\vec{u}_m)\,\vec{a}_m \tag{38}$$

where $\hat{i}_m\hat{i}_m$ and $\vec{u}_m\vec{u}_m$ are dyadics. The unit vector $\hat{i}_m$ is in the direction of $\vec{a}_m$.

Equation (38) can then be written in matrix notation as follows;

$$([I] - 2\,\{u\}_m\{u\}^T{}_m)\{a\}_m = |\{a\}_m|\{v\}_m \tag{39}$$

which is equal to equation (31). In matrix or tensor notation it becomes evident that the dimensions of $\{a\}_m$, $\{v\}_m$, and $\{u\}_m$ are not limited to three.

$$\alpha_m = |\{a_m\}| \tag{40}$$

and

$$\mu_m = 2\,\{a_m\}^T\{a_m\} \tag{41}$$

Then if equation (31) is premultiplied by $\{a_m\}^T$

$$\{a_m\}^T\{a_m\} - 2\{a_m\}^T\{u_m\}\{u_m\}^T\{a_m\} = |\{a_m\}|\{a_m\}^T\{v_m\} \tag{42}$$

And substituting α_m and μ_m into equation (42).

$$\alpha^2_m - \frac{1}{2}\mu^2_m = \alpha_m \{a_m\}^T\{v_m\} \tag{43}$$

or

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \{a_m\}^T\{v_m\})} \tag{44}$$

In vector notation equation (44) is seen to be equal to;

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)} \tag{45}$$

Also, if equation (44) is substituted back into equation (31)

$$\{a_m\} - \sqrt{2\alpha_m(\alpha_m - \{a_m\}^T\{v_m\})}\,\{a_m\} = \alpha_m\{v_m\} \tag{46}$$

Therefore;

$$\{u_m\} = \frac{\{a_m\} - \alpha_m\{v_m\}}{\sqrt{2\alpha_m(\alpha_m - \{a_m\}^T\{v_m\})}} \tag{47}$$

or in vector notation

$$\vec{u}_m = \frac{\vec{a}_m - \alpha_m \vec{v}_m}{\sqrt{2\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)}} \tag{48}$$

Appendix B

SKEWED SOURCE-VORTEX LATTICE INFLUENCE EQUATIONS

The general form of the skewed source and vortex lattice influence equations was derived in appendices B and C of reference (26), respectively. These equations will be specialized to the case where the inboard and outboard sweep angles for a given horseshoe are equal. The perturbation velocity due to a skewed vortex is then given by the following equations.

$$\frac{U_L}{V_\infty} = \frac{U_V}{V_\infty} \tag{1}$$

$$\frac{V_L}{V_\infty} = \frac{\Delta Y_V \left(\frac{V_V}{V_\infty}\right)}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} - \frac{\Delta Z_V \left(\frac{W_V}{V_\infty}\right)}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} \tag{2}$$

$$\frac{W_L}{V_\infty} = \frac{\Delta Z_V \left(\frac{V_V}{V_\infty}\right)}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} + \frac{\Delta Y_V \left(\frac{W_V}{V_\infty}\right)}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} \tag{3}$$

Where ΔY_V and ΔZ_V are the changes in Y and Z across the width of the horseshoe vortex, respectively.

$$\frac{U_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_{u_V}}{4\pi} \tag{4}$$

$$\frac{V_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_{v_V}}{4\pi} \tag{5}$$

$$\frac{W_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_{w_V}}{4\pi} \tag{6}$$

Where

$$E_{u_V} = \frac{\bar{Z}}{R_2^{\,2}}(I_2 + I_3) \tag{7}$$

$$E_{v_V} = \bar{Z}\left[\frac{(I_1 + 1)}{R_1^{\,2}} - \frac{(I_4 + 1)}{R_3^{\,2}} - \frac{\bar{T}}{R_2^{\,2}}(I_2 + I_3)\right] \tag{8}$$

$$E_{w_V} = \frac{(\bar{Y} - \beta\ y_\nu)(I_4 + 1)}{R_3^{\,2}} - \frac{(\bar{Y} + \beta\ y_\nu)(I_1 + 1)}{R_1^{\,2}} - \frac{(\bar{X} - \bar{Y}\,\bar{T})(I_2 + I_3)}{R_2^{\,2}} \tag{9}$$

Let

$$\text{term 1} = \frac{(I_2 + I_3)}{R_2^{\,2}} \tag{10}$$

$$\text{term 2} = \frac{I_1 + 1}{R_1^{\,2}} \tag{11}$$

$$\text{term 3} = \frac{I_4 + 1}{R_3^{\,2}} \tag{12}$$

Then

$$E_{u_V} = \bar{Z}\ (\text{term 1}) \tag{13}$$

$$E_{v_V} = \bar{Z}\left[-\bar{T}(\text{term 1}) + (\text{term 2}) - (\text{term 3})\right] \tag{13}$$

$$E_{w_V} = -(\bar{X} - \bar{T}\,\bar{Y})(\text{term 1}) - (\bar{Y} + \beta\ y_\nu)(\text{term 2}) + (\bar{Y} - \beta\ y_\nu)(\text{term 3}) \tag{14}$$

Where

$$I_1 = \frac{\bar{X} + T y_\nu}{\left[(\bar{X} + T\ y_\nu)^2 + (\bar{Y} + \beta\ y_\nu)^2 + \bar{Z}^2\right]^{1/2}} = \frac{\bar{X} + T y_\nu}{R_5} \tag{15}$$

$$I_2 = \frac{\bar{Y} + \bar{T}\,\bar{X} + \beta\, y_\nu\ (1 + \bar{T}^2)}{\left[(\bar{X} + T\ y_\nu)^2 + (\bar{Y} + \beta\ y_\nu)^2 + \bar{Z}^2\right]^{1/2}} = \frac{\bar{Y} + \bar{T}\,\bar{X} + \beta\, y_\nu\ (1 + \bar{T}^2)}{R_5} \tag{16}$$

$$I_3 = \frac{\bar{Y} + \bar{T}\,\bar{X} - \beta\, y_\nu\ (1 + \bar{T}^2)}{\left[(\bar{X} - T\ y_\nu)^2 + (\bar{Y} - \beta\ y_\nu)^2 + \bar{Z}^2\right]^{1/2}} = -\ \frac{\bar{Y} + \bar{T}\,\bar{X} - \beta\, y_\nu\ (1 + \bar{T}^2)}{R_4} \tag{17}$$

$$I_4 = \frac{\bar{X} - T\, y_\nu}{\left[(\bar{X} - T\ y_\nu)^2 + (\bar{Y} - \beta\ y_\nu)^2 + \bar{Z}^2\right]^{1/2}} = \frac{\bar{X} - T\, y_\nu}{R_4} \tag{18}$$

$$R_1^2 = (\bar{Y} + \beta y_\nu)^2 + \bar{Z}^2 \tag{19}$$

$$R_2^2 = (\bar{X} - TY)^2 + \bar{Z}^2(1 + \bar{T}^2) \tag{20}$$

$$R_3^2 = (\bar{Y} - \beta y_\nu)^2 + \bar{Z}^2 \tag{21}$$

$$R_4^2 = (\bar{X} - T y_\nu)^2 + (\bar{Y} - \beta y_\nu)^2 + \bar{Z}^2 \tag{22}$$

$$R_5^2 = (\bar{X} + T y_\nu)^2 + (\bar{Y} + \beta y_\nu)^2 + \bar{Z}^2 \tag{23}$$

and

$$\bar{X} = X_q - X_V \tag{24}$$

$$\bar{Y} = \beta \frac{\Delta Y_V(Y_q - Y_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \beta \frac{\Delta Z_V(Z_q - Z_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \tag{25}$$

$$\bar{Z} = -\beta \cdot \frac{\Delta Y_V(Y_q - Y_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \beta \frac{\Delta Z_V(Z_q - Z_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \tag{26}$$

$$y_\nu = 1/2 \sqrt{\Delta Y_V^2 + \Delta Z_V^2} \tag{27}$$

$$\bar{T} = \frac{\tan \Lambda}{\beta} \tag{28}$$

where (X_V, Y_V, Z_V) and (X_q, Y_q, Z_q) are the locations of the influencing point and the point being influenced respectively.

The elements of $[A_X]$, $[A_Y]$, and $[A_Z]$ are computed for a unit strength of Γ/V_∞.

Similarly, the perturbation velocity due to a skewed source line is given by;

$$\frac{U_t}{V_\infty} = \frac{U_S}{V_\infty} \tag{29}$$

$$\frac{V_t}{V_\infty} = \frac{\Delta Y_V \dfrac{V_S}{V_\infty}}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} - \frac{\Delta Z_V \dfrac{W_S}{V_\infty}}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} \tag{30}$$

$$\frac{W_t}{V_\infty} = \frac{\Delta Z_V \dfrac{V_S}{V_\infty}}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} + \frac{\Delta Y_V \dfrac{W_S}{V_\infty}}{\sqrt{\Delta Y_V^{\,2} + \Delta Z_V^{\,2}}} \tag{31}$$

where

$$\frac{U_S}{V_\infty} = \frac{(\Sigma / V_\infty) E u_S}{4\pi} \tag{32}$$

$$\frac{V_S}{V_\infty} = \frac{(\Sigma / V_\infty) E v_S}{4\pi} \tag{33}$$

$$\frac{W_S}{V_\infty} = \frac{(\Sigma / V_\infty) E w_S}{4\pi} \tag{34}$$

and

$$E_{u_s} = \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} (\text{term } 4) + \frac{1}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY)(\text{term } 1) \tag{35}$$

$$E_{v_s} = \frac{1}{\sqrt{1 + \bar{T}^2}} (\text{term } 4) - \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY)(\text{term } 1) \tag{36}$$

$$E_{w_s} = \sqrt{1 + \bar{T}^2}\, \bar{Z} \; (\text{term } 1) \tag{37}$$

and where

$$\text{term } 4 = \frac{1}{R_4} - \frac{1}{R_5} \tag{38}$$

The elements of $[S_X]$, $[S_Y]$, and $[S_Z]$ are computed for a unit strength of Σ / V_∞. The influences of both the vortices and sources at the quarter chord of the subpanel are computed simultaneously due to the similarity in the vortex and source influence equations.

For the case where $|\bar{Z}| \leq 2\, y_\nu$, set $\bar{Z} = 0$ and if both

$$(\bar{X} - TY)^2 / \left[(X + T\, y_\nu)^2 + (\bar{Y} + \beta\, y_\nu)^2 \right] \tag{39}$$

and

$$(\bar{X} - TY)^2 / \left[(X - T\, y_\nu)^2 + (\bar{Y} - \beta\, y_\nu)^2 \right] \tag{40}$$

are less than $(0.08716)^2$, and $|\bar{Y}| > \beta y_\nu$, then use;

$$\frac{1}{2\sqrt{1+\bar{T}^2}} \left| \frac{1}{(\bar{X} - T y_\nu)^2 + (\bar{Y} - \beta y_\nu)^2} - \frac{1}{(\bar{X} + T y_\nu)^2 + (\bar{Y} + \beta y_\nu)^2} \right| \tag{41}$$

in place of (term 1)

If $|\bar{Y}| \le \beta y_\nu$ and $|\bar{X} - TY| < \bar{\ell}_M/4$ set (term 1) equal to zero.

Appendix C

QUADRILATERAL VORTEX INFLUENCE EQUATIONS

The Biot-Savart law can be used to calculate the influence of a finite vortex segment on a point in three-dimensional space. The incremental change in induced velocity at a point in space due to an incremental change in length of a finite vortex is given by the following expression.

$$dq = \frac{K \cos \phi \; d\phi}{4\pi h} \tag{1}$$

where;

- K = Vortex strength
- h = Perpendicular distance from the vortex segment to the point in space.
- ϕ = Angle between the line formed by h and a line from the field point to a point on the vortex segment.
- q = Velocity induced by the finite vortex segment perpendicular to the plane formed by h and the vortex segment.

A vector expression for q can be determined from figure (D-1):

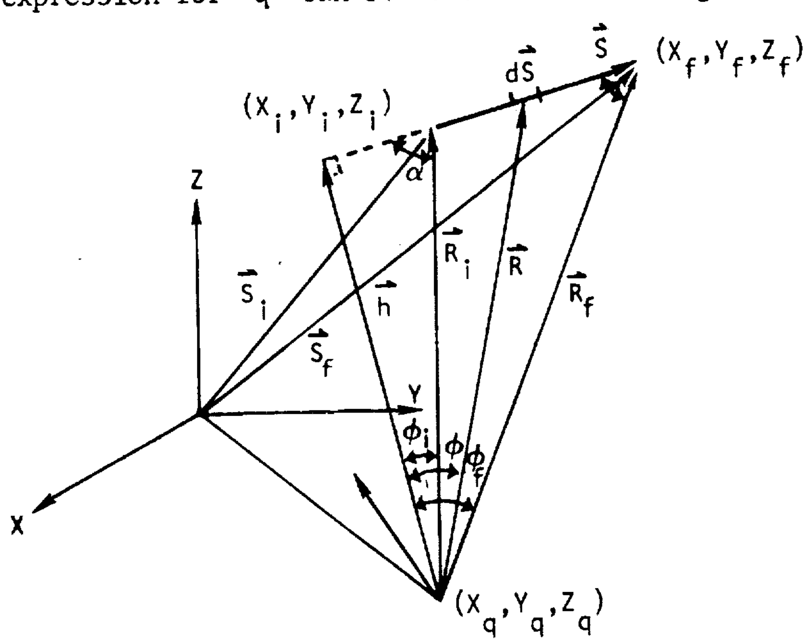


Figure C-1.- Velocity induced by finite vortex segment.

The magnitude of the velocity $\vec{q}$ induced at (X_q, Y_q, Z_q) by the vortex segment $\vec{s}$ is given by the following equation after equation (1) has been integrated from ϕ_i to ϕ_f.

$$\left|\vec{q}\right| = \frac{k}{4\pi h}(\cos\beta - \cos\alpha) \tag{2}$$

where

$$\cos\beta = \frac{\vec{s}\cdot\vec{R}_f}{\left|\vec{s}\right|\left|\vec{R}_f\right|},$$

$$\cos\alpha = \frac{\vec{s}\cdot\vec{R}_i}{\left|\vec{s}\right|\left|\vec{R}_i\right|}.$$

The vector $\vec{h}$ is determined such as it satisfies the conditions of being perpendicular to $\vec{s}$ and equal to the vector sum $\vec{h} = \vec{R}_f - a\vec{s}$ where "a" defines the length of $\vec{h}$.

Since;

$$\vec{h} = \vec{R}_f - a\vec{s} \tag{3}$$

and

$$\vec{h}\cdot\vec{s} = 0 \tag{4}$$

then

$$\vec{h}\cdot\vec{s} = \vec{R}_f\cdot\vec{s} - a\vec{s}\cdot\vec{s} = 0 \tag{5}$$

therefore;

$$a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} \tag{6}$$

After substituting "a" into equation (3), $\vec{h}$ is defined as;

$$\vec{h} = \vec{R}_f - \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} \vec{s} \tag{7}$$

Also, a unit vector $\hat{q}$ in the direction of $\vec{q}$ is seen to be equal to;

$$\hat{q} = \frac{\vec{R}_f \times \vec{s}}{\left|\vec{R}_f \times \vec{s}\right|} \tag{8}$$

The magnitude and direction of $\vec{q}$ are then expressed in terms of the coordinates of the control point (X_q, Y_q, Z_q) and the end points of the vortex segment (X_i, Y_i, Z_i) and (X_f, Y_f, Z_f). If $\hat{i}$, $\hat{j}$, and $\hat{k}$ are defined as unit vectors in the X, Y, and Z directions respectively, then;

$$\begin{aligned} \vec{s} &= \vec{s}_f - \vec{s}_i = (X_f - X_i)\,\hat{i} + \beta\,(Y_f - Y_i)\,\hat{j} + \beta\,(Z_f - Z_i)\,\hat{k} \\ \vec{R}_i &= \vec{s}_i - \vec{Q} = (X_i - X_q)\,\hat{i} + \beta\,(Y_i - Y_q)\,\hat{j} + \beta\,(Z_i - Z_q)\,\hat{k} \end{aligned} \tag{9}$$

and

$$\vec{R}_f = \vec{s}_f - \vec{Q} = (X_f - X_q)\,\hat{i} + \beta\,(Y_f - Y_q)\,\hat{j} + \beta\,(Z_f - Z_q)\,\hat{k} \tag{10}$$

The value of "a" is then expressed as;

$$a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} = \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \tag{11}$$

and the components of $\vec{h}$ by,

$$h_x = (X_f - X_q) - \left[\frac{(X_f - X_q)(X_f - X_i) + (Y_f - Y_q)(Y_f - Y_i)\beta^2 + (Z_f - Z_q)(Z_f - Z_i)\beta^2}{(X_f - X_i)^2 + (Y_f - Y_i)^2\beta^2 + (Z_f - Z_i)^2\beta^2}\right](X_f - X_i)$$

$$h_y = (Y_f - Y_q) - \left[\frac{(X_f - X_q)(X_f - X_i) + (Y_f - Y_q)(Y_f - Y_i)\beta^2 + (Z_f - Z_q)(Z_f - Z_i)\beta^2}{(X_f - X_i)^2 + (Y_f - Y_i)^2\beta^2 + (Z_f - Z_i)^2\beta^2}\right](Y_f - Y_i) \tag{12}$$

$$h_z = (Z_f - Z_q) - \left[\frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2}\right](Z_f - Z_i)$$

The magnitude of $\vec{q}$ is found by substituting

$$h = |\vec{h}| = \sqrt{h_x^2 + h_y^2 + h_z^2}$$

and the following expressions for $\cos\alpha$ and $\cos\beta$ into equation (2).

$$\cos\alpha = \frac{(X_f - X_i)(X_i - X_q) + \beta^2(Y_f - Y_i)(Y_i - Y_q) + \beta^2(Z_f - Z_i)(Z_i - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2}\ \sqrt{(X_i - X_q)^2 + \beta^2(Y_i - Y_q)^2 + \beta^2(Z_i - Z_q)^2}} \tag{13}$$

$$\cos\beta = \frac{(X_f - X_i)(X_f - X_q) + \beta^2(Y_f - Y_i)(Y_f - Y_q) + \beta^2(Z_f - Z_i)(Z_f - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2\ \beta^2(Z_f - Z_i)^2}\ \sqrt{(X_f - X_q)^2 + \beta^2(Y_f - Y_q)^2 + \beta^2(Z_f - Z_q)^2}}$$

The components of the vector $\vec{q}$ are then given by the multiplication of the components of equation (8) by $|q|$.

$$q_X = \frac{|\vec{q}|\left[(Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i)\right]\ \beta^2}{|\vec{R}X\vec{S}|}$$

$$q_X = \frac{|\vec{q}|\left[(X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q)\right]\ \beta^2}{|\vec{R}X\vec{S}|} \tag{14}$$

$$q_X = \frac{|\vec{q}|\left[(X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i)\right]\ \beta^2}{|\vec{R}X\vec{S}|}$$

where

$$|\vec{R}\times\vec{S}| = \Bigg\{\Big[(Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i)\Big]^2 \beta^2 + \Big[(X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q)\Big]^2 \beta$$

$$+ \Big[(X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i)\Big]\beta^2\Bigg\}^{1/2} \tag{15}$$

The velocity induced at a control point by a vortex segment is then given by equation (14). Since a curved vortex can be represented by a number of straight segments, this equation can be used to compute the induced flow produced by a vortex of arbitrary shape.

The components of velocity induced by a quadrilateral vortex can be written as ratios computed by the product of influence matrices and the vortex strengths.

$$\left\{\frac{u}{V_\infty}\right\} = \Big[\ A_X\ \Big]\left\{\frac{K}{V_\infty}\right\} \tag{16}$$

$$\left\{\frac{v}{V_\infty}\right\} = \Big[\ A_Y\ \Big]\left\{\frac{K}{V_\infty}\right\} \tag{17}$$

and

$$\left\{\frac{w}{V_\infty}\right\} = \Big[\ A_Z\ \Big]\left\{\frac{K}{V_\infty}\right\} \tag{18}$$

where the elements of A_X, A_Y, and A_Z, are computed from the following equations.

$$A_X = \sum \frac{\beta^2\left[\cos\beta \quad - \cos\alpha\right]\left[(Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i)\right]}{4\pi \;|\vec{h}||\vec{R}X\vec{S}|} \tag{19}$$

$$A_Y = \sum \frac{\beta\left[\cos\beta \quad - \cos\alpha\right]\left[(X_f-X_i)(Z_f-Z_q) - (Z_f-Z_i)(X_f-X_q)\right]}{4\pi \;|\vec{h}||\vec{R}X\vec{S}|} \tag{20}$$

and

$$A_Z = \sum \frac{\beta\left[\cos\beta \quad - \cos\alpha\right]\left[(X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i)\right]}{4\pi \;|\vec{h}||\vec{R}X\vec{S}|} \tag{21}$$

The Σ sign indicates that the contributions from all of the sides of the quadrilateral vortex are summed.

Appendix D

WOODWARD'S DISTRIBUTED PANEL INFLUENCE EQUATIONS

The equations will be derived for supersonic flow first, then for subsonic flow in subappendix A.

Preliminaries

Generalized potential function. If

$$\beta^2 \Omega_{xx} = \Omega_{yy} + \Omega_{zz}$$

Then

$$\Omega(x, y, z) = -\frac{1}{2\pi}\frac{\partial}{\partial x}\iint\left(\frac{\partial \Omega}{\partial \nu} + \frac{\partial \Omega'}{\partial \nu'}\right)\sigma \, dS + \frac{1}{2\pi}\frac{\partial}{\partial x}\iint(\Omega - \Omega')\frac{\partial \sigma}{\partial \nu}\, dS \qquad (1)$$

where

$$\sigma = \cosh^{-1}\frac{x-\xi}{\beta\sqrt{(y-\eta)^2 + (z-\zeta)^2}}$$

and

$$\vec{\nu} = \left(-\beta^2 n_1, n_2, n_3\right) = -\vec{\nu}\,'$$

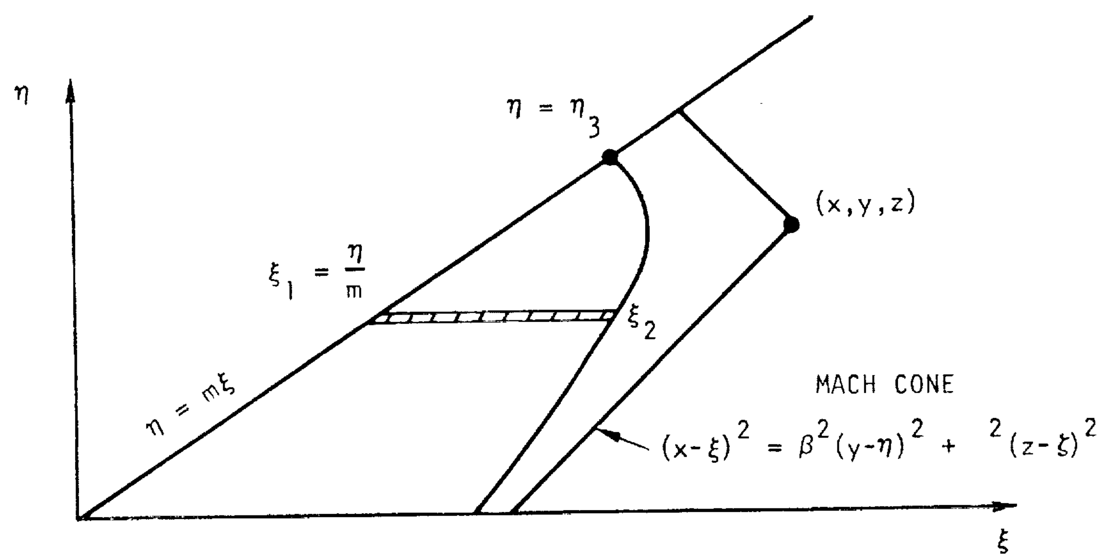
with $n = (n_1, n_2, n_3)$ the unit normal to surface S

Region of integration. - In the (ξ, η, ζ) coordinate system the plane of the semi-infinite triangular surface is determined by $\zeta = a\xi$. The lines $\eta = o$ and $\eta = m\xi$ are the projections in the ξ, η plane of the triangle edges. The area of integration in equation (1) lies on the semi-infinite triangle and is within the Mach forecone from the point (x, y, z) given by $\xi < x$ and $(x-\xi)^2 > \beta^2(y-\eta)^2 + \beta^2(z-\zeta)^2$. The surface integral is carried out by integrating first over ξ and then over η.

The ξ integration goes from the leading edge $\xi = \xi_1 = \eta/m$ to the intersection of the Mach forecone with the semi-infinite triangle, $\xi = \xi_2(\eta)$, where since $\zeta = a\xi$, $(x-\xi_2)^2 = \beta^2(y-\eta)^2 + \beta^2(z-a\xi_2)^2$. The η integration goes from $\eta = o$ to the intersection of the Mach forecone the leading edge where $\eta = \eta_3$. Thus,

$$\left(x - \frac{\eta_3}{m}\right)^2 = \beta^2\left(y - \eta_3\right)^2 + \beta^2\left(z - \frac{a\eta_3}{m}\right)^2.$$

Looking down on ζ axis



Thus

$$\xi_1 = \frac{\eta}{m} \tag{2}$$

$$\left(x - \xi_2\right)^2 = \beta^2(y - \eta)^2 + \beta^2 (z - a\xi_2)^2 \tag{3}$$

$$(\eta_3 - mx)^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 (a\eta_3 - mz)^2 \tag{4}$$

These relations may be manipulated into other forms.

$$\left[(\eta_3 - y) - (mx - y)\right]^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 \left[a(\eta_3 - y) + (ay - mz)\right]^2$$

or

$$(\eta_3 - y)^2 \left|1 - \beta^2(a^2 + m^2)\right| - 2\left|(mx - y) + \beta^2 a(ay - mz)\right|(\eta_3 - y)$$

$$+ (mx - y)^2 - \beta^2 (ay - mz)^2 = 0$$

Thus on the forecone

$$\left|1 - \beta^2(a^2 + m^2)\right| (\eta_3 - y) = (mx - y) + \beta^2 a(ay - mz)$$

$$- \sqrt{\left|(mx - y) + \beta^2 a(ay - mz)\right|^2 - \left|1 - \beta^2(a^2 + m^2)\right|\left|(mx - y)^2 - \beta^2 (ay - mz)^2\right|}$$

The following relation may be used to manipulate this further.

$$\left|m(x - \beta^2 az) - y(1 - \beta^2 a^2)\right|^2 \equiv \left|(mx - y) + \beta^2 a(ay - mz)\right|^2$$

$$\equiv \beta^2 m^2 (z - ax)^2 + (1 - \beta^2 a^2)\left|(mx - y)^2 - \beta^2 (ay - mz)^2\right| \tag{5}$$

Thus

$$\left|1 - \beta^2(a^2 + m^2)\right|\eta_3 = m(x - \beta^2 az) - \beta^2 m^2 y$$

$$- \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} \tag{6}$$

Or starting from (4) again after multiplying by $(1 - \beta^2 a^2)$

$$(1 - \beta^2 a^2)(\eta_3 - mx)^2 = \beta^2 m^2(1 - \beta^2 a^2)(\eta_3 - y)^2$$

$$+ \beta^2(1 - \beta^2 a^2)\left|a(\eta_3 - mx) - m(z - ax)\right|^2$$

which becomes

$$(1 - \beta^2 a^2)^2(\eta_3 - mx)^2 + 2\beta^2 am(z - ax)(1 - \beta^2 a^2)(\eta_3 - mx)$$

$$+ \left|\beta^2 am\right|^2(z - ax)^2 = \beta^2 m^2\left|(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2\right|$$

or

$$\left|(1 - \beta^2 a^2)(\eta_3 - mx) + \beta^2 am(z - ax)\right|^2$$

$$= \beta^2 m^2\left|(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2\right|$$

or

$$\left|\eta_3(1 - \beta^2 a^2) - m(x - \beta^2 az)\right|^2 = \beta^2 m^2 \left|(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2\right| \tag{7}$$

Surface Distribution of Sources

Boundary conditions. - For a distribution of sources on the plane $\zeta = a\xi$ we will assume

$$\text{(a)} \quad \phi = \phi' \tag{8}$$

$$\text{(b)} \quad \frac{\partial \phi}{\partial \nu} + \frac{\partial \phi'}{\partial \nu'} = \frac{\partial \phi}{\partial \xi}\nu_1 + \frac{\partial \phi}{\partial \zeta}\nu_3 + \frac{\partial \phi'}{\partial \xi}\nu_1' + \frac{\partial \phi'}{\partial \zeta}\nu_3'$$

$$= (u - u')\nu_1 + (w - w')\nu_3$$

$$= \frac{1}{\sqrt{1 + a^2}}\left[\beta^2 a(u - u') + (w - w')\right]$$

$$= \frac{2}{\sqrt{1 + a^2}}\left[\bar{w} + \beta^2 a\bar{u}\right] = \text{const}$$

In equation (1), if we set $\Omega = \phi$, then (a) says the second integral vanishes and (b) means the quantity $\partial\phi/\partial\nu + \partial\phi'/\partial\nu'$ may be removed from the integral. These assumptions may be checked after the integration is performed. The statement Woodward makes on the bottom of page 17, of reference (55), $u' = -u$ and $w' = -w$ is not true and not necessary.

Evaluation of the integral over ξ. - The integral which results for a surface distribution of sources is,

$$\phi(x, y, z) = -\frac{w + \beta^2 a\bar{u}}{\pi} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2 (y-\eta)^2 - \beta^2 (z - a\xi)^2}} \, d\eta \tag{9}$$

But

$$\int \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2 (y-\eta)^2 - \beta^2 (z - a\xi)^2}}$$

$$= \frac{-1}{2\sqrt{1-\beta^2 a^2}} \log \frac{(x-\xi) - \beta^2 a (z - a\xi) + \sqrt{(1-\beta^2 a^2)\left[(x-\xi)^2 - \beta^2(\eta - y)^2 - \beta^2 (z - a\xi)^2\right]}}{(x-\xi) - \beta^2 a (z - a\xi) - \sqrt{(1-\beta^2 a^2)\left[(x-\xi)^2 - \beta^2(\eta - y)^2 - \beta^2 (z - a\xi)^2\right]}}$$

Which may be verified by differentiating

First we let

$$A = (x-\xi) - \beta^2 a (z - a\xi)$$

$$B^2 = \beta^2 \left[(1-\beta^2 a^2)(\eta - y)^2 + (z - ax)^2\right]$$

$$C^2 = (1 - \beta^2 a^2)\left[(x-\xi)^2 - \beta^2 (\eta - y)^2 - \beta^2 (z - a\xi)^2\right]$$

and we note that

$$A^2 = B^2 + C^2$$

To verify the indefinite integral we must evaluate

$$\frac{\partial}{\partial \xi} \log \frac{A+C}{A-C} = \frac{A'+C'}{A+C} - \frac{A'-C'}{A-C}$$

$$= \frac{2(AC'-A'C)}{A^2-C^2}$$

where $'$ denotes $\frac{\partial}{\partial \xi}$

But since $B' = 0$

$$AA' = CC'$$

or $$C' = \frac{AA'}{C}$$

Therefore

$$\frac{-1}{2\sqrt{1-\beta^2 a^2}} \frac{\partial}{\partial \xi} \log \frac{A+C}{A-C} = -\frac{A'(A^2-C^2)}{C(A^2-C^2)\sqrt{1-\beta^2 a^2}}$$

$$= \frac{\sqrt{1-\beta^2 a^2}}{C}$$

$$= \frac{1}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}}$$

Q.E.D.

Now since from Eq (3)

$$(x - \xi_2)^2 = \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi_2)^2$$

and from Eq (2)

$$\xi_1 = \frac{\eta}{m}$$

$$\int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}}$$

$$= \frac{1}{2\sqrt{1-\beta^2 a^2}} \log \frac{(mx-\eta) - \beta^2 a (mz - a\eta) + \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2\right]}}{(mx-\eta) - \beta^2 a (mz - a\eta) - \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2\right]}}$$

(10)

Therefore from Eq (9)

$\phi(x,y,z) =$

$$- \frac{\bar{w} + \beta^2 a\bar{u}}{2\pi\sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \log \frac{(mx-\eta) - \beta^2 a (mz - a\eta) + \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2\right]}}{(mx-\eta) - \beta^2 a (mz - a\eta) - \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2\right]}} \, d\eta$$

(11)

Evaluation of the Integral over η. - To evaluate equation (11) we must first integrate by parts. If we let,

$$f = (mx-\eta) - \beta^2 a(mz - a\eta)$$

$$g^2 = \beta^2 m^2 \left[(1-\beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right] \tag{12}$$

$$h^2 = (1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2 \right]$$

then $\quad f^2 = g^2 + h^2$

Now let

$$u = \frac{1}{2} \log \frac{f+h}{f-h} \qquad dv = d\eta$$

$$du = \frac{fh' - hf'}{f^2 - h^2}\, d\eta = \frac{fhh' - f'h^2}{h(f^2-h^2)}\, d\eta$$

$$= \frac{fhh' - f'(f^2 - g^2)}{hg^2}\, d\eta = \frac{f[hh' - ff'] + f'g^2}{hg^2}\, d\eta$$

$$= \frac{1}{hg^2}\left[f'g^2 - fgg' \right] d\eta \tag{13}$$

and $\quad v = \eta$

but $\quad gg' = \beta^2 m^2 (1-\beta^2 a^2)(\eta - y)$

$\quad\quad f' = -(1-\beta^2 a^2)$

Now we can write

$$- \eta\,[g^2 f' - g' g f]$$

$$= \eta\,(1 - \beta^2 a^2)\,\beta^2 m^2\,\left|(z - ax)^2 + [m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\,(\eta - y)\right|$$

$$= \beta^2 m^2\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\,[1 - \beta^2 a^2)\,(\eta - y)^2 + (z - ax)^2]$$

$$+ \beta^2 m^2\,y\,(1 - \beta^2 a^2)\,\left|\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\,(\eta - y)\right|$$

$$+ \beta^2 m^2\,(z - ax)^2\,\left|\eta\,(1 - \beta^2 a^2) - [m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\right|$$

$$= \beta^2 m^2\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\,[(1 - \beta^2 a^2)\,(\eta - y)^2 + (z - ax)^2]$$

$$+ \beta^2 m^2\,(1 - \beta^2 a^2)\,\left|\,y\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)] + (z - ax)^2\right|\,(\eta - y)$$

$$+ \beta^2 m^2\,(z - ax)^2\,\left|\,y\,(1 - \beta^2 a^2) - [m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\right|$$

And since from the integration by parts

$$\frac{1}{2}\int \log\frac{f + h}{f - h}\,d\eta = \frac{1}{2}\eta\log\frac{f + h}{f - h} + \int \frac{-\eta\,(g^2 f' - gg' f)}{g^2\sqrt{f^2 - g^2}}\,d\eta$$

We get for the integration by parts

$$\frac{1}{2}\int \log \frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}\,d\eta$$

$$= \frac{\eta}{2}\log \frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

$$+ \left[m(x-\beta^2az)-y(1-\beta^2a^2)\right]\int \frac{d\eta}{\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}}$$

$$-\int \frac{(1-\beta^2a^2)\left\{y\left[m(x-\beta^2az)-y(1-\beta^2a^2)\right]+(z-ax)^2\right\}(\eta-y)+(z-ax)^2\left\{y(1-\beta^2a^2)-\left[m(x-\beta^2az)-y(1-\beta^2a^2)\right]\right\}}{\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}}\,d\eta$$

(14)

The first integral in equation (14) can be evaluated using the relation

$$\int \frac{d\eta}{\sqrt{A\eta^2+2B\eta+C}} = \frac{-1}{2\sqrt{A}}\log\frac{-(A\eta+B)+\sqrt{A\left[A\eta^2+2B\eta+C\right]}}{-(A\eta+B)-\sqrt{A\left[A\eta^2+2B\eta+C\right]}}$$

(15)

In our case expanding the denominator gives

$$A = (1 - \beta^2 a^2)\,[1 - \beta^2 (a^2 + m^2)]$$

$$B = [-m (x - \beta^2 az) + \beta^2 m^2 y]\,(1 - \beta^2 a^2)$$

$$C = m^2 (x - \beta^2 az)^2 - \beta^2 m^2 [y^2 (1 - \beta^2 a^2) + (z - ax)^2]$$

and since we can write

$$-(A\eta + B) = [(mx - \eta) - \beta^2 a (mz - a\eta) + \beta^2 m^2 (\eta - y)]\,(1 - \beta^2 a^2)$$

$$A[A\eta^2 + 2B\eta + C] = (1-\beta^2 a^2)[1-\beta^2(a^2+m^2)]\left\{[m(x-\beta^2 az) - \eta(1-\beta^2 a^2)]^2 - \beta^2 m^2[(1-\beta^2 a^2)(\eta - y)^2 + (z-ax)^2]\right\}$$

$$= [1-\beta^2(a^2+m^2)](1-\beta^2 a^2)^2\left[(mx-\eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2\right]$$

Therefore

$$\int \frac{d\eta}{\sqrt{[m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 \left[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2\right]}}$$

$$= \frac{-1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)]\,(1 - \beta^2 a^2)}} \log \frac{-(A\eta + B) + \sqrt{A[A\eta^2 + 2B\eta + C]}}{-(A\eta + B) - \sqrt{A[A\eta^2 + 2B\eta + C]}} \qquad (16)$$

$$= \frac{-1}{2\sqrt{[1-\beta^2(a^2+m^2)](1-\beta^2 a^2)}} \log \frac{(mx-\eta) - \beta^2 a(mz - a\eta) + \beta^2 m^2(\eta - y) + \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}{(mx-\eta) - \beta^2 a(mz - a\eta) + \beta^2 m^2(\eta - y) - \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}$$

The last integral in equation (14) may be evaluated using the method described in Appendix A of reference (55).

From Appendix A of reference (55);

If

$$\gamma^4 + (c - ae^2)\,\gamma^2 - b^2 e^2 = 0 \qquad (17)$$

then

$$\int \frac{(A + Bv)\,dv}{(v^2 + e^2)\sqrt{\hat{a}v^2 + 2bv + c}}$$

$$= \frac{Ab\gamma + B\gamma^3}{\gamma^4 + b^2e^2} \tan^{-1} \frac{\gamma\sqrt{\hat{a}v^2 + 2bv + c}}{\gamma^2 - bv}$$

$$+ \frac{-A\frac{\gamma^3}{e} + Bbe\gamma}{2(\gamma^4 + b^2e^2)} \log \frac{-(v\gamma^2 + be^2) + \gamma e\sqrt{\hat{a}v^2 + 2bv + c}}{-(v\gamma^2 + be^2) - \gamma e\sqrt{\hat{a}v^2 + 2bv + c}} \tag{18}$$

From equation (14) we can write

$$[m(x - \beta^2 az) - \eta(1 - \beta^2a^2)]^2 - \beta^2m^2[(1 - \beta^2a^2)(\eta - y)^2 + (z - ax)^2]$$

$$= (1 - \beta^2a^2)[1 - \beta^2(a^2 + m^2)](\eta - y)^2$$

$$- 2[m(x - \beta^2az) - y(1 - \beta^2a^2)](\eta - y)(1 - \beta^2a^2)$$

$$+ [m(x - \beta^2az) - y(1 - \beta^2a^2)]^2 - \beta^2m^2(z - ax)^2$$

Therefore if we factor out a $(1 - \beta^2 a^2)$ we can write the denominator as

$$[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}$$

$$= (1 - \beta^2 a^2)^{3/2} (v^2 + e^2) \sqrt{\hat{a} v^2 + 2bv + c} \tag{19}$$

where

$$\hat{a} = [1 - \beta^2 (a^2 + m^2)]$$

$$b = -[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]$$

$$c = \left| [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 \right| (1 - \beta^2 a^2)^{-1}$$

$$e = (z - ax)(1 - \beta^2 a^2)^{-1/2}$$

$$v = \eta - y$$

Referring to equation (17)

$$c - \hat{a}e^2 = \frac{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 - [1 - \beta^2 (a^2 + m^2)](z - ax)^2}{(1 - \beta^2 a^2)}$$

$$= \frac{b^2}{(1 - \beta^2 a^2)} - e^2 (1 - \beta^2 a^2)$$

$$\gamma^4 + (c - \hat{a}e^2) - b^2 e^2 = \left[\gamma^2 + \frac{B^2}{(1 - \beta^2 a^2)}\right]\left[\gamma^2 - e^2 (1 - \beta^2 a^2)\right] = 0$$

Therefore we can choose

$$\gamma^2 = e^2 (1 - \beta^2 a^2)$$

or

$$\gamma = (z - ax) \tag{20}$$

Using this the numerator of the second integral in (12) becomes

$$(z - ax)^2 \left| y (1 - \beta^2 a^2) - [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \right| + (1 - \beta^2 a^2) \left| y [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] + (z - ax)^2 \right| (\eta - y)$$

$$= \gamma^2 \left| y (1 - \beta^2 a^2) + b \right| + (1 - \beta^2 a^2) \left| -yb + \gamma^2 \right| v$$

Therefore from (18) the coefficient of $\tan^{-1}$ is

$$\frac{b\gamma^3 \left| y (1 - \beta^2 a^2) + b \right| + (1 - \beta^2 a^2) \gamma^3 \left| -yb + \gamma^2 \right|}{(1 - \beta^2 a^2)^{3/2} (\gamma^4 + b^2 e^2)} = \frac{\gamma (1 - \beta^2 a^2) \left| \gamma^4 + \frac{b^2 \gamma^2}{1 - \beta^2 a^2} \right|}{(1 - \beta^2 a^2)^{3/2} \left| \gamma^4 + \frac{b^2 \gamma^2}{1 - \beta^2 a^2} \right|}$$

$$= \frac{\gamma}{\sqrt{1 - \beta^2 a^2}}$$

$$= \frac{z - ax}{\sqrt{1 - \beta^2 a^2}}$$

since $\gamma = (z - ax)$ and $\gamma^2 = e^2 (1 - \beta^2 a^2)$

Using (19) and (18) again the coefficient of $\frac{1}{2} \log$ is

$$\frac{- \gamma^5 [y (1 - \beta^2 a^2) + b] + be^2 \gamma (1 - \beta^2 a^2) [-yb + \gamma^2]}{e (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)^{3/2}}$$

$$= \frac{[- \gamma^5 - b^2 e^2 \gamma] \, y (1 - \beta^2 a^2)}{(z - ax) (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)} = -y$$

And from (19)

$$(1 - \beta^2 a^2)\ [\hat{a}v^2 + 2bv + c]$$

$$= [m\,(x - \beta^2 az) - (v + y)\,(1 - \beta^2 a^2)]^2 - \beta^2 m^2\,[1 - \beta^2 a^2)v^2 + (z - ax)^2]$$

$$\gamma^2 - bv = (z - ax)^2 + v\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)] - (v\gamma^2 + be^2)$$

$$-(v\gamma^2 + be^2) = \left|\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)] - v\,(1 - \beta^2 a^2)\right| \frac{(z - ax)^2}{(1 - \beta^2 a^2)}$$

Therefore

$$\tan^{-1} \frac{\gamma\sqrt{\hat{a}v^2 + 2bv + c}}{\gamma^2 - bv} = \tan^{-1} \frac{(z - ax)\ \sqrt{[m\,(x - \beta^2 az) - \eta\,(1 - \beta^2 a^2)]^2 - \beta^2 m^2\,[(1 - \beta^2 a^2)\,(\eta - y)^2 + (z - ax)^2]}}{\sqrt{1 - \beta^2 a^2}\ \left|(z - ax)^2 + (\eta - y)\,[m\,(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]\right|}$$

$$\log \frac{-(v\gamma^2 + be^2) + \gamma e\sqrt{\hat{a}v^2 + 2bv + c}}{-(v\gamma^2 + be^2) - \gamma e\sqrt{\hat{a}v^2 + 2bv + c}}$$

$$= \log \frac{m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) + \sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}}{m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) - \sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}}$$

$$= \log \frac{(mx - \eta) - \beta^2 a(mz - a\eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2(\eta - y)^2 - \beta^2(mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a(mz - a\eta) - \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2(\eta - y)^2 - \beta^2(mz - a\eta)^2]}}$$

because from (12)

$$[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2\,[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]$$

$$= (1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2(\eta - y)^2 - \beta^2(mz - ay)^2]$$

Therefore combining terms

$$\frac{1}{2}\int \log \frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}\,d\eta$$

$$=\frac{1}{2}(\eta-y)\log \frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}$$

$$+\frac{y(1-\beta^2a^2)-m(x-\beta^2az)}{2\sqrt{(1-\beta^2a^2)[1-\beta^2(a^2+m^2)]}}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}$$

$$+\frac{(z-ax)}{\sqrt{1-\beta^2a^2}}\tan^{-1}\frac{(z-ax)\sqrt{[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}}{\sqrt{1-\beta^2a^2}\,\left|(z-ax)^2+(\eta-y)[m(x-\beta^2az)-y(1-\beta^2a^2)]\right|} \tag{21}$$

Differentiation of the indefinite integral over η. - Equation (21) may be checked by differentiation

To differentiate the first log term we can refer back to the integration by parts of the integral over η. As before let

$$f = (mx-\eta)-\beta^2a(mz-a\eta)$$

$$g^2 = \beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]$$

$$h^2 = (1-\beta^2a^2)[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-ay)^2]$$

then $$f^2 = g^2+h^2$$

and $$\frac{d}{d\eta}\,\frac{1}{2}\log\frac{f+h}{f-h} = \frac{1}{hg^2}[f'g^2-fgg'] = \frac{f'}{h}-\frac{fgg'}{hg^2}$$

Then since

$$f' = -(1-\beta^2 a^2)$$

$$gg' = \beta^2 m^2 (1-\beta^2 a^2)(\eta - y)$$

we can write

$$\frac{1}{2}\frac{\partial}{\partial\eta}(\eta-y)\log\frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right.}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right.}}$$

$$= \frac{1}{2}\log\frac{(mx-\eta)-\beta^2 a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2 a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

$$\frac{-(\eta-y)^2(1-\beta^2a^2)}{\sqrt{[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}\;[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}$$

$$\frac{-(\eta-y)(1-\beta^2a^2)}{\sqrt{[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}} \tag{22}$$

To differentiate the second log term in (21) we will first perform the following manipulation,

$$\left\{ m(x-\beta^2 a z) - \beta^2 m^2 y \;-\; [1-\beta^2(a^2+m^2)]\eta \right\}^2$$

$$= \left\{ m(x-\beta^2 a z) - y(1-\beta^2 a^2) \;-\; [1-\beta^2(a^2+m^2)](\eta-y) \right\}^2$$

$$= \beta^2 m^2 (z-ax)^2 + (1-\beta^2 a^2)\left[(mx-y)^2 - \beta^2(ay-mz)^2\right]$$

$$-2\left[(mx-y) - \beta^2 a(mz-ay)\right]\left[1-\beta^2(a^2+m^2)\right](\eta-y)$$

$$+ (1-\beta^2 a^2)\left[1-\beta^2(a^2+m^2)\right](\eta-y)^2$$

$$- \beta^2 m^2\left[1-\beta^2(a^2+m^2)\right](\eta-y)^2$$

$$= \beta^2 m^2\left[(mx-y)^2 + (z-ax)^2 - \beta^2(ay-mz)^2\right]$$

$$+\left[1-\beta^2(a^2+m^2)\right]\Big\{(mx-y)^2 - 2(\eta-y)(mx-y) + (\eta-y)^2$$

$$-\beta^2\left[(ay-mz)^2 - 2a(mz-ay)(\eta-y) + a^2(\eta-y)^2\right]$$

$$-\beta^2 m^2(\eta-y)^2 \Big\}$$

$$= \beta^2 m^2\left[(mx-y)^2 + (z-ax)^2 - \beta^2(ay-mz)^2\right]$$

$$+ \left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2\right]$$

Therefore if we let

$$f = m(x-\beta^2 a z) - \beta^2 m^2 y - \left[1-\beta^2(a^2+m^2)\right]\eta$$

$$g^2 = \beta^2 m^2\left[(mx-y)^2 + (z-ax)^2 - \beta^2(ay-mz)^2\right]$$

$$h^2 = \left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2 - \beta^2 m^2(\eta-y) - \beta^2(mz-a\eta)^2\right]$$

then $f^2 = g^2 + h^2$ (23)

Since in this case $\frac{d}{d\eta}\frac{1}{2}\log\frac{f+h}{f-h} = \frac{f'}{h}$ [using (13)]

we can write

$$f' = -\left[1-\beta^2(a^2+m^2)\right]$$

and

$$\frac{1}{2}\frac{d}{d\eta}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}$$

$$= \frac{-[1-\beta^2(a^2+m^2)]}{\sqrt{\left|m(x-\beta^2az)-\beta^2m^2y-\eta[1-\beta^2(a^2+m^2)]\right|^2-\beta^2m^2[(mx-y)^2+(z-ax)^2-\beta^2(ay-mx)^2]}}$$

$$= \frac{-\sqrt{[1-\beta^2(a^2+m^2)](1-\beta^2a^2)}}{\sqrt{[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}} \tag{24}$$

This is because, using (5)

$$(1-\beta^2a^2)\left\{\left|m(x-\beta^2az)-\beta^2m^2y-\eta[1-\beta^2(a^2+m^2)]\right|^2-\beta^2m^2[(mx-y)^2+(z-ax)^2-\beta^2(ay-mz)^2]\right\}$$

$$= (1-\beta^2a^2)\left|m(x-\beta^2az)-y(1-\beta^2a^2)-[1-\beta^2(a^2+m^2)](\eta-y)\right|^2-\beta^2m^2(1-\beta^2a^2)(z-ax)^2$$

$$-\beta^2m^2[m(x-\beta^2az)-y(1-\beta^2a^2)]^2+\beta^4m^4(z-ax)^2$$

$$= [1-\beta^2(a^2+m^2)]\Big|[m(x-\beta^2az)-y(1-\beta^2a^2)]^2-2(1-\beta^2a^2)[m(x-\beta^2az)-y(1-\beta^2a^2)](\eta-y)$$

$$+(1-\beta^2a^2)[1-\beta^2(a^2+m^2)](\eta-y)^2-\beta^2m^2(z-ax)^2\Big|$$

$$= [1-\beta^2(a^2+m^2)]\left|[m(x-\beta^2az)-y(1-\beta^2a^2)-(1-\beta^2a^2)(\eta-y)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]\right|$$

$$= [1-\beta^2(a^2+m^2)]\left|[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]\right|$$

We can also write,

$$\frac{d}{d\eta}\tan^{-1}\frac{f}{g} = \frac{\frac{f'}{g}-\frac{g'f}{g^2}}{\left(\frac{f}{g}\right)^2+1} = \frac{gff'-g'f^2}{f(f^2+g^2)}$$

$$f = (z - ax) \sqrt{[m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2 (\eta - y)^2 + (z - ax)^2]}$$

$$ff^1 = -(z - ax)^2 (1 - \beta^2 a^2) \Big| [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)] + \beta^2 m^2 (\eta - y) \Big|$$

$$g = \sqrt{1 - \beta^2 a^2} \Big| (z - ax)^2 + (\eta - y) [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big|$$

$$g^1 = \sqrt{1 - \beta^2 a^2} [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]$$

$$- \frac{gff' - g'f^2}{(z - ax)^2 \sqrt{1 - \beta^2 a^2}} = (1 - \beta^2 a^2) [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2) + \beta^2 m^2 (\eta - y)] \Big| (z - ax)^2 + (\eta - y) [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big|$$

$$+ [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big| [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \Big|$$

$$= (z - ax)^2 \Big| [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)] (1 - \beta^2 a^2) + \beta^2 m^2 (1 - \beta^2 a^2) (\eta - y) - \beta^2 m^2 [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big|$$

$$+ [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)] [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big| m (x - \beta^2 az) - \eta (1 - \beta^2 a^2) + (\eta - y) (1 - \beta^2 a^2) \Big|$$

$$= [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)] \Big| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \Big|$$

$$f^2 + g^2 = (z - ax)^2 \Big| [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \Big|$$

$$+ (1 - \beta^2 a^2) \Big| (z - ax)^2 + (\eta - y) [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \Big|^2$$

$$= (z - ax)^2 \Big| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 + (\eta - y)^2 (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] - \beta^2 m^2 (z - ax)^2 \Big|$$

$$+ (1 - \beta^2 a^2 \Big| (z - ax)^4 + (\eta - y)^2 [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 \Big|$$

$$= (z - ax)^2 \Big| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \Big|$$

$$+ (1 - \beta^2 a^2) (\eta - y)^2 \Big| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \Big|$$

$$= \Big| (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \Big| \Big| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \Big|$$

Therefore

$$\frac{d}{d\eta}\tan^{-1}\frac{(z-ax)\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}}{\sqrt{1-\beta^2a^2}\,\left\{(z-ax)^2+(\eta-y)\left[m(x-\beta^2az)-y(1-\beta^2a^2)\right]\right\}}$$

$$=\frac{-(z-ax)\sqrt{1-\beta^2a^2}\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]}{\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}}$$

(25)

Combining (22), (24), and (25)

$$\frac{d}{d\eta}\left\{\frac{1}{2}(\eta-y)\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}\right.$$

$$+\frac{y(1-\beta^2a^2)-m(x-\beta^2az)}{2\sqrt{(1-\beta^2a^2)\left[1-\beta^2(a^2+m^2)\right]}}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{\left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{\left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

$$\left.+\frac{(z-ax)}{\sqrt{1-\beta^2a^2}}\tan^{-1}\frac{(z-ax)\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}}{\sqrt{1-\beta^2a^2}\,\left|(z-ax)^2+(\eta-y)\left|m(x-\beta^2az)-y(1-\beta^2a^2)\right|\right|}\right\}$$

$$=\frac{1}{2}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

$$+\Big\{-(\eta-y)(1-\beta^2a^2)\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]$$

$$-(1-\beta^2a^2)(\eta-y)^2\left[m(x-\beta^2az)-y(1-\beta^2a^2)-(\eta-y)(1-\beta^2a^2)\right]+\left[m(x-\beta^2az)-y(1-\beta^2a^2)\right]\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]$$

$$-(z-ax)^2\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]\Big\}\times\left\{\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]\sqrt{\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]}\right\}^{-1}$$

$$=\frac{1}{2}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

Evaluation of the definite integral. - From (6) and (7)

$$m\,(x - \beta^2 az) - \eta_3\,(1 - \beta^2 a^2) = \beta m \sqrt{(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2} \tag{6}$$

$$m\,(x - \beta^2 az) - \beta^2 m^2 y - [1 - \beta^2(a^2 + m^2)]\,\eta_3 = \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} \tag{7}$$

Therefore using (6), (7) and (23) both of the log terms in equation (21) are zero when $\eta = \eta_3$. However the $\tan^{-1}$ term can be either zero or $\pm\pi$ when $\eta = \eta_3$, depending upon whether the denominator is greater or less than zero. If it is greater than zero the limit is zero as $\eta \to \eta_3$. In (21) when $\eta = \eta_3$ using (6) and then (5) and (7),

$$(z - ax)^2 + (\eta_3 - y)\,[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]$$

$$= \frac{[1 - \beta^2(a^2 + m^2)](z - ax)^2 + [m(x - \beta^2 az) - y(1 - \beta^2 a^2) - \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}\,][m(x - \beta^2 az) - y(1 - \beta^2 a^2)]}{1 - \beta^2(a^2 + m^2)}$$

$$= \frac{\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}\,\left\{(1 - \beta^2 a^2)\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} - \beta m[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]\right\}}{[1 - \beta^2(a^2 + m^2)]} \tag{26}$$

Now if we examine the numerator of (26) and use (5)

$$(1 - \beta^2 a^2)^2\,[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2] - \beta^2 m^2\,[m(x - \beta^2 az) - y\,(1 - \beta^2 a^2)]^2$$

$$= (1 - \beta^2 a^2)\,[1 - \beta^2(a^2 + m^2)]\,[(mx - y)^2 - \beta^2(ay - mz)^2] + [(1 - \beta^2 a^2)^2 - \beta^4 m^4]\,(z - ax)^2$$

$$= [1 - \beta^2(a^2 + m^2)]\,\left|\,(1 - \beta^2 a^2)\,[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2] + \beta^2 m^2\,(z - ax)^2\right| \tag{27}$$

because

$$(1 - \beta^2a^2)^2 - \beta^4m^4 = (1 - \beta^2a^2 - \beta^2m^2)(1 - \beta^2a^2 + \beta^2m^2)$$

But if $1 - \beta^2(a^2 + m^2) > 0$ then from (27)

$$(1 - \beta^2a^2)\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} > \beta m\,|m(x - \beta^2az) - y(1 - \beta^2a^2)|$$

and (26) shows that the denominator of the $\tan^{-1}$ term is > 0. But if $1 - \beta^2(a^2 + m^2) < 0$ then (27) shows

$$(1 - \beta^2a^2)\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} < \beta m\,\left|m(x - \beta^2az) - y(1 - \beta^2a^2)\right|$$

and the denominator of the $\tan^{-1}$ term is > 0 if and only if

$$m(x - \beta^2az) - y(1 - \beta^2a^2) > 0$$

But

$$m(x - \beta^2az) - y(1 - \beta^2a^2) = m[x - \beta\sqrt{y^2 + z^2}] + \beta m\sqrt{y^2 + z^2} - \beta^2amz - y(1 - \beta^2a^2)$$

But we can also write

$$\beta^2m^2(y^2 + z^2) - [\beta^2amz + y(1 - \beta^2a^2)]^2$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2m^2z^2 - \beta^2m^2\beta^2a^2z^2 - 2\beta^2(1 - \beta^2a^2)amyz + \beta^2a^2(1 - \beta^2a^2)y^2$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2(1 - \beta^2a^2)(m^2z^2 - 2\,amyz + a^2y^2)$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2(1 - \beta^2a^2)(mz - ay)^2 > 0 \text{ if } 1 - \beta^2(a^2 + m^2) < 0$$

And therefore

$$\beta m \sqrt{y^2 + z^2} - \beta^2 amz - y(1 - \beta^2 a^2) > 0$$

and

$$m(x - \beta^2 az) - y(1 - \beta^2 a^2) > 0$$

if

$$1 - \beta^2(a^2 + m^2) < 0$$

and

$$x > \beta(y^2 + z^2)$$

Therefore inside the Mach cone from the origin $x^2 > \beta^2(y^2 + z^2)$ we have

$$(z - ax)^2 + (\eta_3 - y)\ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] > 0 \tag{28}$$

and therefore because of (6) and (28)

$$\tan^{-1} \frac{(z - ax)\sqrt{[m(x - \beta^2 az) - \eta_3(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2]}}{\sqrt{1 - \beta^2 a^2}\ \left|(z - ax)^2 + (\eta_3 - y)[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]\right|} = 0$$

If we change the sign of the denominator we would get π for $z > ax$ and $-\pi$ for $z < ax$. However when we evaluate the term when $\eta = 0$ and subtract, the result is the same.

Therefore if we evaluate (21) for $\eta = 0$ we get for (11)

$$\phi(x, y, z) = -\frac{\bar{w} + \beta^2 a\bar{u}}{2\pi\sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \log \frac{(mx-\eta) - \beta^2 a(mz - a\eta) + \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2(\eta - y)^2 - \beta^2(mz - a\eta)^2\right]}}{(mx-\eta) - \beta^2 a(mz - a\eta) - \sqrt{(1-\beta^2 a^2)\left[(mx-\eta)^2 - \beta^2 m^2(\eta - y)^2 - \beta^2(mz - a\eta)^2\right]}} \, d\eta$$

$$= \frac{\bar{w} + \beta^2 a\bar{u}}{\pi} \left\{ \frac{(z - ax)}{1 - \beta^2 a^2} \tan^{-1} \frac{m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2} \right.$$

$$+ \frac{y(1 - \beta^2 a^2) - m(x - \beta^2 az)}{2(1 - \beta^2 a^2)\sqrt{1 - \beta^2(a^2 + m^2)}} \log \frac{x - \beta^2(az + my) + \sqrt{[x^2 - \beta^2(y^2 + z^2)][1 - \beta^2(a^2 + m^2)]}}{x - \beta^2(az + my) - \sqrt{[x^2 - \beta^2(y^2 + z^2)][1 - \beta^2(a^2 + m^2)]}}$$

$$\left. - \frac{y}{2\sqrt{1 - \beta^2 a^2}} \log \frac{x - \beta^2 az + \sqrt{[x^2 - \beta^2(y^2 + z^2)](1 - \beta^2 a^2)}}{x - \beta^2 az - \sqrt{[x^2 - \beta^2(y^2 + z^2)](1 - \beta^2 a^2)}} \right\} \tag{29}$$

<u>Evaluation of the velocity components.</u> - The velocity components may now be obtained by differentiation of the velocity potential. However, whoever attempts to take partial derivatives of (29) is in for a long days work. But the partial derivatives of (29) may be obtained quite simply if it is noted that all terms in the expressions for the velocity components must have coefficients involving either log or $\tan^{-1}$. This is easily shown by differentiating the expression for ϕ given by (11).

$$u = \frac{\partial\phi}{\partial x} = -\frac{\bar{w}+\beta^2 a\bar{u}}{2\pi\sqrt{1-\beta^2a^2}}\frac{\partial}{\partial x}\int_0^{\eta_3} \log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}\,d\eta$$

$$= -\frac{\bar{w}+\beta^2 a\bar{u}}{2\pi\sqrt{1-\beta^2a^2}}\left\{\frac{\partial\eta_3}{\partial x}\log\frac{(mx-\eta_3)-\beta^2a(mz-a\eta_3)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta_3)^2-\beta^2m^2(\eta_3-y)^2-\beta^2(mz-a\eta_3)^2\right]}}{(mx-\eta_3)-\beta^2a(mz-a\eta_3)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta_3)^2-\beta^2m^2(\eta_3-y)^2-\beta^2(mz-a\eta_3)^2\right]}}\right.$$

$$\left.+\int_0^{\eta_3}\frac{\partial}{\partial x}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)-\sqrt{(1-\beta^2a^2)\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}\,d\eta\right\}$$

$$= -\frac{\bar{w}+\beta^2a\bar{u}}{\pi\sqrt{1-\beta^2a^2}}\int_0^{\eta_3}\frac{m-\dfrac{(\eta-y)[m(x-\beta^2az)-\eta(1-\beta^2a^2)](1-\beta^2a^2)}{(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2}}{\sqrt{[m(x-\beta^2az)-\eta(1-\beta^2a^2)]^2-\beta^2m^2[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2]}}\,d\eta$$

because from (6) and (23a) the log term is zero when $\eta = \eta_3$

However this integral expression for u involves integrals of the same form as in (13), (16), or (18). A closer examination shows that the resulting log or $\tan^{-1}$ terms will have arguments which are identical with those for ϕ, although the coefficients may be different. Therefore all expressions must involve either log or $\tan^{-1}$ terms. The same arguments can be made for the v and w velocity components.

With this in mind the partial derivatives of ϕ can be obtained from (29) by differentiating only the coefficients of the log and $\tan^{-1}$ terms. Differentiation of the $\tan^{-1}$ or log terms will not yield other $\tan^{-1}$ or log terms and therefore the sum of these terms must give zero. Therefore

$$u = \frac{\partial\phi}{\partial x} = -\frac{\bar{w}+\beta^2a\bar{u}}{\pi(1-\beta^2a^2)}\left\{a\tan^{-1}\frac{m(z-ax)\sqrt{x^2-\beta^2(y^2+z^2)}}{y[(y-mx)-\beta^2a(ay-mz)]+(z-ax)^2}\right.$$

$$\left.+\frac{m}{2\sqrt{1-\beta^2(a^2+m^2)}}\log\frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}\right\}$$

Constant Pressure Surface

Boundary Conditions. - In (1) we can set $\Omega = u$

Assume that on S

$$\text{(a)} \qquad u - u' = \text{const}$$

$$\text{(b)} \qquad \left[\frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'}\right] = 0 \qquad (30)$$

Now since

$$\frac{\partial u}{\partial \nu} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{1}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \zeta}$$

$$\frac{\partial u'}{\partial \nu'} = \frac{-\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{-1}{1 + a^2} \frac{\partial u}{\partial \zeta}$$

and from (b)

$$\frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial}{\partial \xi}(u - u') + \frac{1}{\sqrt{1 + a^2}} \frac{\partial}{\partial \zeta}(u - u') = 0$$

Now from (a), $u - u' = \text{const}$ on S implies that on S

$$\frac{\partial}{\partial \eta}(u - u') = 0$$

and

$$\frac{\partial}{\partial \xi}(u - u') + a\frac{\partial}{\partial \zeta}(u - u') = 0$$

This means that since

$$\beta^2 a \frac{\partial}{\partial \xi}(u - u') + \frac{\partial}{\partial \zeta}(u - u') = 0$$

and

$$\frac{\partial}{\partial \xi}(u - u') + a\frac{\partial}{\partial \zeta}(u - u') = 0$$

then if $$1-\beta^2 a \neq 0$$

$$\frac{\partial}{\partial \xi}(u - u') = \frac{\partial}{\partial \eta}(u - u') = \frac{\partial}{\partial \zeta}(u - u') = 0 \text{ on } S$$

Note that assumptions (a) and (b) in (30) are independent assumptions.

(a) Says that (u - u') is constant on S

(b) Effectively says that the derivative of u - u' in a direction normal to S is zero.

Woodward assumes, in reference (55), that (a) implies (b), which is not true.

If we examine the normal velocity on S

$$u_n = \frac{-au + w}{\sqrt{1 + a^2}} \qquad u'_n = \frac{au' - w}{\sqrt{1 + a^2}}$$

then since due to irrotationality $\dfrac{\partial w}{\partial \xi} = \dfrac{\partial u}{\partial \zeta}$

$$\sqrt{1 + a^2}\frac{\partial}{\partial \xi}u_n = -a\frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \xi} = -a\frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \zeta}$$

and

$$\sqrt{1 + a^2}\,\frac{\partial}{\partial \xi}(u_n + u'_n) = -a\frac{\partial}{\partial \xi}(u - u') + \frac{\partial}{\partial \xi}(u - u') = 0$$

and therefore the source strength, if any, is constant.

Evaluation of the integral over ξ. - The integral to be evaluated is,

$$\phi(x,y,z) = \frac{\Delta P}{4\pi q_\infty}\int_0^{\eta_3}\int_{\xi_1}^{\xi_2}\frac{\beta^2 a - \dfrac{(x-\xi)(z-a\xi)}{(y-\eta)^2 + (z-a\xi)^2}}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}}\,d\xi\,d\eta$$

$$= \frac{\Delta P}{4\pi a q_\infty}\int_0^{\eta_3}\int_{\xi_1}^{\xi_2}\frac{\beta^2 a^2\left[(z-a\xi)^2 + (y-\eta)^2\right] - (z-a\xi)^2 + (z-a\xi)(z-ax)}{\left[(z-a\xi)^2 + (y-\eta)^2\right]\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}}\,d\xi\,d\eta$$

$$= \frac{\Delta P}{4\pi a q_\infty}\int_0^{\eta_3}\int_{\xi_1}^{\xi_2}\frac{(z-a\xi)(z-ax) + (y-\eta)^2}{\left[(z-a\xi)^2 + (y-\eta)^2\right]\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}}\,d\xi\,d\eta$$

$$- \frac{(1-\beta^2a^2)\,\Delta P}{4\pi a q_\infty}\int_0^{\eta_3}\int_{\xi_1}^{\xi_2}\frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}}\,d\eta$$

Except for the coefficient, the second integral is identical to the integral obtained for a surface distribution of sources. It has already been evaluated [see (9)]. The first integral over ξ is easy to evaluate,

$$\int \frac{[(z-ax)(z-a\xi)+(y-\eta)^2]\,d\xi}{[(y-\eta)^2+(z-a\xi)^2]\sqrt{(x-\xi)^2-\beta^2(y-\eta)^2-\beta^2(z-a\xi)^2}}$$

$$= -\frac{1}{2}\log\frac{(x-\xi)+\sqrt{(x-\xi)^2-\beta^2[(z-a\xi)^2+(y-\eta)^2]}}{(x-\xi)-\sqrt{(x-\xi)^2-\beta^2[(z-a\xi)^2+(y-\eta)^2]}}$$

This can be verified by differentiating. Let,

$$f = x-\xi$$

$$g^2 = \beta^2[(z-a\xi)^2+(y-\eta)^2]$$

$$h^2 = (x-\xi)^2-\beta^2[(z-a\xi)^2+(y-\eta)^2]$$

and $\displaystyle \frac{\partial}{\partial\xi}\,\frac{1}{2}\log\frac{f+h}{f-h} = \frac{1}{hg^2}\left[f'g^2-fgg'\right]$ from (13)

$$f' = -1 \qquad gg' = -\beta^2 a(z-a\xi)$$

Therefore

$$-\frac{1}{2}\frac{\partial}{\partial\xi}\log\frac{(x-\xi)+\sqrt{(x-\xi)^2-\beta^2[(z-a\xi)^2+(y-\eta)^2]}}{(x-\xi)-\sqrt{(x-\xi)^2-\beta^2[(z-a\xi)^2+(y-\eta)^2]}}$$

$$= \frac{(z-a\xi)^2+(y-\eta)^2-a(x-\xi)(z-a\xi)}{[(z-a\xi)^2+(y-\eta)^2]\sqrt{(x-\xi)^2-\beta^2(z-a\xi)^2-\beta^2(y-\eta)^2}}$$

$$= \frac{(z-ax)(z-a\xi)+(y-\eta)^2}{[(z-a\xi)^2+(y-\eta)^2]\sqrt{(x-\xi)^2-\beta^2(z-a\xi)^2-\beta^2(y-\eta)^2}}$$

Since from (3)

$$(x - \xi_2)^2 = \beta^2 \left[(z - a\,\xi_2)^2 + (y - \eta)^2 \right]$$

$$\log \frac{(x-\xi_2) + \sqrt{(x-\xi_2)^2 - \beta^2[(z-a\xi_2)^2 + (y-\eta)^2]}}{(x-\xi_2) - \sqrt{(x-\xi_2)^2 - \beta^2[(z-a\xi_2)^2 + (y-\eta)^2]}} = 0$$

and from (2) $\qquad \xi_1 = \dfrac{\eta}{m}$

$$\int_{\xi_1}^{\xi_2} \frac{(z - ax)(z - a\xi) + (y - \eta)^2}{\left[(y-\eta)^2 + (z - a\xi)^2\right] \sqrt{(x-\xi)^2 - \beta^2 (y-\eta)^2 - \beta^2 (z - a\xi)^2}}\, d\xi$$

$$= \frac{1}{2} \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2[(a\eta - mz)^2 + m^2(\eta - y)^2]}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2[(a\eta - mz)^2 + m^2(\eta - y)^2]}}$$

therefore

(32)

$$\phi(x,y,z) = \frac{1}{8\pi a}\,\frac{\Delta P}{q_\infty} \int_0^{\eta_3} \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}\, d\eta$$

$$- \frac{1}{8\pi a}\,\frac{\Delta P}{q_\infty} \sqrt{1-\beta^2 a^2} \int_0^{\eta_3} \log \frac{(mx-\eta) - \beta^2 a(mz-a\eta) + \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}{(mx-\eta) - \beta^2 a(mz-a\eta) - \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}\, d\eta$$

The second integral was evaluated previously for a surface distribution of sources.

Evaluation of the indefinite integral over η. - The first integral in (32) must first be integrated by parts.

let $$f = mx - \eta$$

$$g^2 = \beta^2 m^2 (y-\eta)^2 + \beta^2 (mz - a\eta)^2$$

$$h^2 = (mx-\eta)^2 - \beta^2 m^2 (y-\eta)^2 - \beta^2 (mz - a\eta)^2$$

then let

$$u = \frac{1}{2} \log \frac{f+h}{f-h} \qquad dv = d\eta$$

$$du = \frac{1}{hg^2}\left[f'g^2 - gg'f\right] d\eta \qquad v = \eta$$

$$f' = -1$$

$$gg' = \beta^2 \left[a\,(a\eta - mz) + m^2\,(\eta - y)\right]$$

$$-\left[f'\,g^2 - gg'f\right] = \beta^2 \left\{(a\eta - mz)^2 + m^2\,(\eta - y)^2 + (mx - \eta)\left[a\,(a\eta - mz) + m^2\,(\eta - y)\right]\right\} \tag{33}$$

$$= \beta^2 \left\{(a\eta - mz)\left[(a\eta - mz) + a\,(mx - \eta)\right] + m^2\,(\eta - y)\left[(\eta - y) + (mx - \eta)\right]\right\}$$

$$= \beta^2 \left\{-m\,(a\eta - mz)\,(z - ax) + m^2\,(\eta - y)\,(mx - y)\right\}$$

$$-\eta\left[f'g^2 - gg'f\right] = \beta^2 \left| m\left[m\,(mx - y) - a\,(z - ax)\right]\eta^2 + m^2\left[\,z\,(z - ax) - y\,(mx - y)\right]\eta \right|$$

$$= \beta^2 \left| \frac{m\,[m\,(mx - y) - a\,(z - ax)]}{(a^2 + m^2)}\left[(a\,\eta - mz)^2 + m^2\,(\,\eta - y)^2\right] \right.$$

$$+ \frac{2m^2\left[m\,(mx - y) - a\,(z - ax)\right](az + my)\,\eta + m^2\,(a^2 + m^2)\left[z\,(z - ax) - y\,(mx - y\right]\eta}{(a^2 + m^2)}$$

$$\left. - \frac{m\left[m\,(mx - y) - a\,(z - ax)\right]m^2\,(y^2 + z^2)}{(a^2 + m^2)} \right|$$

But

$$\left[m\,(mx - y) - a\,(z - ax)\right](az + my) + (a^2 + m^2)\left[z\,(z - ax) - y\,(mx - y)\right]$$

$$= \left[m\,(az + my) - y\,(z^2 + m^2)\right](mx - y) + \left[z\,(z^2 + m^2) - a\,(az + my)\right](z - ax)$$

$$= -a\,(ay - mz)\,(mx - y) - m\,(ay - mz)\,(z - ax)$$

$$= (ay - mz)^2$$

Therefore

$$\frac{1}{2}\int \log \frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}\, d\eta$$

$$= \frac{1}{2}\,\eta \log \frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}$$

$$- \frac{m\left[(az+my)-(a^2+m^2)x\right]}{(a^2+m^2)}\int \frac{d\eta}{\sqrt{(mx-\eta)^2-\beta^2(a\eta-mz)^2-\beta^2m^2(\eta-y)^2}}$$

$$+ \frac{m^2}{a^2+m^2}\int \frac{\left\{(ay-mz)^2+(my+az)\left[m(mx-y)-a(z-ax)\right]\right\}\eta - m(y^2+z^2)\left[m(mx-y)-a(z-ax)\right]}{\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]}}\, d\eta$$

(34)

The first integral in (34) may be evaluated using (16) because,

$$\left[m(x-\beta^2az)-\eta(1-\beta^2a^2)\right]^2-\beta^2m^2\left[(1-\beta^2a^2)(\eta-y)^2+(z-ax)^2\right]$$

$$= (1-\beta^2a^2)\left|(mx-\eta)^2-\beta^2(a\eta-mz)^2-\beta^2m^2(\eta-y)^2\right|$$

(35)

Therefore using (16) and (35)

$$\int \frac{d\eta}{\sqrt{(mx-\eta)^2-\beta^2(a\eta-mz)^2-\beta^2m^2(\eta-y)^2}} =$$

$$\frac{\mp 1}{2\sqrt{1-\beta^2(a^2+m^2)}}\log \frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}$$

(36)

The last term in equation (34) must be changed to the form of equation (18) in order to be evaluated. We can write

$$(a\eta - mz)^2 + m^2(\eta - y)^2 = (a^2 + m^2)\eta^2 - 2m(az + my)\eta + m^2(y^2 + z^2)$$

$$= (a^2 + m^2)\left[\eta - \frac{m(az + my)}{a^2 + m^2}\right]^2 + \frac{m^2\left[(a^2 + m^2)(y^2 + z^2) - (az + my)^2\right]}{a^2 + m^2}$$

$$= (a^2 + m^2)\left[\eta - \frac{m(az + my)}{a^2 + m^2}\right]^2 + \frac{m^2(ay - mz)^2}{(a^2 + m^2)}$$

Now we introduce a change of variables. Let

$$u = \eta - \frac{m(az + my)}{a^2 + m^2}$$

Therefore

$$(a\eta - mz)^2 + m^2(\eta - y)^2 = (a^2 + m^2)\left\{u^2 + \frac{m^2(ay - mz)^2}{(a^2 + m^2)^2}\right\} \tag{37}$$

$$\eta - mx = u + \frac{m(az + my) - mx(a^2 + m^2)}{(a^2 + m^2)} \tag{38}$$

$$(\eta - mx)^2 - \beta^2 (a\eta - mz)^2 - \beta^2 m^2 (\eta - y)^2$$

$$= \left[1 - \beta^2 (a^2 + m^2)\right] u^2 + 2\,\frac{m\,(az + my) - mx\,(a^2 + m^2)}{(a^2 + m^2)}\, u$$

$$- \frac{\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x\,(a^2 + m^2)\right]^2}{(a^2 + m^2)^2} \tag{39}$$

Now (18) can be used. Let

$$\hat{a} = 1 - \beta^2 (a^2 + m^2)$$

$$b = \frac{m\left[(az + my) - x\,(a^2 + m^2)\right]}{(a^2 + m^2)} = \frac{m\left[a\,(z - ax) - m\,(mx - y)\right]}{(a^2 + m^2)}$$

$$c = \frac{-\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x\,(a^2 + m^2)\right]^2}{(a^2 + m^2)^2}$$

$$e = \frac{m\,(ay - mz)}{(a^2 + m^2)} \tag{40}$$

and referring to (17)

$$c - \hat{a}e^2 = \frac{- m^2 (ay - mz)^2 + m^2 \left[m (mx - y) - a (z - ax)\right]^2}{(a^2 + m^2)^2}$$

$$b^2e^2 = \frac{m^2 \left[m (mx - y) - a (z - ax)\right]^2 m^2 (ay - mz)^2}{(a^2 + m^2)^4}$$

$$\gamma^4 + (c - \hat{a}e^2)\ \gamma^2 - b^2e^2$$

$$= \left\{ \gamma^2 - \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\} \left\{ \gamma^2 + \frac{m^2 \left[m (mx - y) - a (z - ax)\right]^2}{(a^2 + m^2)^2} \right\} = 0$$

Therefore we can choose

$$\gamma = \frac{m (ay - mz)}{(a^2 + m^2)} = e \tag{41}$$

The numerator of the second integral in equation (34) can be written,

$$\left\{ (a^2 + m^2)\,\gamma^2 - m\,(az + my)b \right\} \left\{ u + \frac{m\,(az + my)}{(a^2 + m^2)} \right\} + m^2\,(y^2 + z^2)b$$

$$= \left\{ (a^2 + m^2)\,\gamma^2 - m\,(az + my)b \right\} u + m\,(az + my)\,\gamma^2 + \frac{m^2 b\,(ay - mz)^2}{(a^2 + m^2)}$$

$$= \left\{ (a^2 + m^2)\,\gamma^2 - m\,(az + my)b \right\} u + \gamma^2 \left[m\,(az + my) + b\,(a^2 + m^2) \right]$$

Now referring to (18) we can write

$$A = \frac{\gamma^2 \left[m\,(az + my) + b\,(a^2 + m^2) \right]}{a^2 + m^2}$$

$$B = \frac{(a^2 + m^2)\,\gamma^2 - m\,(az + my)\,b}{a^2 + m^2}$$

The $(a^2 + m^2)$ in the denominator of A and B comes from (37) which is in the denominator of (34). From (18) and (41)

$$\frac{Ab\gamma + B\gamma^3}{\gamma^4 + b^2 e^2} = \frac{b\gamma \left[m\,(az + my) + b\,(a^2 + m^2) \right] + \left[(a^2 + m^2)\,\gamma^2 - m\,(az + my)b \right] \gamma}{(a^2 + m^2)(\gamma^2 + b^2)}$$

$$= \frac{\gamma\,(a^2 + m^2)}{(a^2 + m^2)} = \frac{m\,(ay - mz)}{(a^2 + m^2)}$$

and

$$\frac{-A\,\frac{\gamma^3}{e} + Bbe\gamma}{\gamma^4 + b^2e^2} = \frac{-\gamma^2\left[m\,(az + my) + b\,(a^2 + m^2)\right] + \left[(a^2 + m^2)\,\gamma^2 - bm\,(az + my)\right] b}{(a^2 + m^2)(\gamma^2 + b^2)}$$

$$= \frac{-m\,(az + my)}{a^2 + m^2}$$

From (39) and (40)

$$\hat{a}u^2 + 2bu + c = (mx - \eta)^2 - \beta^2\,(a\cdot\eta - mz)^2 - \beta^2 m^2\,(\eta - y)^2$$

$$\gamma^2 - bu = \gamma^2 - b\left[\eta - \frac{m\,(az + my)}{a^2 \quad m^2}\right] = \frac{m\left[m\,(mx - y) - a\,(z - ax)\right]\eta}{(a^2 + m^2)} + \left[\gamma^2 + b\,\frac{m\,(az + my)}{a^2 + m^2}\right]$$

$$= \frac{m\left[m\,(mx - y) - a\,(z - ax)\right]\eta}{a^2 + m^2} + \frac{m^2\,(ay^2 - 2amyz + m^2z^2) + m^2\left[a\,(z - ax) - m\,(mx \quad y)\right](az + my)}{(a^2 + m^2)^2}$$

$$= \frac{m}{a^2 + m^2}\left\{\left[m\,(mx - y) - a\,(z - ax)\right]\eta + m\left[z\,(z - ax) - y\,(mx - y)\right]\right\}$$

$u + b = \eta - mx$ from (38) and (40).

Therefore using (18), and (41) we get from (34)

$$\frac{m^2}{a^2+m^2}\int \frac{\left\{(ay-mz)^2+(my+az)\left[m(mx-y)-a(z-ax)\right]\right\}\eta - m\left[m(mx-y)-a(z-ax)\right](y^2+z^2)}{\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]}}\,d\eta$$

$$=\frac{m(ay-mz)}{a^2+m^2}\tan^{-1}\frac{(ay-mz)\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]}}{\left[m(mx-y)-a(z-ax)\right]\eta+m\left[z(z-ax)-y(mx-y)\right]}$$

$$-\frac{m(az+my)}{2(a^2+m^2)}\log\frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}} \tag{42}$$

Therefore substituting (36), and (42) in (34)

$$\frac{1}{2}\int\log\frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}\,d\eta =$$

$$\frac{1}{2}\left\{\eta-\frac{m(az+my)}{a^2+m^2}\right\}\log\frac{mx-\eta+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{mx-\eta-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}$$

$$+\frac{m\left[(az+my)-(a^2+m^2)x\right]}{2(a^2+m^2)}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{\left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{\left[1-\beta^2(a^2+m^2)\right]\left[(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2\right]}}$$

$$+\frac{m(mz-ay)}{a^2+m^2}\tan^{-1}\frac{(mz-ay)\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]}}{\left[m(mx-y)-a(z-ax)\right]\eta+m\left[z(z-ax)-y(mx-y)\right]} \tag{43}$$

Differentiation of the indefinite integral over η. - From (33)

$$\frac{1}{2}\frac{d}{d\eta}\log\frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}} =$$

$$\frac{m^2\left[y\,(mx-y)-z\,(z-ax)\right]+m\left[a\,(z-ax)-m\,(mx-y)\right]\eta}{\left[(a\eta-mz)^2+m^2\,(\eta-y)^2\right]\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2\,(\eta-y)^2\right.}} \tag{44}$$

From (24) using (35)

$$\frac{1}{2}\frac{d}{d\eta}\log\frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2]}}$$

$$= \frac{-\sqrt{1-\beta^2\,(a^2+m^2)}}{\sqrt{(mx-\eta)^2-\beta^2\left[(a\eta-mz)^2+m^2\,(\eta-y)^2\right]}} \tag{45}$$

and

$$\frac{d}{d\eta}\tan^{-1}\frac{f}{g} = \frac{\frac{f'}{g}-\frac{g'f}{g^2}}{\left(\frac{f}{g}\right)^2+1} = \frac{gff'-g'f^2}{f\,(f^2+g^2)}$$

where

$$f = (mz - ay) \sqrt{(mx - \eta)^2 \quad \beta^2 \left[(a\eta - mz)^2 + m^2 (\eta - y)^2 \right]}$$

$$g = \left[m (mx - y) - a (z - ax) \right] \eta + m \left[z (z - ax) - y (mx - y) \right] \tag{46}$$

$$ff' = (mz - ay)^2 \left[(\eta - mx) - \beta^2 a (a\eta - mz) - \beta^2 m^2 (\eta - y) \right]$$

$$g' = \left[m (mx - y) - a (z - ax) \right]$$

$$\begin{aligned}
f^2 + g^2 = {} & (mz - ay)^2 \left\{ (\eta - mx)^2 - \beta^2 \left[(a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \right\} \\
& + \left\{ \left[m (mx - y) - a (z - ax) \right] \eta + m \left[z (z - ax) - y (mx - y) \right] \right\}^2 \\
= {} & - \beta^2 (mz - ay)^2 \left[(a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \\
& + \left[m (z - ax) + a (mx - y) \right]^2 (\eta - mx)^2 \\
& + \left[m (mx - y) (\eta - y) - (z - ax) (a\eta - mz) \right]^2 \\
= {} & - \beta^2 (mz - ay)^2 \left[(a\eta - mx)^2 + m^2 (\eta - y)^2 \right] \\
& + \left[a^2 (mx - y)^2 + 2\,am (mx - y) (z - ax) + m^2 (z - ax)^2 \right] (\eta - mx)^2 \\
& + m^2 (mx - y)^2 (\eta - y)^2 - 2m (mx - y) (z - ax) (\eta - y) (a\eta - mz) + (2 - ax)^2 (a\eta - mz)^2
\end{aligned} \tag{47}$$

$$= -\beta^2 (mz - ay)^2 \left[(a\eta - mz)^2 + m^2 (\eta - y)^2\right]$$

$$+ (mx - y)^2 \left[(a\eta - mz) + m(z - ax)\right]^2 + m^2 (z - ax)^2 \left[(\eta - y) - (mx - y)\right]^2$$

$$- 2m (mx - y)(z - ax) \left[(\eta - y) - (mx - y)\right] \left[(a\eta - mx) + m(z - ax)\right]$$

$$+ m^2 (mx - y)^2 (\eta - y)^2 - 2m (mx - y)(z - ax)(\eta - y)(a\eta - mz) + (z - ax)^2 (a\eta - mz)^2$$

$$= \left[(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2\right] \left[(a\eta - mz)^2 + m^2 (\eta - y)^2\right]$$

$$\frac{gff' - f^2g'}{(ay - mz)^2} = \left\{ \left[m(mx - y) - a(z - ax)\right](\eta - mx) + m\left[(mx - y)^2 + (z - ax)^2\right] \right\} \left\{ \left[1 - \beta^2 (a^2 + m^2)\right](\eta - mx) + \beta^2 m \left[a(z - ax) - m(mx - y)\right] \right\}$$

$$+ \left[a(z - ax) - m(mx - y)\right] \left\{ (\eta - mx)^2 - \beta^2 \left[a(\eta - mx) - m(z - ax)\right]^2 - \beta^2 m^2 \left[(\eta - mx) + (mx - y)\right]^2 \right\}$$

$$= \left\{ \left[m(mx - y) - a(z - ax)\right](\eta - mx) + m\left[(mx - y)^2 + (z - ax)^2\right] \right\} \left\{ \left[1 - \beta^2 (a^2 + m^2)\right](\eta - mx) + \beta^2 m \left[a(z - ax) - m(mx - y)\right] \right\}$$

$$+ \left[a(z - ax) - m(mx - y)\right] \left\{ (\eta - mx)^2 \left[1 - \beta^2 (a^2 + m^2)\right] + 2\beta^2 m \left[a(z - ax) - m(mx - y)\right](\eta - mx) - \beta^2 m^2 \left[(z - ax)^2 + (mx - y)^2\right] \right\}$$

$$= m(\eta - mx) \left\{ \left[1 - \beta^2 (a^2 + m^2)\right] \left[(mx - y)^2 + (z - ax)^2\right] + \beta^2 \left[a(z - ax) - m(mx - y)\right]^2 \right\}$$

$$= m(\eta - mx) \left\{ (mx - y)^2 + (z - ax)^2 - \beta^2 \left[m^2 (z - ax)^2 + 2am(z - ax)(mx - y) + a^2 (mx - y)^2\right] \right\}$$

$$= m(\eta - mx) \left\{ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mx)^2 \right\}$$

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Therefore

$$\frac{d}{d\eta}\tan^{-1}\frac{(ay - mz)\sqrt{(\eta - mx)^2 - \beta^2\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]}}{\left[m(mx - y) - a(z - ax)\right]\eta + m\left[z(z - ax) - y(mx - y)\right]}$$

$$= \frac{m(ay - mz)(\eta - mx)}{\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]\sqrt{(\eta - mx)^2 - \beta^2\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]}} \tag{48}$$

Therefore combining terms in (43), (46), (47), and (48)

$$\frac{1}{2}\frac{d}{d\eta}\int \log\frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}\, d\eta \quad =$$

$$\frac{1}{2}\log\frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2}}$$

$$+ \left\{\eta - \frac{m(az + my)}{a^2 + m^2}\right\}\frac{m^2\left[y(mx - y) - z(z - ax)\right] + m\left[a(z - ax) - m(mx - y)\right]}{\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]\sqrt{(\eta - mx)^2 - \beta^2\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]}}$$

$$- \frac{m\left[(az + my) - (a^2 + m^2)x\right]}{(a^2 + m^2)\sqrt{(mx - \eta)^2 - \beta^2\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]}}$$

$$+ \frac{m^2(ay - mz)^2(\eta - mx)}{(a^2 + m^2)\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]\sqrt{(mx - \eta)^2 - \beta^2\left[(a\eta - mz)^2 + m^2(\eta - y)^2\right]}}$$

Arranging all terms over a common denominator

$$= \frac{1}{2}\log\frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}} + \left\{\frac{m}{(a^2+m^2)\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]\sqrt{(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(mz-a\eta)^2}}\right\}$$

$$\times\left\{ \left\{(a^2+m^2)\eta-m(az+my)\right\}\left\{m\left[y(mx-y)-z(z-ax)\right]+\left[a(z-ax)-m(mx-y)\right]\eta\right\}\right.$$

$$\left. -\left[a(z-ax)-m(mx-y)\right]\left[(a\eta-mz)^2+m^2(\eta-y)^2\right]+m(ay-mz)^2(\eta-mx)\right\}$$

The numerator can be written

$$\eta^2\left\{(a^2+m^2)\left[a(z-ax)-m(mx-y)\right]-(a^2+m^2)\left[a(z-ax)-m(mx-y)\right]\right\}$$

$$+\ \eta\left\{-m(az+my)\left[a(z-ax)-m(mx-y)\right]+(a^2+m^2)\,m\left[y(mx-y)-z(z-ax)\right]\right.$$

$$\left.+\ 2\left[a(z-ax)-m(mx-y)\right]\left[az+my\right]m+m(ay-mz)^2\right\}$$

$$-\ m^2(az+my)\left[y(mx-y)-z(z-ax)\right]-m^2\left[a(z-ax)-m(mx-y)\right](y^2+z^2)-m^2x(ay-mz)^2$$

$$= m\eta\left\{(z-ax)\left[-a(az+my)-(a^2+m^2)z+2a\left[az+my\right]-m(ay-mz)\right]\right.$$

$$\left.+\ (mx-y)\left[m(az+my)+(a^2+m^2)y-2m\left[az+my\right]-a(ay-mz)\right]\right\}$$

$$+\ m^2\left\{(z-ax)\left[z(az+my)-a(y^2+z^2)+mx(ay-mz)\right]\right.$$

$$\left.+\ (mx-y)\left[-y(az+my)+m(y^2+z^2)+ax(ay-mx)\right]\right\}$$

$$= m^2\left\{(z-ax)\left[-zm(mx-y)+ay(mx-y)\right]\right.$$

$$\left.+\ (mx-y)\left[-ay(z-ax)+mz(z-ax)\right]\right\} = 0$$

Therefore (43) is verified.

Evaluation of the definite integral over η. - Because of (4) and (6) and analogous to the section in the evaluation of the definite integral for the surface distribution of sources, all of the terms in (43) will be zero when $\eta = \eta_3$ provided the denominator of the $\tan^{-1}$ term is greater than zero. At $\eta = \eta_3$, using (6), and then (5),

$$\left[m(mx - y) - a(z - ax)\right]\eta_3 + m\left[z(z - ax) - y(mx - y)\right]$$

$$= \frac{m\left[m(mx - y) - a(z - ax)\right]\left\{(x - \beta^2 az) - \beta^2 my - \beta\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mx)^2}\right\}}{\left[1 - \beta^2(a^2 + m^2)\right]}$$

$$+ m\left[z(z - ax) - y(mx - y)\right]$$

$$= m\left\{\left[m(x - \beta^2 az) - y(1 - \beta^2 a^2)\right](mx - y) + \left[(1 - \beta^2 m^2)z - a(x - \beta^2 my)\right](z - ax)\right. \tag{49}$$

$$\left. - \beta\left[m(my - x) - a(z - ax)\right]\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}\right\}\left[1 - \beta^2(a^2 + m^2)\right]^{-1}$$

$$= \frac{m\left\{\left[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2\right] - \beta\left[m(mx - y) - a(z - ax)\right]\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}\right\}}{1 - \beta^2(a^2 + m^2)}$$

$$= \frac{m\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}}{1 - \beta^2(a^2 + m^2)}\left\{\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} - \beta\left[m(mx - y) - a(z - ax)\right]\right\}$$

But

$$\left[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2\right] - \beta^2\left[m(mx - y) - a(z - ax)\right]^2$$

$$= \left[1 - \beta^2(a^2 + m^2)\right]\left[(mx - y)^2 + (z - ax)^2\right] + \beta^2 a^2(mx - y)^2 + \beta^2 m^2(z - ax)^2 \tag{50}$$

$$+ 2\beta^2 am(mx - y)(z - ax) - \beta^2(ay - mz)^2$$

$$= \left[1 - \beta^2(a^2 + m^2)\right]\left[(mx - y)^2 + (z - ax)^2\right] + \beta^2\left[a(mx - y) + m(z - ax)\right]^2 - \beta^2(ay - mz)^2$$

$$= \left[1 - \beta^2(a^2 + m^2)\right]\left[(mx - y)^2 + (z - ax)^2\right]$$

Therefore if $1 - \beta^2 (a^2 + m^2) > 0$

$$\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} > \beta \left| m (mx - y) - a (z - ax) \right|$$

and from (49)

$$\Big[m (mx - y) - a (z - ax) \Big] \eta_3 + m \Big[z (z - ax) - y (mx - y) \Big] > 0$$

However also,

$$m (mx - y) - a (z - ax) = (a^2 + m^2) x - (my + az)$$

and

$$(a^2 + m^2) x^2 - (my + az)^2 = (a^2 + m^2)^2 x^2 - (a^2 + m^2)(y^2 + z^2) + (ay - mz)^2$$

$$= \frac{a^2 + m^2}{\beta^2} \Big\{ \Big[\beta^2 (a^2 + m^2) - 1 \Big] x^2 + \Big[x^2 - \beta^2 (y^2 + z^2) \Big] \Big\} + (ay - mz)^2 > 0$$

if

$$1 - \beta^2 (a^2 + m^2) < 0$$

and

$$X^2 > \beta^2 (y^2 + z^2)$$

Therefore if $x > 0$ also

$$(a^2 + m^2 X) > |my + az|$$

and

$$m(mx - y) - a(z - ax) > 0$$

if

$$1 - \beta^2 (a^2 + m^2) < 0 \qquad X^2 > \beta^2 (y^2 + z^2)$$

Therefore from (50) if $1 - \beta^2 (a^2 + m^2) < 0$

$$\left[\beta^2 (a^2 + m^2) - 1\right]^{-1} \left\{ \beta \left[m(mx - y) - a(z - ax)\right] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \right\} > 0$$

and from (49), if $x^2 > \beta^2 (y^2 + z^2)$

$$\left[m(mx - y) - a(z - ax)\right] \eta_3 + m\left[z(z - ax) - y(mx - y)\right] > 0$$

and therefore, using (4) and (54), the $\tan^{-1}$ term in (45) is zero when $\eta = \eta_3$.

Therefore substituting $\eta = 0$ into (43), and from (32),

$$\phi(x,y,z) = \frac{\frac{\Delta P}{q_\infty}}{8\pi a}\left\{ \int_0^{\eta_3} \log \frac{(mx-\eta)+\sqrt{(mx-\eta)^2-\beta^2 m^2(\eta-y)^2-\beta^2(a\eta-mz)^2}}{(mx-\eta)-\sqrt{(mx-\eta)^2-\beta^2 m^2(\eta-y)^2-\beta^2(a\eta-mz)^2}}\, d\eta \right.$$

$$\left. -\sqrt{1-\beta^2a^2}\int_0^{\eta_3} \log \frac{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)+\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(a\eta-mz)^2]}}{(mx-\eta)-\beta^2a(mz-a\eta)+\beta^2m^2(\eta-y)-\sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2-\beta^2m^2(\eta-y)^2-\beta^2(a\eta-mz)^2]}}\, d\eta \right\}$$

$$= \frac{1}{4\pi a}\,\frac{\Delta P}{q_\infty}\left\{ \frac{m(az+my)}{2(a^2+m^2)}\log\frac{x+\sqrt{x^2-\beta^2(y^2+z^2)}}{x-\sqrt{x^2-\beta^2(y^2+z^2)}} \right.$$

$$-\frac{m[(az+my)-(a^2+m^2)x]}{2(a^2+m^2)\sqrt{1-\beta^2(a^2+m^2)}}\log\frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}$$

$$+\frac{m(ay-mz)}{a^2+m^2}\tan^{-1}\frac{(mz-ay)\sqrt{x^2-\beta^2(y^2+z^2)}}{z(z-ax)-y(mx-y)}$$

$$+(z-ax)\tan^{-1}\frac{m(z-ax)\sqrt{x^2-\beta^2(y^2+z^2)}}{y[(y-mx)-\beta^2a(ay-mz)]+(z-ax)^2}$$

$$-\frac{m(x-\beta^2az)-y(1-\beta^2a^2)}{2\sqrt{1-\beta^2(a^2+m^2)}}\log\frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}$$

$$\left. -\frac{1}{2}y\sqrt{1-\beta^2a^2}\log\frac{(x-\beta^2az)+\sqrt{(1-\beta^2a^2)[x^2-\beta^2(y^2+z^2)]}}{(x-\beta^2az)-\sqrt{(1-\beta^2a^2)[x^2-\beta^2(y^2+z^2)]}} \right\}$$

Therefore combining terms

$$
\begin{aligned}
\phi(x,y,z) = \frac{1}{4\pi a}\left(\frac{\Delta P}{q_\infty}\right)\Bigg\{ & \frac{m(az+my)}{2(a^2+m^2)} \log \frac{x+\sqrt{x^2-\beta^2(y^2+z^2)}}{x-\sqrt{x^2-\beta^2(y^2+z^2)}} \\
& + \frac{a(ay+mz)\sqrt{1-\beta^2(a^2+m^2)}}{2(a^2+m^2)} \log \frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}} \\
& - \frac{m(mz-ay)}{a^2+m^2} \tan^{-1} \frac{(mz-ay)\sqrt{x^2-\beta^2(y^2+z^2)}}{z(z-ax)-y(mx-y)} \\
& + (z-ax)\tan^{-1} \frac{m(z-ax)\sqrt{x^2-\beta^2(y^2+z^2)}}{y\left[(y-mx)-\beta^2 a(ay-mz)\right]+(z-ax)^2} \\
& - \frac{1}{2}\, y\sqrt{1-\beta^2 a^2}\, \log \frac{x-\beta^2 az+\sqrt{(1-\beta^2a^2)[x^2-\beta^2(y^2+z^2)]}}{x-\beta^2 az-\sqrt{(1-\beta^2a^2)[x^2-\beta^2(y^2+z^2)]}}
\end{aligned}
\qquad (51)
$$

Evaluation of the velocity components. - Using the arguments presented in the section covering the evaluation of the velocity components for the source distribution, the partial derivatives of (51) may be obtained by differentiating only the coefficients of each term.

Modifications and Regions of Validity of the Velocity Potential Functions

Region of validity of ϕ. - All of the integrations were performed without regard to the existence of the limits of integration, negative square roots, etc. Therefore the functions in (29) and (51) must be examined and possibly modified for some regions of (x, y, z) space.

Since all of the terms contain $\sqrt{x^2-\beta^2(y^2+z^2)}$ the formula for $\phi(x,y,z)$ given by (51) is only valid when $x^2 > \beta^2(y^2+z^2)$, which means inside the Mach cone from the origin.

If $1-\beta^2(a^2+m^2) < 0$ (supersonic leading edge) the log term which contains the square root of this quantity must be modified. We can write

$$\frac{i}{2} \log \frac{f+ig}{f-ig} = -\tan^{-1} \frac{g}{f} \tag{52}$$

and therefore if $1-\beta^2(a^2+m^2) < 0$

$$\frac{1}{2}\sqrt{1-\beta^2(a^2+m^2)} \log \frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}$$

$$= -\sqrt{\beta^2(a^2+m^2)-1} \tan^{-1} \frac{\sqrt{[\beta^2(a^2+m^2)-1][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)}. \tag{53}$$

If $1-\beta^2(a^2+m^2) = 0$ there is a sonic leading edge. For the case where $1-\beta^2(a^2+m^2) \to 0$ we can write

$$\lim_{1-\beta^2(a^2+m^2)\to 0} \frac{1}{2}\sqrt{1-\beta^2(a^2+m^2)} \log \frac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}$$

$$= [1-\beta^2(a^2+m^2)] \frac{\sqrt{x^2-\beta^2(y^2+z^2)}}{x-\beta^2(az+my)} \tag{54}$$

All of the log terms appear to potentially involve logarithms of negative numbers. This difficulty may be avoided by taking the absolute value of the arguments. This is allowable since any log of -1 would have canceled in the definite integral over η. The argument could not have passed through zero and therefore must have been always positive or always negative.

Since certain functions occur repeatedly we define

$$F1 = \tan^{-1} \frac{m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2}$$

$$F2 = \frac{1}{2\sqrt{1 - \beta^2(a^2 + m^2)}} \log \frac{x - \beta^2(my + az) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(my + az) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}$$

$$F3 = \tan^{-1} \frac{(mz - ay)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \tag{56}$$

$$F6 = \sqrt{1 - \beta^2 a^2}\ \frac{1}{2} \log \frac{x - \beta^2 az + \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2 az - \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}}$$

<u>Evaluation of the $\tan^{-1}$ functions F1 and F3.</u> - The terms F1 or F3 are always real inside the Mach cone from the origin,

$$x^2 - \beta^2(y^2 + z^2) \geq 0$$

However as the argument of these functions go to zero the functions may take on different values depending upon how zero is approached. Corresponding to each of the four quadrants, if

$$\theta = \tan^{-1} \frac{f}{g}$$

then

$$f \geq 0 \quad g > 0 \quad \text{means} \quad 0 \leq \theta < \frac{\pi}{2}$$

$$f \geq 0 \quad g < 0 \quad \text{means} \quad \frac{\pi}{2} < \theta < \pi$$

$$f \leq 0 \quad g > 0 \quad \text{means} \quad 0 \geq \theta > -\frac{\pi}{2}$$

$$f \leq 0 \quad g < 0 \quad \text{means} \quad -\frac{\pi}{2} > \theta \geq -\pi \tag{57}$$

Therefore from (52) and (57) as $z \to ax$

$$\lim_{z \to ax} F1 = \begin{cases} 0 & y < 0 \text{ or } y > mx \\ \pi & 0 < y < mx,\ z > ax \\ -\pi & 0 < y < mx,\ z > ax \end{cases} \tag{58}$$

and as $mz \to ay$

$$\lim_{mz \to ay} F3 = \begin{cases} 0 & y < 0 \text{ or } y > mx \\ \pi & 0 < y < mx \quad mz > ay \\ -\pi & 0 < y < mx \quad mz < ay \end{cases} \tag{59}$$

Velocity potential on $x^2 = \beta^2(y^2+z^2)$ if $1-\beta^2(a^2+m^2) > 0$. Referring to (52) F2 and F6 are zero if $x^2-\beta^2(y^2+z^2) = 0$. The functions F1 and F3 will be zero if and only if the denominators of their arguments are greater than zero[see (57)]. First we note the following

$$
\begin{aligned}
x^2 - [\beta^2(my + az)]^2 &= [x^2 - \beta^2(y^2 + z^2)] + \beta^2(y^2 + z^2) - \beta^4[m^2y^2 + 2amyz + a^2z^2] \\
&= [x^2 - \beta^2(y^2 + z^2)] + \beta^2[1 - \beta^2(a^2 + m^2)]\,(y^2 + z^2) + \beta^4[a^2y^2\ 2amyz + m^2z^2] \\
&= [x^2 - \beta^2(y^2 + z^2)] + \beta^2[1 - \beta^2(a^2 + m^2)](y^2 + z^2) + \beta^4(ay - mz)^2 \\
&= [x - \beta^2(my + az)][x + \beta^2(my + az)]
\end{aligned}
\tag{60}
$$

which means $\quad x > |\beta^2(my + az)|$

if $\quad x^2-\beta^2(y^2+z^2) \geq 0 \qquad 1-\beta^2(a^2+m^2) > 0$

For F1 the denominator is

$$y\ \ [(y - mx) - \beta^2 a\ (ay - mz)]\ + (z - ax)^2$$

$$= y^2\ [1 - \beta^2\ (a^2 + m^2)]\ - my\,[x - \beta^2\ (az + my)\,]\ + (z - ax)^2$$

> 0 if $y < 0$ from (60)

$$y \; [(y - mx) - \beta^2 a \; (ay - mz)] + (z - ax)^2$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 \; (ay - mz)^2 + mx \; (y - mx) - \beta^2 mz \; (ay - mz)$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 \; (ay - mz)^2 + mxy - m^2 \; (x^2 - \beta^2 z^2) - \beta^2 amyz$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 \; (ay - mz)^2 + my \, [x - \beta^2 \; (az + my)]$$

$$- m^2 \, [x^2 - \beta^2 \; (y^2 + z^2)] > 0$$

if $x^2 - \beta^2 \; (y^2 + z^2) = 0, \; y > 0$ due to (50) and (60)

Therefore

$$y \, [(y - mx) - \beta^2 a \; (ay - mz)] + (z - ax)^2 > 0$$

on

$$x^2 = \beta^2 \; (y^2 + z^2) \tag{61}$$

if

$$1 - \beta^2 \; (a^2 + m^2) > 0$$

Therefore due to (57) and (61)

$$\lim F1 = 0$$

$$\text{as } x^2 - \beta^2 \; (y^2 + z^2) \to 0$$

if

$$1 - \beta^2 \; (a^2 + m^2) > 0$$

or F3

$$(z - ax) - y (mx - y) = y^2 + z^2 - x (az + my) \tag{62}$$

$$= - [x^2 - \beta^2 (y^2 + z^2)]\frac{1}{\beta^2} + \frac{x}{\beta^2} [x - \beta^2 (az + my)] > 0$$

if

$x^2 - \beta^2 (y^2 + z^2) = 0$, and $1 - \beta^2 (a^2 + m^2 > 0$ due to (54c) if $x > 0$

Therefore examining (29) and (56) shows that for a surface distribution of sources or a constant pressure surface

$$\lim \phi = 0$$

$$\text{as } x^2 - \beta^2 (y^2 + z^2) \rightarrow 0$$

if

$$1 - \beta^2 (a^2 + m^2) \rightarrow 0 . \tag{63}$$

Therefore ϕ will be continuous, as it must be, if we define ϕ to be zero outside the Mach cone from the origin.

$$\phi = 0$$

if

$$1 - \beta^2 (a^2 + m^2) > 0 \text{ and } x^2 < \beta^2 (y^2 + z^2) \tag{64}$$

Supersonic leading edge and the mach cone envelope. With a supersonic leading edge, $1 - \beta^2 (a^2 + m^2) < 0$, all functions in the equations for ϕ will be shown to go to zero for points on the Mach cone from the origin, except for a region on this Mach cone which borders the envelope of Mach cones from the supersonic leading edge. Inside this envelope of Mach cones the functions F1, F2 and F3 will be shown to have constant values. However on the outer boundary of this envelope of Mach cones ϕ will go to zero, and therefore all functions may be defined to be zero outside this envelope.

The envelope of Mach cones from the leading edge is illustrated on p-29 of Reference (55). The Mach cone from any point x_o, y_o, z_o can be written,

$$(x - x_o)^2 = \beta^2 [(y - y_o)^2 + (z - z_o)^2]$$

on the leading edge $mx_o = y_o$ and $ax_o = z_o$

Therefore

$$(mx - y_o)^2 = \beta^2 [m^2 (y - y_o)^2 + (mz - ay_o)^2] \tag{65}$$

The Mach cone envelope is determined by the maximum values for z, at a given x and y, obtained by a variation of y_o. Therefore differentiating (65) with respect to y_o, holding x and y constant, and setting $dz/dy_o = 0$

gives

$$(mx - y_o) = \beta^2 m^2 (y - y_o) + \beta^2 a (mz - ay_o)$$

or

$$y_o [1 - \beta^2 (a^2 + m^2)] = m [x - \beta^2 (my + az)] \tag{66}$$

If $y_o = 0$ (65) and (66) give

$$x^2 = \beta^2 (y^2 + z^2) \text{ and } x - \beta^2 (my + az) = 0 \tag{67}$$

or

$$x^2 = \beta^2 y^2 + \beta^2 \left[\frac{x - \beta^2 my}{\beta^2 a} \right]^2$$

$$\beta^2 a^2 x^2 = \beta^4 a^2 y^2 + x^2 - 2\beta^2 mxy + \beta^4 m^2 y^2$$

$$\beta^4 (a^2 + m^2) y^2 - 2\beta^2 mxy + x^2 (1 - \beta^2 a^2) = 0$$

or, if $y_o = 0$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m \pm a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \qquad (68)$$

where

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m - a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \qquad z > 0$$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \qquad z < 0$$

For $y_o > 0$ we can eliminate y_o from (65) and (66).

From (65)

$$\left[1 - \beta^2 (a^2 + m^2)\right] y_o^2 - 2m\left[x - \beta^2 (az + my)\right] y_o + m^2\left[x^2 - \beta^2(y^2 + z^2)\right] = 0$$

or

$$y_o\left[1 - \beta^2 (a^2 + m^2)\right] = m\left[x - \beta^2 (az + my)\right]$$

$$\pm m \sqrt{\left[x - \beta^2 (az + my)\right]^2 - \left[1 - \beta^2 (a^2 + m^2)\right]\left[x^2 - \beta^2 (y^2 + z^2)\right]}$$

Therefore from (66)

$$\left|x - \beta^2 (az + my)\right|^2 - \left|1 - \beta^2 (a^2 + m^2)\right| \left|x^2 - \beta^2 (y^2 + z^2)\right| = 0$$

or

$$\beta^2 (a^2 + m^2) x^2 - 2 x \beta^2 (az + my) - \beta^4 (ay - mz)^2 + \beta^2 (y^2 + z^2) = 0$$

and solving for x

$$x = \frac{(az + my) \pm \sqrt{(az + my)^2 + (a^2 + m^2) \left[(ay - mz)^2 \beta^2 - (y^2 + z^2) \right]}}{(a^2 + m^2)}$$

$$= \frac{(az + my) \pm \sqrt{(ay - mz)^2 \left[\beta^2 (a^2 + m^2) - 1 \right]}}{(a^2 + m^2)}$$

$$= \frac{(az + my) \pm |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}$$

If we write

$$x = \frac{(az + my) \pm (mz - ay) \sqrt{\beta^2 (a^2 + m^2) -1}}{(a^2 + m^2)}$$

or

$$\frac{\partial z}{\partial y}\left[a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right] = \left[-m \pm a \sqrt{\beta^2 (a^2 + m^2) - 1} \right]$$

$$\frac{\partial z}{\partial y}(a^2 + m^2)\left[1 - \beta^2 m^2\right]$$

$$= \left(- a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1}\right)\left(- m \pm a \sqrt{\beta^2 (a^2 + m^2) -1}\right)$$

$$= (a^2 + m^2)\left[- \beta^2 am \pm \sqrt{\beta^2 (a^2 + m^2) - 1}\right]$$

Therefore use

$+$ for $mz > ay$

$-$ for $mz < ay$

or

$$x = \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \tag{69}$$

This is presented on page 29 of reference (55).

If we set

$$z = y \tan \theta \quad \text{on } x^2 = \beta^2 (y^2 + z^2)$$

then

$$x^2 = \beta^2 y^2 (1 + \tan^2 \theta)$$

or

$$x = \frac{\beta y}{\cos\theta} \qquad \beta y = x \cos\theta \tag{70}$$

Therefore (67) gives for the two points where the envelope of Mach cones from the leading edge is on the Mach cone from the origin,

$$(y^2 + z^2) = \beta^2 (my + az)^2$$

or

$$\left[1 + \tan^2\theta\right] = \beta^2 m^2 + 2\beta^2 am \tan\theta + \beta^2 a^2 \tan^2\theta$$

which means

$$(1 - \beta^2 a^2)\tan\theta = \beta^2 am \pm \sqrt{\beta^4 a^2 m^2 - (1 - \beta^2 a^2)(1 - \beta^2 m^2)}$$

or

$$(1 - \beta^2 a^2)\tan\theta = \beta^2 am \pm \sqrt{\beta^2 (a^2 + m^2) - 1} \tag{71}$$

but

$$x - \beta^2 (my + az) = 0$$

means

$$\frac{\beta a}{\cos\theta} - \beta^2 a (m + a \tan\theta) = 0 \tag{72}$$

Therefore the two points corresponding to (68) are

$$\tan \theta_1 = \frac{\beta a}{\cos \theta_1} - \sqrt{\beta^2 (a^2 + m^2) - 1}$$

$$\tan \theta_2 = \frac{\beta a}{\cos \theta_2} + \sqrt{\beta^2 (a^2 + m^2) - 1} \tag{73}$$

(assume $a > 0$ and $\theta_2 > \theta_1$)

From (56) and (57) the value of F1 or F3 as we approach the Mach cone from the origin depends on the sign of the denominator of its argument.

For

F 1 on $x^2 = \beta^2 (y^2 + z^2)$ we can write

$$D1 = \frac{1}{y^2} \left\{ y (y - mx) - \beta^2 a (ay - mz) + (z - ax)^2 \right\}$$

$$= (1 - \beta^2 a^2) - \frac{\beta m}{\cos \theta} + \beta^2 am \tan \theta + \left[\tan \theta - \frac{\beta a}{\cos \theta} \right]^2$$

$$= 1 - \beta^2 (a^2 + m^2) + \left[\tan \theta - \frac{\beta a}{\cos \theta} \right]^2 - \beta m \left[\beta m + \frac{1}{\cos \theta} - \beta a \tan \theta \right]$$

$= 0$ if $\theta = \theta_1$, or $\theta = \theta_2$ from (72) and (73)

If we differentiate this with respect to θ we get

$$\frac{dD1}{d\theta} = 2\left[\tan\theta - \frac{\beta a}{\cos\theta}\right]\left[\sec^2\theta - \frac{\beta a \sin\theta}{\cos^2\theta}\right] + \beta m\left[\beta a \sec^2\theta - \frac{\sin\theta}{\cos^2\theta}\right]$$

$$= \frac{1}{\cos^2\theta}\left[\tan\theta - \frac{\beta a}{\cos\theta}\right] \left| 2\,(1 - \beta a \sin\theta) - \beta m \cos\theta \right|$$

$$= \frac{1}{\cos^2\theta}\left[\tan\theta - \frac{\beta a}{\cos\theta}\right] \left| 2\,(1 - \beta a \sin\theta - \beta m \cos\theta\,) + \beta m \cos\theta \right|$$

From (72)

$$1 - \beta a \sin\theta - \beta m \cos\theta = 0 \quad \text{at} \quad \theta = \theta_1,\ \theta_2$$

and since

$$\cos\theta_1, > 0,\ \cos\theta_2 > 0 \text{ and } \beta m > 0$$

and using (73) we get

$$\frac{dD1}{d\theta} < 0 \qquad \theta = \theta_1$$

$$\frac{dD1}{d\theta} > 0 \qquad \theta = \theta_2$$

and therefore using (57) if

$$1 - \beta^2 (a^2 + m^2) < 0$$

$$\lim_{x^2 \to \beta^2 (y^2 + z^2)} F1 = \begin{cases} 0 & \theta > \theta_2 \quad \theta < \theta_1 \\ \pi & z > ax \quad \theta_1 < \theta < \theta_2 \\ -\pi & z < zx \quad \theta_1 < \theta < \theta_2 \end{cases} \tag{74}$$

For

$$F3 \text{ on } x^2 = \beta^2 (y^2 + z^2)$$

let

$$\begin{aligned} D3 &= z (z - ax) - y (mx - y) \\ &= y^2 (1 + \tan^2 \theta) - yx (m + a \tan \theta) \\ &= \frac{x^2}{\beta^2} \left[1 - \beta \cos \theta \ (m + a \tan \theta) \right] \\ &= \frac{x^2}{\beta^2} \left[1 - (\beta m \cos \theta + \beta a \sin \theta) \right] \\ &= 0 \text{ if } \theta = \theta_1 \text{ or } \theta = \theta_2 \text{ from (72)} \end{aligned}$$

$$\frac{d}{d\theta}\, D3 = \frac{x^2}{\beta}\left[m \sin\theta - a\cos\theta\right]$$

$$= \frac{x^2 \cos\theta}{\beta\,(1 - \beta^2 a^2)}\left[m\,(1 - \beta^2 a^2)\tan\theta - a\,(1 - \beta^2 a^2\right]$$

$$= \frac{x^2 \cos\theta}{\beta(1 - \beta^2 a^2)}\left[a\,\beta^2 m^2 - a\,(1 - \beta^2 a^2) \pm m\sqrt{\beta^2\,(a^2 + m^2) - 1}\right]$$

$$= \frac{x^2 \cos\theta}{\beta(1 - \beta^2 a^2)}\left\{- a\left[1 - \beta^2\,(a^2 + m^2)\right] \pm m\sqrt{\beta^2\,(a^2 + m^2) - 1}\right\}$$

$$= \frac{x^2 \cos\theta\,\sqrt{\beta^2\,(a^2 + m^2) - 1}}{\beta\,(1 - \beta^2 a^2)}\left\{a\sqrt{\beta^2\,(a^2 + m^2) - 1} \pm m\right\}$$

> 0 for θ_2 $(+m)$ [see (73)]

< 0 for θ_1 $(-m)$ (assuming $a > 0$)

because

$$m^2 > a^2\left[\beta^2\,(a^2 + m^2) - 1\right]$$

since

$$(a^2 + m^2)\,(1 - \beta^2 a^2) > 0$$

Therefore since

$$\frac{d}{d\theta} D3 \begin{array}{ll} >0 & \theta = \theta_2 \\ <0 & \theta = \theta_1 \end{array}$$

and therefore

$$D3 \begin{array}{ll} >0 & \theta > \theta_2 \text{ or } \theta < \theta_1 \\ <0 & \theta_1 < \theta < \theta_2 \end{array} \tag{75}$$

We can say

$$\lim_{x^2 \to \beta^2 (y^2 + z^2)} F\,3 = \begin{array}{lll} 0 & \theta > \theta_2 \;\; \theta < \theta_1 & \\ \pi & mz > ay & \theta_1 < \theta < \theta_2 \\ -\pi & mz < ay & \theta_1 < \theta < \theta_2 \end{array} \tag{76}$$

Also on the plane

$$mz = ay$$

$$D3 = z\,(z - ax) - y\,(mx-y) = y^2 \left[1 + \frac{a^2}{m^2}\right] - xy \left[m + \frac{a^2}{m}\right]$$

$$= \frac{y\,(y-mx)\,(a^2 + m^2)}{m^2}$$

and therefore

$$\lim_{(ay-mz) \to 0} F3 = \begin{array}{lll} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & mz > ay \\ -\pi & 0 < y < mx & mz < ay \end{array} \tag{77}$$

To find the limit of F2 as $x^2 \to \beta^2 (y^2 + z^2)$ when $1 - \beta^2 (a^2 + m^2) < 0$ [see (53) for F2] we note that

$$\lim_{x^2 \to \beta^2 (y^2 + z^2)} \left| \frac{x - \beta^2 (my + az)}{\beta \sqrt{(mx-y)^2 + (z-ax)^2 - \beta^2 (ay-mz)^2}} \right| = 1$$

because

$$\beta^2 \left|(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2\right| = \beta^2 (a^2 + m^2) x^2 + \beta^2 (y^2 + z^2) - 2x\beta^2 (my + az) + 2\beta^4 amyz - \beta^2 a^2 y^2 - \beta^4 m^2 z^2$$

$$= [\beta^2 (a^2 + m^2) - 1]\, [x^2 - \beta^2 (y^2 + z^2)] + x^2 - 2x\beta^2 (my + az) + 2\beta^4 amyz + \beta^4 m^2 y^2 + \beta^4 a^2 z^2$$

$$= [\beta^2 (a^2 + m^2) - 1]\, [x^2 - \beta^2 (y^2 + z^2)] + [x - \beta^2 (my + az)]^2$$

Now

$$x - \beta^2 (my + az) = x \left[1 - \beta^2 m \frac{y}{x} - \beta^2 a \frac{y}{z} \tan \theta \right]$$

$$= x\, [1 - \beta m \cos \theta - \beta a \sin \theta] = \frac{\beta^2}{x} D3$$

where D3 is defined by (75). Therefore from (75)

$$\lim_{x^2 \to \beta^2 (y^2 + z^2)} F2 = \begin{cases} 0 & \theta_1 < \theta \quad \theta > \theta_2 \\ \dfrac{\pi}{\sqrt{\beta^2 (a^2 + m^2) - 1}} & \theta_1 < \theta < \theta_1 \end{cases} \tag{78}$$

Therefore combining these results

$$F1 = \tan^{-1} \frac{m (z-ax) \sqrt{x^2 - \beta^2 (y^2 + z^2)}}{y\, [(y-mx) - \beta^2 a (ay-mz)] + (z-ax)^2} \qquad x^2 > \beta^2 (y^2 + z^2)$$

If
$$1 - \beta^2 (a^2 + m^2) > 0$$

$$F1 = 0 \qquad x^2 \leq \beta^2 (y^2 + z^2) \tag{79}$$

$$\lim_{z \to ax} F1 = \begin{matrix} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & z > ax \\ \pi & 0 < y < mx & z < ax \end{matrix}$$

Then from (69) and (74) if

$$1 - \beta^2 (a^2 + m^2) < 0$$

and

$$x^2 \leq \beta^2 (a^2 + m^2)$$

$$F1 = 0 \qquad \theta > \theta_2 \quad \theta < \theta_1 \tag{80}$$

$$F1 = \pi \operatorname{sgn} (z-ax)$$

if
$$\theta_1 < \theta < \theta$$

and

$$x > \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} = 0$$

and $$F1 = 0$$

if $$\theta_1 < \theta < \theta_2$$

and $$x > \frac{(az + my) + mz + ay\sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}$$

and where from (71)

$$(1 - \beta^2 a^2) \tan \theta_1 = \beta^2 am - \sqrt{\beta^2 (a^2 + m^2) - 1}$$

$$(1 - \beta^2 a^2) \tan \theta_2 = \beta^2 am + \sqrt{\beta^2 (a^2 + m^2) - 1}$$

For

$$x^2 > \beta^2 (y^2 + z^2)$$

from (53), (54), and (56)

$$F2 = \begin{cases} \dfrac{1}{2\sqrt{1-\beta^2(a^2+m^2)}} \log \dfrac{x-\beta^2(az+my)+\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)-\sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}} & 1-\beta^2(a^2+m^2) > 0 \\ \dfrac{\sqrt{x^2-\beta^2(y^2+z^2)}}{x-\beta^2(az+my)} & 1-\beta^2(a^2+m^2) = 0 \\ \dfrac{-1}{\sqrt{\beta^2(a^2+m^2)-1}} \tan^{-1} \dfrac{\sqrt{[\beta^2(a^2+m^2)-1][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)} & 1-\beta^2(a^2+m^2) < 0 \end{cases} \tag{81}$$

and from (78)

$$F2 = 0 \qquad x^2 \leq \beta^2 (y^2+z^2) \qquad 1 - \beta^2 (a^2+m^2) > 0$$

$$= 0 \qquad x^2 \leq \beta^2 (y^2+z^2) \qquad \theta > \theta_2, \quad \theta > \theta_1$$

$$= \pi \qquad x^2 \leq \beta^2 (y^2+z^2) \qquad \theta_1 < \theta < \theta_2 \qquad 1 - \beta^2 (a^2+m^2) < 0$$

$$= 0 \qquad \text{outside envelope of Mach cones.} \tag{82}$$

For

$$x^2 > \beta^2 (y^2+z^2)$$

From (56) and (58)

$$F3 = \tan^{-1} \frac{(mz-ay) \sqrt{x^2 - \beta^2 (y^2+z^2)}}{z (z-ax) - y (mx-y)} \tag{83}$$

and

$$\lim_{(mz-ay) \to 0} F3 = \begin{array}{lll} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & mz > ay \\ -\pi & 0 < y < mx & mz < ay \end{array}$$

and for

$$x^2 \leq \beta^2 (y^2+z^2)$$

from (76)

$$
\begin{aligned}
F3 &= 0 \qquad \theta < \theta_1 \qquad \theta > \theta_2 \\
&= \pi \qquad mz > ay \qquad \theta_1 < \theta < \theta_2 \\
&= -\pi \qquad mz < ay \qquad \theta_1 < \theta < \theta_2 \\
&= 0 \qquad \text{outside envelope of Mach cones.}
\end{aligned}
\tag{84}
$$

<u>Value of ϕ on the envelope of mach cones</u>. For a supersonic leading edge $[1 - \beta^2 (a^2+m^2) < 0]$, in the region inside the envelope of mach cones from the leading edge and outside the mach cone from the origin, we have for a surface distribution of sources [from (29), (79), (80), and (81)]

$$
\begin{aligned}
\sqrt{\beta^2 (a^2+m^2) - 1}\,(1-\beta^2 a^2)\,\frac{\bar{W} + \beta^2 a\bar{u}'}{\pi}\,\phi_s &= (z-ax)\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1} + y\,(1-\beta^2 a^2) - m\,(x-\beta^2 az) \\
&= -x\left[m + a\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1}\right] + z\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1} + y\,(1-\beta^2 a^2) + \beta^2 amz \\
&= \frac{-x\,(a^2+m^2)\,(1-\beta^2 a^2) + [m - a\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1}]\,[z\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1} + y\,(1-\beta^2 a^2) + \beta^2 amz]}{m - a\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1}} \\
&= \frac{(1-\beta^2 a^2)\left\{-x\,(a^2+m^2) + (my+az) + \mathrm{sgn}\,(z-ax)\,(mz-ay)\sqrt{\beta^2 (a^2+m^2)-1}\right\}}{m - a\,\mathrm{sgn}\,(z-ax)\sqrt{\beta^2 (a^2+m^2)-1}}
\end{aligned}
$$

On the lines

$$x = \frac{(my + az) + |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}$$

$$\operatorname{sgn}(z - ax)(mz - ay) = |mz - ay|$$ [See figure in Woodward]

Therefore on the envelope of mach cones from the leading edge

$$\phi_s = 0$$

In this same region for a constant pressure surface [from (71)] and

$$\phi_p \left[\frac{1}{4\pi a} \left(\frac{\Delta P}{q_\infty}\right)\right]^{-1} (a^2 + m^2) \, \pi^{-1}$$

$$= -a(ay - mz)\sqrt{\beta^2 (a^2 + m^2) - 1} + m(ay - mz)\operatorname{sgn}(mz - ay) + (a^2 + m^2)(z - ax)\operatorname{sgn}(z - ax)$$

and in the region where

$$\operatorname{sgn}(z - ax) = \operatorname{sgn}(mz - ay)$$

$$= \frac{+ a\,|mz - ay|\,\sqrt{\beta^2(a^2 + m^2) - 1} + m\,(ay - mz) + (a^2 + m^2)\,(z - ax)}{\operatorname{sgn}(z - ax)}$$

$$= \operatorname{sgn}(z - ax)\, a \left| - x\,(a^2 + m^2) + (my + az) + |mz - ay|\,\sqrt{\beta^2\,(a^2 + m^2) - 1}\right|$$

$$= 0 \text{ on the lines } x = \frac{(my + az) + |mz - ay|\,\sqrt{\beta^2\,(a^2 + m^2) - 1}}{a^2 + m^2}$$

Therefore $\phi_p = 0$ on the envelope of Mach cones from the leading edge.

Verification of the imposed boundary conditions. Now (8) can be verified for the case of a surface distribution of sources. From (29) and (79), on $z = ax$, since the $\cosh^{-1}$ terms are continuous

$$\phi = \phi' \text{ or } \phi - \phi' = 0$$

For (8b) using (29) and (79) and the results of 2.6

$$u = -\frac{\bar{w} + \beta^2 a\bar{u}}{\pi(1 - \beta^2 a^2)} \left|\pi a + mF2\right| \qquad 0 < y < mx \quad z > ax$$

$$u' = -\frac{\bar{w} + \beta^2 a\bar{u}}{\pi(1 - \beta^2 a^2)} \left|-\pi a + mF2\right| \qquad 0 < y < mx \quad z < ax$$

$$w = \frac{\bar{w} + \beta^2 a\bar{u}}{\pi(1 - \beta^2 a^2)} \left|\pi + \beta^2 amF2\right| \qquad 0 < y < mx \quad z > ax$$

$$w' = \frac{\bar{w} + \beta^2 a\bar{u}}{\pi(1 - \beta^2 a^2)} \left|-\pi + \beta^2 amF2\right| \qquad 0 < y < mx \quad z < ax$$

Therefore

$$(w - w') + \beta^2 a(u - u') = 2\frac{\bar{w} + \beta^2 a\bar{u}}{\pi(1 - \beta^2 a^2)} \pi[1 - \beta^2 a^2] = 2[\bar{w} + \beta^2 a\bar{u}] = \text{const}$$

which agrees with (8b)

For

$$y < 0$$

or

$$y > mx$$

where

$$f = m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}$$

$$g = y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2$$

and since

$$f = 0 \text{ on } S$$

when

$$z = ax$$

$$\frac{\partial u}{\partial x} = \frac{-am\sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{y(y - mx)(1 - \beta^2 a^2)}$$

and likewise on

$$z = ax$$

$$\frac{\partial u}{\partial y} = 0$$

$$\frac{\partial u}{\partial z} = \frac{m\sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{y(y - mx)(1 - \beta^2 a^2)}$$

Therefore

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \text{ and } \frac{\partial u}{\partial z}$$

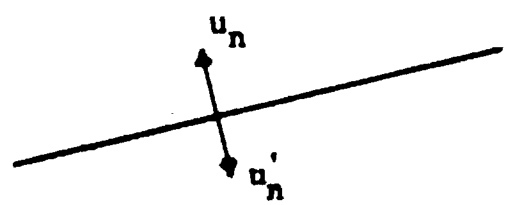
are continuous on S (except possibly at the edges) and therefore

$$\frac{\partial}{\partial x}(u - u') = \frac{\partial}{\partial y}(u - u') = \frac{\partial}{\partial z}(u - u') = 0$$

and (30b) is verified.

Surface velocities on a constant pressure surface. - Flow normal to surface U_n

$$u_n = \frac{1}{\sqrt{1 + a^2}}(-au + w)$$



$$u_n' = \frac{1}{\sqrt{1 + a^2}}(au' - w')$$

$$\text{Flow through surface} = \frac{1}{2}(u_n - u_n') = \frac{\frac{1}{2}}{\sqrt{1 + a^2}}\left|-a(u + u') + (w + w')\right|$$

on

$$(z = ax) \quad u + u' = 0$$

from (51) and (79)

$$\frac{1}{2}(w + w') = \frac{1}{4\pi a}\left(\frac{\Delta P}{q_\infty}\right)\left\{\frac{am}{(a^2 + m^2)}\ \frac{1}{2}\log\frac{x + \sqrt{x^2(1-\beta^2a^2) - \beta^2y^2}}{x - \sqrt{x^2(1-\beta^2a^2) - \beta^2y^2}}\right.$$

$$\frac{-am}{2(a^2 + m^2)}\sqrt{1 - \beta^2(a^2 + m^2)}\ \log\frac{x(1-\beta^2a^2) - \beta^2my + \sqrt{[1-\beta^2(a^2+m^2)][x^2(1-\beta^2a^2) - \beta^2y^2]}}{x(1-\beta^2a^2) - \beta^2my - \sqrt{[1-\beta^2(a^2+m^2)][x^2(1-\beta^2a^2) - \beta^2y^2]}}$$

$$\left.\frac{-m^2}{a^2 + m^2}\tan^{-1}\frac{(mz - ay)\sqrt{x^2(1 - \beta^2a^2) - \beta^2y^2}}{-y(mx - y)}\right\}$$

$$= \frac{1}{2}\sqrt{1 + a^2}\,(u_n - u_n')$$

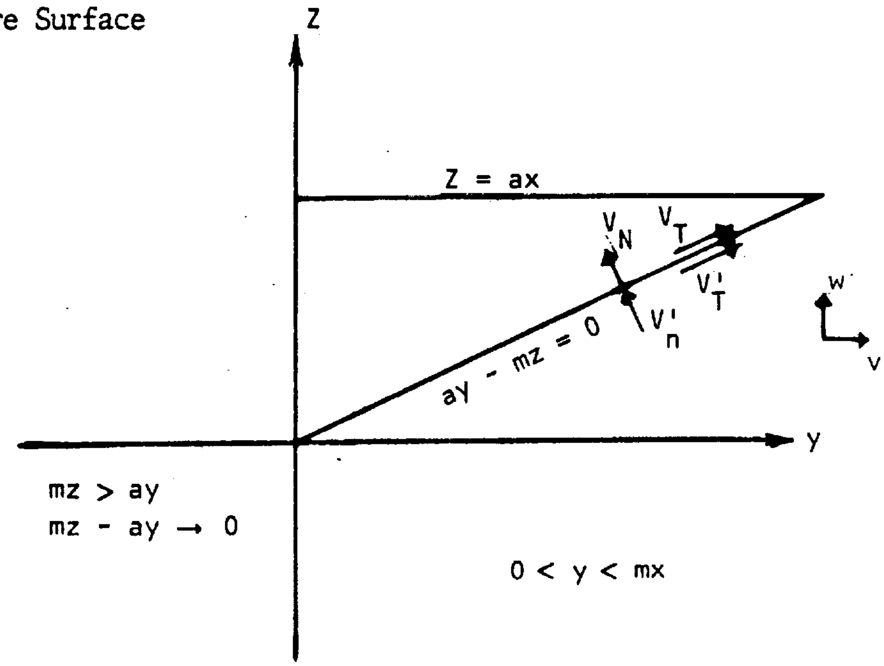
$$\text{Source strength} = u_n + u_n' = \frac{1}{\sqrt{1 + a^2}}\left|-a(u - u') + (w - w')\right|$$

$$= \frac{(1 + a^2)}{\sqrt{1 + a^2}}\ \frac{2\Delta P}{4\pi a q_\infty}\pi$$

$$= 2\frac{\sqrt{1 + a^2}\,\Delta P}{4aq_\infty} \tag{86}$$

Velocity discontinuity in the leading edge wake. -

Constant Pressure Surface



$$V_T = \frac{v + \frac{a}{m} w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$

$$V_n = \frac{-\frac{a}{m} v + w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$

$$V_n - V_n' = \frac{-\frac{a}{m}(v - v') + (w - w')}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$

$$= \frac{1}{4\pi a\sqrt{1 + \left(\frac{a}{m}\right)^2}} \left(\frac{\Delta P}{q_\infty}\right) 2\pi \left[\left(-\frac{a}{m}\right) am + (-m^2)\right] (a^2 + m^2)^{-1}$$

$$= \frac{-2m}{4a\sqrt{a^2 + m^2}} \left(\frac{\Delta P}{q_\infty}\right)$$ Source Distribution! [see (86)]

$$V_T - V_T' = \frac{2}{4\pi a\sqrt{1 + \left(\frac{a}{m}\right)^2}} \frac{\Delta P}{q_\infty} \pi \left|am - m^2 \frac{a}{m}\right| (a^2 + m^2)^{-1} = 0$$

Alternate sign choice of $\tan^{-1}$ denominator. -

$$\frac{m(ay - mz)}{(a^2 + m^2)} \tan^{-1} \frac{(mz - ay)\sqrt{(mx - \eta)^2 - \beta^2[(a\eta - mz)^2 + m^2(\eta - y)^2]}}{[a(z - ax) - m(mx - y)]\eta + m[y(mx - y) - z(z - ax)]}$$

[see (45) and note sign change of denominator and since

$$[a(z - ax) - m(mx - y)]\eta_3 + m[y(mx - y) - z(z - ax)] < 0$$

The above term when evaluated at

$$\eta = \eta_3$$

is

$$\frac{m(ay - mz) \quad \pi \operatorname{sgn}(mz - ay)}{(a^2 + m^2)}$$

Evaluated at $\eta = 0$ the term becomes

$$\frac{m(ay - mz)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{[y(mx - y) - z(z - ax)]}$$

If

$$mz = ay \qquad y(mx - y) - z(z - ax) = \frac{y(mx - y)(a^2 + m^2)}{m^2}$$

which means that on $mz = ay$ and $\eta = 0$

$$\tan^{-1} = \begin{cases} \pi \operatorname{sgn}(mz - ay) & y < 0 \quad y > mx \\ 0 & 0 < y < mx \end{cases}$$

$$\tan^{-1} \Big|_{0}^{\eta_3} = \begin{cases} 0 & y < 0 \quad y > mx \\ \pi \operatorname{sgn}(mz - ay) & 0 < y < mx \end{cases}$$

Subappendix A - Woodward's Subsonic Equations

If $\Omega(x, y, z)$ satisfies

$$\left\{\beta^2 \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right\} \Omega(x, y, z) = 0 \qquad \beta^2 = 1 - M^2$$

Then we can write the solution for $\Omega(x, y, z)$ as

$$\Omega(x,y,z) = \iint_S \left[\Omega(\xi, \eta, \zeta) \frac{\partial}{\partial \nu} - \frac{\partial\Omega(\xi, \eta, \zeta)}{\partial \nu}\right] \frac{1}{4\pi r} \, dS \tag{A1}$$

where

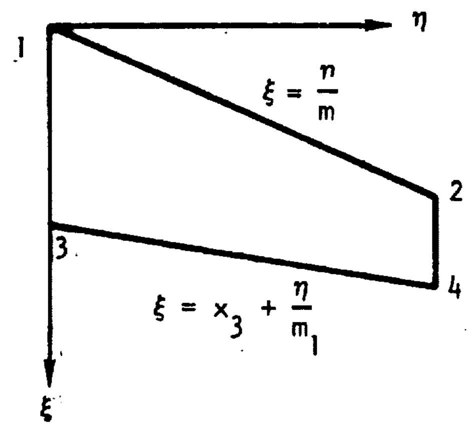
$$r = \sqrt{(x-\xi)^2 + \beta^2 (y-\eta)^2 + \beta^2 (z-\zeta)^2}$$

and

$$\frac{\partial}{\partial \nu} = \left\{ \beta^2 n_\xi \frac{\partial}{\partial \xi} + n_\eta \frac{\partial}{\partial \eta} + \eta_\zeta \frac{\partial}{\partial \zeta} \right\}$$

$\vec{n} = (n_\xi, \eta_\eta, \eta_\zeta)$ is the unit normal on S

We will assume S is the surface $\zeta = a\xi$ and



$$(x_1, y_1, z_1) = (o, o, o)$$

$$y_2 = y_4 = m_1 (x_4 - x_3) = mx_2$$

$$z_3 = ax_3 \quad z_2 = ax_2 \quad z_4 = ax_4$$

$$\therefore dS = \sqrt{1 + a^2}\ d\xi\, d\eta$$

Surface distribution of sources. - Let $\Omega = \phi$ and as for supersonic flow assume that on S $\sqrt{1 + a^2}\ \vec{\nu} = (-\beta^2 a, 0, 1)$

a) $\phi = \phi'$

b) $\frac{\partial\phi}{\partial\nu} + \frac{\partial\phi}{\partial\nu'} = \frac{2}{\sqrt{1 + a^2}} (\bar{w} - \beta^2 a\bar{u}) = \text{const.} \quad \bar{w} = w - w^1$

the primed quantities refer to $z < ax$ or $\zeta < a\xi$

Therefore since on S $\zeta = a\xi$ we get

$$\phi(x, y, z) = \frac{\bar{w} - \beta^2 a\bar{u}}{2\pi} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{d\xi}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi)^2}}\, d\eta \tag{A2}$$

This is the same as (9) for the supersonic case except for the limits, a factor of 1/2, and the fact that β^2 is replaced by $-\beta^2$. Therefore the integral over ξ may be performed by using the equation below (9) for the supersonic case, and replacing β^2 by $-\beta^2$.

$$\int \frac{d\xi}{\sqrt{(x-\xi)^2+\beta^2(y-\eta)^2+\beta^2(z-a\xi)^2}} = \tag{A3}$$

$$= \frac{-1}{2\sqrt{1+\beta^2a^2}} \log \frac{\sqrt{(1+\beta^2a^2)[(x-\xi)^2+\beta^2(y-\eta)^2+\beta^2(z-a\xi)^2]}+(x-\xi)+\beta^2a(z-a\xi)}{\sqrt{(1+\beta^2a^2)[(x-\xi)^2+\beta^2(y-\eta)^2+\beta^2(z-a\xi)^2]}-(x-\xi)-\beta^2a(z-a\xi)}$$

A negative sign was included in the argument of the logarithm. This was possible since the derivative of log(-1) is zero. Therefore

$$\phi(x,y,z) = \frac{\bar{w}-\beta^2a\bar{u}}{4\pi\sqrt{1+\beta^2a^2}} \int_0^{y_2} \log \frac{\sqrt{(1+\beta^2a^2)[(mx-\eta)^2+\beta^2m^2(\eta-y)^2+\beta^2(mz-a\eta)^2]}+(mx-\eta)+\beta^2a(mz-a\eta)}{\sqrt{(1+\beta^2a^2)[(mx-\eta)^2+\beta^2m^2(\eta-y)^2+\beta^2(mz-a\eta)^2]}-(mx-\eta)-\beta^2a(mz-a\eta)}\, d\eta \tag{A4}$$

$$+ \frac{\bar{w}-\beta^2a\bar{u}}{4\pi\sqrt{1+\beta^2a^2}} \int_0^{y_2} \log \frac{\sqrt{(1+\beta^2a^2)[(m_1\hat{x}-\eta)^2+\beta^2m_1^2(\eta-y)^2+\beta^2(m_1\hat{z}-a\eta)^2]}+(m_1\hat{x}-\eta)+\beta^2a(m_1\hat{z}-a\eta)}{\sqrt{(1+\beta^2a^2)[(m_1\hat{x}-\eta)^2+\beta^2m_1^2(\eta-y)^2+\beta^2(m_1\hat{z}-a\eta)^2]}-(m_1\hat{x}-\eta)-\beta^2a(m_1\hat{z}-a\eta)}\, d\eta$$

where $\hat{x} = x - x_3$ and $\hat{z} = z - z_3 = z - ax_3$

The second integral is the same as the first except that x, z and m are replaced by $x-x_3$, $z-z_3$ and m_1. These integrals are the same as for supersonic flow, given by equation (21), if β^2 is replaced by $-\beta^2$ and a factor of 1/2 is added.

When the limit $\eta = y_2$ is calculated it can be seen that it will be the same as when $\eta = 0$ if x, y and z are replaced by $x - x_2$ $y - y_2$ and $z - z_2$.

Therefore

$$\phi_o(x, y, z, m) = \frac{\bar{w} - \beta^2 a\bar{u}}{2\pi} \left\{ \frac{z - ax}{1 + \beta^2 a^2} \tan^{-1} \frac{m(z - ax)\sqrt{x^2 + \beta^2(y^2 + z^2)}}{y\left[(y - mx) + \beta^2 a(ay - mz)\right] + (z - ax)^2} \right.$$

$$+ \frac{y(1 + \beta^2 a^2) - m(x + \beta^2 az)}{2(1 + \beta^2 a^2)\sqrt{1 + \beta^2(a^2 + m^2)}} \log \frac{\sqrt{\left[1 + \beta^2(a^2 + m^2)\right]\left[x^2 + \beta^2(y^2 + z^2)\right]} + \left[x + \beta^2(az + my)\right]}{\sqrt{\left[1 + \beta^2(a^2 + m^2)\right]\left[x^2 + \beta^2(y^2 + z^2)\right]} - \left[x + \beta^2(az + my)\right]}$$

$$\left. - \frac{y}{2\sqrt{1 + \beta^2 a^2}} \log \frac{\sqrt{(1 + \beta^2 a^2)\left[x^2 + \beta^2(y^2 + z^2)\right]} + (x + \beta^2 az)}{\sqrt{(1 + \beta^2 a^2)\left[x^2 + \beta^2(y^2 + z^2)\right]} - (x + \beta^2 az)} \right\} \qquad \text{(A5)}$$

Now we can write the result of (A4) as

$$\phi(x, y, z) = \phi_o(x, y, z, m) - \phi_o(x - x_2, y - y_2, z - z_2, m)$$

$$- \phi_o(x - x_3, y - y_3, z - z_3, m_1) + \phi_o(x - x_4, y - y_4, z - z_4, m_1)$$

This means that (A5) may be interpreted as the velocity potential for an infinite panel with leading edge slope $y = mx$, and at angle of attack a.

Constant pressure surface. - In (A1) we will use $u(x, y, z) = \Omega(x, y, z)$ and

a) $\dfrac{\partial u}{\partial \nu} + \dfrac{\partial u'}{\partial \nu'} = 0$ on S

b) $u - u' = \text{const} = \Delta u$ on S

and where S is the same as previously defined.

Therefore (A1) becomes

$$u(x, y, z) = \frac{\Delta u}{4\pi} \iint_S \frac{-\beta^2 a(x - \xi) + \beta^2 (z - a\xi)}{\left[(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi)\right]^{3/2}} \, d\xi \, d\eta$$

$$= -\frac{\Delta u}{4\pi} \frac{\partial}{\partial x} \iint_S \frac{-\beta^2 a - \dfrac{(x - \xi)(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2}}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi)^2}} \, d\xi \, d\eta \qquad \text{(A6)}$$

Now we can write, using $\Delta P/2q_\infty = -\Delta u$

$$\phi(x, y, z) = \int_{-\infty}^{x} u(x', y, z)\, dx$$

$$= \frac{\Delta P}{8\pi q_\infty} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1}+x_3} \frac{-\beta^2 a - \dfrac{(x-\xi)(z-a\xi)}{(y-\eta)^2+(z-a\xi)^2}}{\sqrt{(x-\xi)^2+\beta^2(y-\eta)^2+\beta^2(z-a\xi)^2}}\, d\xi\, d\eta$$

$$- \frac{\Delta P}{8\pi q_\infty} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1}+x_3} \frac{(z-a\xi)}{(y-\eta)^2+(z-a\xi)^2}\, d\xi\, d\eta \qquad \text{(A7)}$$

The first term, except for the limits of integration and an additional factor of 1/2, is the same as the result for supersonic flow, which is given by the first equation of the section covering the evaluation of the integral over ξ for the constant pressure surface, with $-\beta^2$ in place of β^2. Therefore, as in the case of a constant distribution of sources, we can use the supersonic result for subsonic flow if we substitute $-\beta^2$ for β^2.

The second integral is new and does not occur for supersonic flow. This integral will now be evaluated.

$$\int \frac{(z-a\xi)\,d\xi}{(y-\eta)^2+(z-a\xi)^2} = -\frac{1}{2a} \log\left[(y-\eta)^2+(z-a\xi)^2\right] \qquad \text{(A8)}$$

Therefore

$$\int_0^{y_2}\int_{\frac{\eta}{m}}^{\frac{\eta}{m_1}+x_3} \frac{(z - a\xi)\,d\xi}{(y-\eta)^2 + (z - a\xi)^2}\,d\eta = \frac{1}{2a}\int_0^{y_2} \log\left\{\left[m^2(y-\eta)^2 + (mz - a\eta)^2\right]\frac{1}{m^2}\right\} d\eta$$

$$\frac{1}{2a}\int_0^{y_2} \log\left\{\left[m_1^{\,2}(y-\eta)^2 + \left(m_1\left[z - z_3\right] - a\eta\right)^2\right]\frac{1}{m_1^{\,2}}\right\} d\eta$$

note that:

$$mz - a\eta = (mz - ay) - a(\eta - y) = m(z - ax) + a(mx - y) - a(\eta - y) \quad \text{(A9)}$$

$$\int \log\left\{\frac{m^2(\eta - y)^2 + (a\eta - mz)^2}{m^2}\right\} d\eta$$

$$= \left[\eta - \frac{m(my + az)}{a^2 + m^2}\right]\left[\log\left\{\frac{m^2(\eta - y)^2 + (a\eta - mz)^2}{m^2}\right\} - 2\right] \quad \text{(A10)}$$

$$- \frac{2m(mz - ay)}{a^2 + m^2}\tan^{-1}\frac{m(mz - ay)}{(\eta - y)(a^2 + m^2) - a(mz - ay)}$$

When this is evaluated at $\eta = 0$ we get from (A7), (A9), and (A10)

$$\phi_{oo}(x, y, z, m) = \frac{\Delta P}{8\pi q_\infty a} \left\{ \frac{-m(az + my)}{(a^2 + m^2)} \left[\frac{1}{2}\log(y^2 + z^2) - 1\right] - \frac{m(mz - ay)}{(a^2 + m^2)} \tan^{-1} \frac{(mz - ay)m}{-a(mz - ay) - (a^2 + m^2)y} \right\} \tag{A11}$$

Since all of the terms in (A10) may be written using only the variables, $(mx - y)$, $(z - ax)$, m, and $(\eta - y)$, the additional term which must be added for subsonic flow is,

$$\begin{aligned} \phi_1(x, y, z) = {} & \phi_{oo}(x, y, z, m) - \phi_{oo}(x - x_2, y - y_2, z - z_2, m) \\ & - \phi_{oo}(x - x_3, y - y_3, z - z_3, m_1) \\ & + \phi_{oo}(x - x_4, y - y_4, z - z_4, m_1) \end{aligned} \tag{A12}$$

Therefore using (51) we can write:

$$
\begin{aligned}
\phi(x, y, z) = \frac{\Delta p}{8\pi q_{\infty} a} \Bigg\{ & \frac{m(az + my)}{a^2 + m^2} \, \frac{1}{2} \log \frac{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x} \\
& + \frac{a(ay - mz)}{2(a^2 + m^2)} \sqrt{1 + \beta^2(a^2 + m^2)} \, \log \frac{\sqrt{[1+\beta^2(a^2+m^2)][x^2+\beta^2(y^2+z^2)]} + [x + \beta^2(az+my)]}{\sqrt{[1+\beta^2(a^2+m^2)][x^2+\beta^2(y^2+z^2)]} - [x + \beta^2(az+my)]} \\
& - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay)\sqrt{x^2 + \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \\
& + (z - ax) \tan^{-1} \frac{m(z - ax)\sqrt{x^2 + \beta^2(y^2 + z^2)}}{y\left[(y - mx) + \beta^2 a(ay - mz)\right] + (z - ax)^2} \\
& - y\sqrt{1 + \beta^2 a^2} \, \frac{1}{2} \log \frac{\sqrt{(1 + \beta^2 a^2)\left[x^2 + \beta^2(y^2 + z^2)\right]} + (x + \beta^2 az)}{\sqrt{(1 + \beta^2 a^2)\left[x^2 + \beta^2(y^2 + z^2)\right]} - (x + \beta^2 az)} \\
& - \frac{m(az + my)}{(a^2 + m^2)} \left[\frac{1}{2} \log (y^2 + z^2) - 1\right] \\
& - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay)m}{a(ay - mz) - y(a^2 + m^2)} \Bigg\}
\end{aligned}
$$

To find the limiting form of ϕ as $a \to 0$ some of the terms must be combined. First, consider the terms

$$\lim_{a\to 0} \frac{y}{2a}\left\{ \log \frac{\sqrt{x^2+\beta^2(y^2+z^2)}+x}{\sqrt{x^2+\beta^2(y^2+z^2)}-x} \right.$$

$$\left. - \sqrt{1+\beta^2a^2}\ \log \frac{\sqrt{(1+\beta^2a^2)\left[x^2+\beta^2(y^2+z^2)\right]}+(x+\beta^2az)}{\sqrt{(1+\beta^2a^2)\left[x^2+\beta^2(y^2+z^2)\right]}-(x+\beta^2az)} \right\}$$

and since only terms up to the first power in a must be considered inside the brackets, this becomes

$$\lim_{a\to 0} -\frac{y}{2a}\left\{ \log \frac{1+\dfrac{\beta^2az}{\sqrt{x^2+\beta^2(y^2+z^2)}+x}}{1-\dfrac{\beta^2az}{\sqrt{x^2+\beta^2(y^2+z^2)}-x}} \right\} = -y\ \frac{z\sqrt{x^2+\beta^2(y^2+z^2)}}{y^2+z^2}$$

since for $\alpha \ll 1 \qquad \log(1+\alpha) = \alpha$

Next consider

$$\lim_{a\to 0} z\left\{ \tan^{-1} \frac{m(z-ax)\sqrt{x^2+\beta^2(y^2+z^2)}}{y\left[(y-mx)+\beta^2a(ay-mz)\right]+(z-ax)^2} \right.$$

$$\left. - \tan^{-1} \frac{(mz-ay)\sqrt{x^2+\beta^2(y^2+z^2)}}{z(z-ax)-y(mx-y)} \right\}$$

For small a we can expand

$$\tan^{-1}\frac{f(a)}{g(a)} = \tan^{-1}\frac{f(0)}{g(0)} + a\frac{\partial}{\partial a}\tan^{-1}\frac{f(a)}{g(a)}\Bigg|_{a=0}$$

$$= \tan^{-1}\frac{f(0)}{g(0)} + a\,\frac{\frac{f'}{g} - \frac{fg'}{g^2}}{1+\left(\frac{f}{g}\right)^2}\Bigg|_{a=0}$$

$$= \tan^{-1}\frac{f(0)}{g(0)} + \frac{a\,[f'g - fg']}{(g^2+f^2)}\Bigg|_{a=0}$$

Now let

$$f = m(z-ax)\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$g = y\left[(y-mx)+\beta^2 a(ay-mz)\right] + (z-ax)^2$$

$$f(0) = mz\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$f'(0) = -mx\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$g(0) = y(y-mx) + z^2$$

$$g'(0) = -\beta^2 m y z - 2xz$$

$$f^2_{(0)} + g^2_{(0)} = m^2z^2\left[x^2+\beta^2(y^2+z^2)\right] + y^2(mx-y)^2 + 2z^2y(y-mx) + z^4$$

$$= y^2\left[(mx-y)^2 + (1+\beta^2m^2)z^2\right] + z^2\left[m^2x^2 + \beta^2m^2z^2 + y^2 - 2mxy + z^2\right]$$

$$= (y^2+z^2)\left[(mx-y)^2 + (1+\beta^2m^2)z^2\right]$$

$$f'g - fg' = \sqrt{x^2+\beta^2(y^2+z^2)}\left\{ -[z^2+y(y-mx)]mx + mz^2[\beta^2 my + 2x] \right\}$$

Now for the second term let

$$f = (mz - ay)\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$g = z(z-ax) - y(mx-y)$$

$$f(0) = mz\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$g(0) = z^2 - y(mx-y)$$

$$f'(0) = -y\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$g'(0) = -xz$$

$$f^2+g^2 = (y^2+z^2)\left[(mx-y)^2 + (1+\beta^2m^2)z^2\right]$$

$$f'g - fg' = \sqrt{x^2+\beta^2(y^2+z^2)}\left\{ -y[z^2 - y(mx-y)] + mz^2x \right\}$$

Therefore

$$\lim_{a\to 0} z\left\{ \tan^{-1}\frac{m(z-ax)\sqrt{x^2+\beta^2(y^2+z^2)}}{y[(y-mx)+\beta^2a(ay-mz)]+(z-ax)^2} - \tan^{-1}\frac{(mz-ay)\sqrt{x^2+\beta^2(y^2+z^2)}}{z(z-ax)-y(mx-y)} \right\}$$

$$= z\left\{ \frac{(y-mx)[z^2+y(y-mx)] + mz^2(\beta^2my + x)}{(y^2+z^2)\left[(mx-y)^2+(1+\beta^2m^2)z^2\right]} \right\}\sqrt{x^2+\beta^2(y^2+z^2)}$$

$$= z\left\{ \frac{y\sqrt{x^2+\beta^2(y^2+z^2)}}{(y^2+z^2)} \right\}$$

To evaluate the additional term ϕ_1, as $a \to o$, we must combine points 1 and 3 and 2 and 4 from (A15)

$$\lim_{a \to o} \left\{ \phi_{oo}(x, y, z, m) - \phi_{oo}(x-x_3, y, z-ax_3, m_1) \right\}$$

$$= \lim_{a \to o} \left\{ -\frac{az+my}{m}\left[\frac{1}{2}\log(y^2+z^2)-1\right] + \frac{a(z-ax_3)+m_1 y}{m_1}\left[\frac{1}{2}\log\left(y^2+(z-ax_3)^2\right)-1\right] \right.$$

$$\left. -\frac{mz-ay}{m}\tan^{-1}\frac{mz-ay}{-(my+az)} + \frac{m_1(z-ax_3)-ay}{m_1}\tan^{-1}\frac{m_1(z-ax_3)-ay}{-[m_1 y+a(z-ax_3)]} \right\} \frac{\Delta P}{8\pi q_\infty a}$$

$$= \frac{\Delta P}{8\pi q_\infty a}\left\{ \left[-\frac{az}{m}+\frac{az}{m_1}\right]\left[\frac{1}{2}\log(y^2+z^2)-1\right] - \frac{ax_3 zy}{y^2+z^2} \right.$$

$$\left. +\left[\frac{ay}{m} - \frac{a(y+m_1x_3)}{m_1}\right]\tan^{-1}\frac{z}{-y} - \frac{az\left(1+\frac{z^2}{y^2}\right)}{m\left(1+\frac{z^2}{y^2}\right)} + \frac{az\left(1+\frac{z^2}{y^2}+\frac{m_1x_3}{y}\right)}{m_1\left(1+\frac{z^2}{y^2}\right)} \right.$$

$$= \frac{\Delta P}{8\pi q_\infty}\left\{ \left[-\frac{z}{m}+\frac{z}{m_1}\right]\left[\frac{1}{2}\log(y^2+z^2)\right] - \left[\frac{(mx-y)}{m} - \frac{[m_1(x-x_3)-y]}{m_1}\right]\tan^{-1}\frac{z}{-y} \right\}$$

where we have used

$$\tan^{-1}(\alpha+\Delta\alpha) = \tan^{-1}\alpha + \frac{\Delta\alpha}{1+\alpha^2}$$

Since we can write

$$\tan^{-1}\frac{z}{-y} = \pi - \tan^{-1}\frac{-y}{z} = \pi + \tan^{-1}\frac{y}{z}$$

Therefore if we let

$$\hat{\phi}_{oo}\,(x,\ y,\ z,\ m) = \frac{-\Delta P}{8\pi q_\infty}\left\{\frac{(mx-y)}{m}\ \tan^{-1}\frac{y}{z} + \frac{z}{m}\,\frac{1}{2}\log\,(y^2+z^2)\right\} \qquad \text{(A15)}$$

Then since $\pi(mx-y)$ will cancel for a complete panel,

$$\lim_{a\to o}\ \phi_1\,(x,y,z) = \hat{\phi}_{oo}\,(x,y,z,m) - \hat{\phi}_{oo}\,(x-x_2,\ y-y_2,\ z,m)$$

$$- \hat{\phi}_{oo}\,(x-x_3,\ y,z,m_1) + \hat{\phi}_{oo}\,(x-x_4,\ y-y_4,z,m_1)$$

And therefore for subsonic flow with a = o

$$\phi(x,y,z) = \frac{-\Delta P}{8\pi q_\infty m}\left\{(mx-y)\ \tan^{-1}\frac{mz\sqrt{x^2+\beta^2\,(y^2+z^2)}}{y\,(y-mx)+z^2} - \frac{z}{2}\log\frac{\sqrt{x^2+\beta^2\,(y^2+z^2)}+x}{\sqrt{x^2+\beta^2\,(y^2+z^2)}-x}\right.$$

$$-y\ \frac{z\sqrt{x^2+\beta^2(y^2+z^2)}}{y^2+z^2} + z\sqrt{1+\beta^2m^2}\ \frac{1}{2}\log\frac{\sqrt{(1+\beta^2m^2)\,[x^2+\beta^2\,(y^2+z^2)]}+(x+\beta^2my)}{\sqrt{(1+\beta^2m^2)\,[x^2+\beta^2\,(y^2+z^2)]}-(x+\beta^2my)}$$

$$\left. + z\ \frac{y\sqrt{x^2+\beta^2(y^2+z^2)}}{y^2+z^2} + (mx-y)\ \tan^{-1}\frac{y}{z} + \frac{z}{2}\log\,(y^2+z^2)\right\}$$

In the form the above equation is written, derivatives with respect to x, y, or z of all but the last two terms may be obtained by differentiating only the coefficients of each term. This was discussed previously when obtaining the velocity components for supersonic flow. If we use $\beta^2 = 1 - M^2$ the above equation can also be used for $M > 1$ if we multiply the expression by 2, take absolute values of the log arguments, and omit the last two terms. Therefore

$$\phi(x,y,z) = \frac{-\Delta P}{8\pi q_\infty m} k \Bigg\{ (mx-y)\tan^{-1}\frac{mz\sqrt{x^2+\beta^2(y^2+z^2)}}{y(y-mx)+z^2} - \frac{z}{2}\log\frac{\sqrt{x^2+\beta^2(y^2+z^2)}+x}{\sqrt{x^2+\beta^2(y^2+z^2)}-x}$$

$$+ z\sqrt{1+\beta^2m^2}\,\frac{1}{2}\log\frac{\sqrt{(1+\beta^2m^2)[x^2+\beta^2(y^2+z^2)]}+(x+\beta^2my)}{\sqrt{(1+\beta^2m^2)[x^2+\beta^2(y^2+z^2)]}-(x+\beta^2my)} \quad y\,\frac{z\sqrt{x^2+\beta^2(y^2+z^2)}}{y^2+z^2}$$

$$+ z\,\frac{y\sqrt{x^2+\beta^2(y^2+z^2)}}{y^2+z^2} + k\left[(mx-y)\tan^{-1}\frac{y}{z} + \frac{z}{2}\log(y^2+z^2)\right]\Bigg\}$$

where we take only the real part and

$$k = \begin{cases} 1 & M \le 1 \\ 2 & M > 1 \end{cases} \qquad k_0 = \begin{cases} 1 & M \le 1 \\ 0 & M > 1 \end{cases} \qquad \beta^2 = M^2 - 1$$

This is equivalent to the form derived by Woodward if it is noted that

$$\log \frac{x+\beta^2 my + \sqrt{(1+\beta^2 m^2)\,[x^2+\beta^2 (y^2+z^2)]}}{\beta\sqrt{(mx-y)^2 + (1+\beta^2 m^2)\; z^2}}$$

$$= \frac{1}{2} \log \frac{\sqrt{(1+\beta^2 m^2)\,[x^2+\beta^2\;(y^2+z^2)]} + (x+\beta^2 my)}{\sqrt{(1+\beta^2 m^2)\,[x^2+\beta^2\;(y^2+z^2)]} - (x+\beta^2 my)}$$

and

$$\frac{1}{2} \log \frac{\sqrt{x^2+\beta^2\;(y^2+z^2)} + x}{\sqrt{x^2+\beta^2\;(y^2+z^2)} - x} = \log \frac{x + \sqrt{x^2+\beta^2\;(y^2+z^2)}}{\beta\sqrt{y^2+z^2}}$$

where $\beta = \sqrt{|\beta^2|}$

If we set a = o in (A10), the additional term which occurs in subsonic flow may be easily obtained. For a = o,

$$\phi_1 (x, y, z) = \frac{\Delta P}{8\pi q_\infty} \int_o^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{z}{(\eta - y)^2 + z^2} d\xi d\eta$$

$$\int_o^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{z}{(\eta - y)^2 + z^2} d\xi d\eta = -\frac{1}{m} \int_o^{y_2} \frac{\eta z}{(\eta - y)^2 + z^2} d\eta + \frac{1}{m_1} \int_o^{y_2} \frac{(\eta + m_1 x_3) z}{(\eta - y)^2 + z^2} d\eta$$

$$= -\frac{z}{m} \left\{ \frac{1}{2} \log \left[(\eta - y)^2 + z^2\right] - \frac{y}{z} \tan^{-1} \frac{z}{\eta - y} \right\} \Bigg|_o^{y_2}$$

$$+ \frac{z}{m_1} \left\{ \frac{1}{2} \log \left[(\eta - y)^2 + z^2\right] - \frac{y + m_1 x_3}{z} \tan^{-1} \frac{z}{\eta - y} \right\} \Bigg|_o^{y_2}$$

$$= \left\{ \frac{z}{2m} \log (y^2 + z^2) - \frac{y}{m} \tan^{-1} \frac{z}{-y} \right\}$$

$$\phi_1 (x,y,z) = \left| \frac{z}{2m_1} \log (y^2+z^2) - \frac{y+m_1x_3}{m_1} \tan^{-1} \frac{z}{-y} \right|$$

$$- \left| \frac{z}{2m} \log [(y-y_2)^2 + z^2] - \frac{y}{m} \tan^{-1} \frac{z}{-(y-y_2)} \right|$$

$$\left| \frac{z}{2m_1} \log [(y-y_2)^2 + z^2] - \frac{y+m_1x_3}{m_1} \tan^{-1} \frac{z}{-(y-y_2)} \right|$$

$$\frac{1}{m} \left| \frac{z}{2} \log (y^2+z^2) + (mx-y) \tan^{-1} \frac{z}{-y} \right|$$

$$- \frac{1}{m_1} \left| \frac{z}{2} \log (y^2+z^2) + [m_1 (x-x_3)-y] \tan^{-1} \frac{z}{-y} \right|$$

$$- \frac{1}{m} \left| \frac{z}{2} \log [(y-y_2)^2 + z^2] + [m (x-x_2) -y-y_2] \tan^{-1} \frac{z}{-(y-y_2)} \right|$$

$$- \frac{1}{m_1} \left| \frac{z}{2} \log [(y-y_2)^2 + z^2] + [m_1 (x-x_4)] - (y-y_4) \tan^{-1} \frac{z}{-(y-y_4)} \right|$$

The above is true since

$$y_2 = mx_2$$

$$y_4 = y_2 = m_1 (x_4-x_3)$$

Therefore, analogous to (A14) or (A18) when a = o

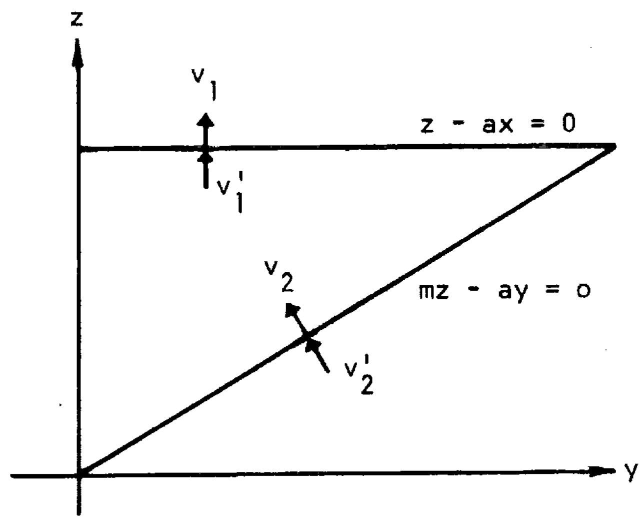
$$\hat{\phi}_{oo}(x,y,z) = \frac{-\Delta P}{8\pi q_\infty m}\left\{(mx-y)\tan^{-1}\frac{z}{-y} + \frac{z}{2}\log(y^2+z^2)\right\}.$$

which is equivalent to [A18] since we can replace

$$\tan^{-1}\frac{z}{-y} \text{ by } \pi + \tan^{-1}\frac{y}{z} \text{ or by } \tan^{-1}\frac{y}{z}$$

since $\pi(mx-y)$, when the contributions from each corner are added, gives zero contribution.

Total Source Strength



$$v_1 - v_1' = K\frac{1}{a}$$

$$v_2 - v_2' = \frac{K}{a}\left\{ \frac{-a}{[a^2+m^2]^{1/2}} \frac{am}{a^2+m^2} + \frac{m}{[a^2+m^2]^{1/2}} \frac{-m^2}{(a^2+m^2)} \right\} = K \frac{m}{a\ (a^2+m^2]^{1/2}}$$

$$L_1 = y_o$$

$$L_2 = y_o\ (a^2+m^2)^{1/2}\ \frac{1}{m}$$

$$\left(v_1 - v_1'\right) L_1 + \left(v_2 - v_2'\right) L_2 = y_o\ K \left[\frac{1}{a} - \frac{1}{a}\right] = 0$$

Point Source at ξ, η, ζ

$$r^2 = (x-\xi)^2 + \beta^2\ [(y-\eta)^2 + (z-\zeta)^2$$

$$\phi(x,y,z) = \frac{m}{4\pi r}$$

Doublet at ξ, η, ζ in $\vec{e}$ direction

$$\phi(x,y,z) = \vec{e} \cdot \nabla_\beta \left[\frac{A}{4\pi r}\right] \qquad \nabla_\beta = \beta^2 \frac{\partial}{\partial \xi} \vec{e}_\xi + \frac{\partial}{\partial \eta} \vec{e}_\eta + \frac{\partial}{\partial \zeta} \vec{e}_\zeta$$

If we integrate a row of these doublets in the direction of $\vec{e}$ we get a source at one end and a sink at the other. Now let $\vec{e}$ be

$$\frac{1}{\sqrt{1+a^2}} \quad [-a\,\vec{e}_\xi + \vec{e}_\zeta]$$

and integrate in the x direction.

$$\phi(x,y,z) = \frac{A}{4\pi\sqrt{1+a^2}} \int_{\xi_o}^{\infty} \left\{ \frac{\beta^2 (z-\zeta)}{\left[(x-\xi)^2 + \beta^2[(y-\eta)^2 + (z-\zeta)^2]\right]^{3/2}} - \beta^2 a \frac{\partial}{\partial \xi} \frac{1}{\sqrt{(x-\xi)^2 + \beta^2\,[(y-\eta)^2 + (z-\zeta)^2]}} \right\} d\xi$$

$$= \frac{+A}{4\pi\sqrt{1+a^2}} \frac{(z-\zeta)}{(y+\eta)^2 + (z-\zeta)^2} \left\{ 1 + \frac{x-\xi_o}{\sqrt{(x-\xi_o)^2 + \beta^2\,(y-\eta)^2 + \beta^2\,(z-\zeta)^2}} \right\} + \frac{A}{4\pi\sqrt{1+a^2}} \frac{\beta^2 a}{\sqrt{(x-\xi_o)^2 + \beta^2\,(y-\eta)^2 + \beta^2\,(z-\zeta)^2}}$$

If we integrate this over S we obtain (A.7). Therefore (A.7) corresponds to a volume of doublets which means there will be a surface of sinks on one side and sources on the other.

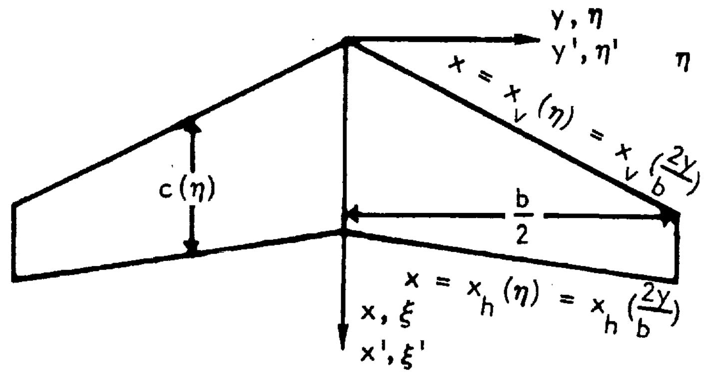
Appendix E

WAGNER'S LIFTING SURFACE EQUATIONS

$$u\ (x,y,z) = \frac{\partial}{\partial x}\Phi(x,y,z)$$

$$v(x,y,z) = \frac{\partial}{\partial y}\Phi(x,y,z)$$

$$w(x,y,z) = \frac{\partial}{\partial z}\Phi(x,y,z)$$



$$\eta = \frac{y}{\frac{1}{2}b}$$

$$\xi = \frac{x - x_v(\eta)}{x_h(\eta) - x_v(\eta)}$$

$$\Phi(x,y,z) = V_\infty \iint_S \frac{z\ \ k\ (x',y')}{4\pi\left[(y-y')^2 + z^2\right]}\left\{1 + \frac{x - x'}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}}\right\} dx'dy' \quad (1)$$

Change of variables

$$x_n(\eta') - x_V(\eta') = c\ (\eta')$$

$$\eta' = \frac{y'}{\frac{1}{2}b}$$

$$y' = \frac{1}{2}\eta' b$$

$$\xi' = \frac{x' - x_V\ (\eta')}{x_h\ (\eta') - x_V\ (\eta')}$$

$$x' = \xi'\ c\ (\eta') + x_V\ (\eta')$$

$$dx'\,dy' = \begin{vmatrix} \dfrac{\partial x'}{\partial \xi'} & \dfrac{\partial x'}{\partial \eta'} \\ \dfrac{\partial y'}{\partial \xi'} & \dfrac{\partial y'}{\partial \eta'} \end{vmatrix} d\xi' d\eta' = \frac{1}{2} bc(\eta')\; d\xi' d\eta'$$

Therefore

$$\Phi(x,y,z) = \frac{V_\infty b}{8\pi} \int_{-1}^{1} \int_{0}^{1} \frac{z\, c(\eta') k\left[x'(\xi',\eta'), y'(\eta')\right]}{\left(y - \frac{1}{2}\eta' b\right)^2 + z^2} \left\{ 1 + \frac{x - x_V(\eta') - \xi' c(\eta')}{\sqrt{\left[x - x_V(\eta') - \xi' c(\eta')\right]^2 + \left(y - \frac{1}{2}\eta' b\right)^2 + z^2}} \right\} d\xi' d\eta' \tag{2}$$

Now if we define

$$\tilde{g}(x,y,z,\eta') = \frac{c(\eta')}{b} \int_{0}^{1} k\left[x'(\xi',\eta'), y'(\eta')\right] \left\{ 1 + \frac{x - x_V(\eta') - \xi' c(\eta')}{\sqrt{\left[x - x_V(\eta') - \xi' c(\eta')\right]^2 + \left(y - \frac{1}{2}\eta' b\right)^2 + z^2}} \right\} d\xi'$$

Then

$$\Phi(x,y,z) = \frac{V_\infty b^2}{8\pi} \int_{-1}^{1} \frac{z\, \tilde{g}(x,y,z,\eta')}{\left(y - \frac{1}{2}\eta' b\right)^2 + z^2}\, d\eta'$$

Near z = 0 we can write

$$\Phi(x,y,z) = \Phi(x,y,0) + z \frac{\partial}{\partial z} \Phi(x,y,z)\Big|_{z=0}$$

$$\tilde{g}\,(x,y,z,\eta') = \tilde{g}\,(x,y,0,\eta') + O\,(z^2)$$

Therefore if we change variables and let (5)

$$\xi = \frac{x - x_V\left(\frac{2y}{b}\right)}{c\left(\frac{2y}{b}\right)}$$

$$\eta = \left(\frac{2y}{b}\right)$$

$$\zeta = \left(\frac{2z}{b}\right)$$

$$\tilde{g}\,(x,y,0,\eta') = g\,(\xi,\eta,\eta')$$

We can say

$$\frac{1}{V_\infty}\quad \Phi(x,y,0) = \varphi(\xi,\eta) = \frac{b}{4\pi}\lim_{\zeta\to 0}\int_{-1}^{1}\frac{\zeta\, g(\xi,\eta,\eta')}{(\eta'-\eta)^2+\zeta^2}\,d\eta' \tag{6}$$

and

$$\lim_{z\to 0}\frac{-1}{V_\infty}\frac{\partial}{\partial z}\,\Phi(x,y,z) = \alpha(\xi,\eta) = -\frac{1}{2\pi}\lim_{\zeta\to 0}\frac{\partial}{\partial\zeta}\int_{-1}^{1}\frac{\zeta\, g(\xi,\eta,\eta')}{(\eta'-\eta)^2+\zeta^2}\,d\eta'$$

(7)

Now integrate (6) by parts $\left[\text{Provided } \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \text{ is continuous}\right]$

$$u = g(\xi, \eta, \eta')$$

$$dv = \frac{\xi \, d\eta'}{(\eta' - \eta)^2 + \xi^2}$$

$$du = \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')$$

$$v = -\tan^{-1} \frac{\xi}{\eta' - \eta}$$

$$\int_{-1}^{1} \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' = -g(\xi, \eta, \eta') \tan^{-1}\left[\frac{\xi}{\eta' - \eta}\right]\Bigg|_{-1}^{1}$$

$$+ \int_{-1}^{1} \tan^{-1}\left[\frac{\xi}{\eta' - \eta}\right]\left[\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')\right] d\eta'$$

$$= \int_{-1}^{1} \tan^{-1}\left[\frac{\xi}{\eta' - \eta}\right]\left[\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')\right] d\eta' \tag{8}$$

where the fact that $g(\xi, \eta, \pm 1) = 0$, because the loading goes to zero at $\eta' = \pm 1$, was used and since

$$\lim_{\xi \to 0} \tan^{-1} \frac{\xi}{\eta' - \eta} = \begin{cases} \pi \operatorname{sgn} \xi & \eta' - \eta < 0 \\ 0 & \eta' - \eta > 0 \end{cases} \tag{9}$$

$$\varphi(\xi,\eta) = \frac{b}{4}\operatorname{sgn}\zeta \quad g(\xi,\eta,\eta) \tag{10}$$

and

$$\alpha(\xi,\eta) = \frac{1}{2\pi}\lim_{\zeta\to 0}\frac{\partial}{\partial\zeta}\int_{-1}^{1}\tan^{-1}\left[\frac{\zeta}{\eta'-\eta}\right]\left[\frac{\partial}{\partial\eta'}g(\xi,\eta,\eta')\right]d\eta'$$

$$= -\frac{1}{2\pi}\lim_{\zeta\to 0}\int_{-1}^{1}\frac{(\eta'-\eta)\frac{\partial}{\partial\eta'}g(\xi,\eta,\eta')}{(\eta'-\eta)^2+\zeta^2}d\eta' \tag{12}$$

and from A1

$$= -\frac{1}{2\pi}\fint_{-1}^{1}\frac{\frac{\partial}{\partial\eta'}g(\xi,\eta,\eta')}{\eta'-\eta}d\eta' = \frac{1}{2\pi}\left\{\frac{2g(\xi,\eta,\eta)}{\epsilon} - \fint_{-1}^{1}\frac{g(\xi,\eta,\eta')}{(\eta'-\eta)^2}d\eta'\right\}$$

If $\frac{\partial}{\partial\eta'}g(\xi,\eta,\eta')$ is not continuous we define

$$\alpha(\xi,\eta) = -\frac{1}{2\pi}\lim_{\zeta\to 0}\frac{\partial}{\partial\zeta}\int_{-1}^{1}\frac{\zeta g(\xi,\eta,\eta')}{(\eta'-\eta)^2+\zeta^2}d\eta' = -\frac{1}{2\pi}\fint_{-1}^{1}\frac{g(\xi,\eta,\eta')}{(\eta'-\eta)^2}d\eta' \tag{13}$$

Now let

$$\frac{c(\eta')}{b}k\left[x'(\xi',\eta'), y'(\eta')\right] = -\sum_{n=0}^{N}f_n(\eta')h_n(\xi')$$

which means

$$\bar{g}(\xi, \eta, \eta') = -\sum_{n=0}^{N} f_n(\eta') H_n(\xi, \eta, \eta')$$

where

$$h_n(\xi') = \frac{1}{\pi}\left[\frac{1-\xi'}{\xi'}\right]^{1/2}\left[\frac{T_n(1-2\xi') + T_{n+1}(1-2\xi')}{1-\xi'}\right]$$

or

$$\hat{h}_n(\Psi) = h_n\left[\frac{1}{2}(1-\cos\Psi)\right] = \frac{2}{\pi}\left[\frac{\cos n\Psi + \cos(n+1)\Psi}{\sin\Psi}\right] = \frac{2}{\pi}\,\frac{\cos\left(n+\frac{1}{2}\right)\Psi}{\sin\frac{\Psi}{2}}$$

and

$$H_n(\xi, \eta, \eta') = \int_0^1 h_n(\xi')\left\{1 + \frac{x - x_v(\eta') - \xi' c(\eta')}{\sqrt{\left[x - x_v(\eta') - \xi' c(\eta')\right] + \left[y - \frac{1}{2}\eta' b\right]^2}}\right\} d\xi'$$

$$= \int_0^1 h_n(\xi')\left\{1 + \frac{\frac{x - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[\frac{x - x_v(\eta')}{c(\eta')} - \xi'\right]^2 + \left[\frac{(\eta - \eta')\frac{1}{2}b}{c(\eta')}\right]^2}}\right\} d\xi' \quad (15)$$

and from (5)

$$= \int_0^1 h_n(\xi') \left\{ 1 + \frac{\dfrac{x_v(\eta) + \xi\, c(\eta) - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[\dfrac{x_v(\eta) + \xi\, c(\eta) - x_v(\eta')}{c\,(\eta')} - \xi'\right]^2 + \left[\dfrac{(\eta - \eta')\,\frac{1}{2}\,b}{c(\eta')}\right]^2}} \right\} d\xi'$$

Now, from (7), for any $g(\xi, \eta, \eta')$ differentiable or not we can write

$$\alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \zeta^2}\, d\eta'$$

$$= \frac{1}{2\pi} \sum_{n=0}^{N} \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta f_n(\eta') \left[H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta')\right]}{(\eta - \eta)^2 + \zeta^2}\, d\eta' \tag{16}$$

where we define

$$H_n(\xi, \eta, \eta') \equiv H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta') \tag{17}$$

or

$$\alpha(\xi, \eta) = \frac{1}{2\pi} \sum_{n=0}^{N} \left\{ H_n(\xi, \eta, \eta) \int_{-1}^{1} \frac{\frac{\partial}{\partial \eta'} f_n(\eta')}{(\eta - \eta')}\, d\eta' + \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \zeta^2}\, d\eta' \right\} \tag{18}$$

where if $\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')$ is continuous, using (11) and (12)

$$\lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta f_n(\eta') K_n(\xi,\eta,\eta')}{(\eta'-\eta)^2 + \zeta^2} d \, ' = \oint_{-1}^{1} \frac{\frac{\partial}{\partial \eta'} \left[f_n(\eta') K_n(\xi, \eta, \eta') \right]}{(\eta - \eta)} d\eta'$$

$$= \oint_{-1}^{1} \frac{f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \tag{19}$$

since $f_n(\pm 1) = 0$

Now referring to (15), let

$$\tilde{H}_n(p,q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p-\xi')^2 + q^2}} \, d \right\} \xi'$$

Then, from (B5) for small $(p-p_o)$ and small q $p_o \neq 0,1$

$$\tilde{H}_n(p,q) = \tilde{H}_n(p_o,0) + 2 h_n(p_o)(p-p_o) - h_n'(p_o) \, q^2 \ln|q| \tag{20}$$

Now set [See (15)]

$$p = \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')}$$

$$p_o = \xi$$

$$q = \frac{1/2 \; b \, (\eta' - \eta)}{c(\eta')} \tag{21}$$

when

$$q = 0$$

$$\eta' = \eta$$

$$p = p_o$$

Then for $(\eta' - \eta) << 1$

$$p - p_o = \frac{\partial}{\partial \eta'}\left[\frac{x_v(\eta) - \xi\, c(\eta) - x_v(\eta')}{c(\eta')}\right]_{\eta'=\eta}(\eta' - \eta) \equiv - \frac{\frac{1}{2} b\,(\eta' - \eta)}{c(\eta)} \tan\phi \tag{21a}$$

where we have defined $\phi(\xi, \eta)$ as the sweep of the constant percent chord lines

$$\tan \phi(\xi, \eta) \equiv - \frac{c(\eta)}{\frac{1}{2} b} \frac{\partial}{\partial \eta'}\left[\frac{x_v(\eta) - \xi\, c(\eta) - x_v(\eta')}{c(\eta')}\right]_{\eta'=\eta} \tag{22}$$

Therefore for $\xi \neq 0,1$

$$\left|\frac{\frac{1}{2} b\,(\eta' - \eta)}{c(\eta')}\right| << 1$$

$$H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) - 2\left[\frac{\frac{1}{2} b\,(\eta' - \eta)}{c(\eta)}\right] h_n(\xi) \tan\phi$$

$$- h_n'(\xi)\left[\frac{\frac{1}{2} b\,(\eta' - \eta)}{c(\eta)}\right]^2 \ln \left|\frac{\frac{1}{2} b\,(\eta' - \eta)}{c(\eta)}\right|$$

Then for small $(\eta' - \eta)$ and $\xi \neq 0,1$

$$K_n(\xi, \eta, \eta') = -2 \left[\frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)}\right] h_n(\xi) \tan \phi - h_n'(\xi) \left[\frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)}\right] \ln \left[\frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)}\right] \tag{23}$$

For $\xi = 0$, from (C2)

$$\tilde{H}_n(p,q) = \tilde{H}_n(0,0) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi}) (p^2 + q^2)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) (p^2 + q^2)^{3/4} \right.$$

where $I_1(\tilde{\phi})$ and $I_2(\tilde{\phi})$ are defined in (C3) and C4) and

$$\tilde{\phi} = \tan^{-1} \frac{-p}{q}$$

At the leading edge $p_o = 0$ and, from (21a)

$$-p = \frac{\frac{1}{2} b\ (\eta' - \eta)}{c(\eta)} \tan \phi_{L.E.} = q \tan \phi_{L.E.}$$

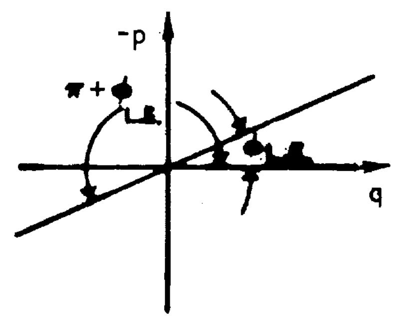
and therefore

$$p^2 + q^2 = \frac{\frac{1}{2} b}{c(\eta)} \left[1 + \tan^2 \phi_{L.E.}\right] (\eta' - \eta)^2 = \frac{1}{\cos^2 \phi_{L.E.}} \left[\frac{\frac{1}{2} b}{c(\eta)}\right]^2 (\eta' - \eta)^2$$

and

$$\tilde{\phi} = \tan^{-1}\left[\tan \phi_{L.E.}\right] = \phi_{L.E.} \qquad q > 0 \text{ or } (\eta'-\eta) > 0$$

$$= \pi + \phi_{L.E.} \qquad (\eta'-\eta) < 0$$



Therefore for $(\eta'-\eta) \ll 1$

$$K_n(0, \eta, \eta') = \varphi_1(\phi)\left[\frac{\frac{1}{2} b\,|\eta'-\eta|}{c(\eta)}\right]^{1/2} - \frac{1}{2}(1+4n+4n^2)\,\varphi_2(\phi)\left[\frac{\frac{1}{2} b\,|\eta'-\eta|}{c(\eta)}\right]^{3/2} \tag{24}$$

where

$$\varphi_1(\phi) = \frac{8\,|\cos\phi|^{3/2}}{\pi} I_1(\phi)$$

$$\varphi_2(\phi) = \frac{8\,|\cos\phi|^{1/2}}{3\pi} I_2(\phi)$$

and

$$\phi = \phi_{L.E.} \qquad (\eta'-\eta) > 0$$

$$\phi = \pi + \phi_{L.E.} \qquad (\eta'-\eta) < 0$$

Using a similar analysis for the trailing edge, $\xi = 1$, we get

$$K_n(1, \eta, \eta') = (-1)^n \varphi_2(\phi)(1 + 2n)\left[\frac{\frac{1}{2} b\,|\eta' - \eta|}{c(\eta)}\right]^{3/2} \qquad (\eta' - \eta) << 1 \quad (25)$$

$$\phi = \phi_{T.E.} + \pi \qquad (\eta' - \eta) > 0$$

$$\phi = \phi_{T.E.} \qquad (\eta' - \eta) < 0$$

For $\xi = 0$

$$\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')$$

is not continuous at $\eta' = \eta$ due to the term involving $(\eta' - \eta)^{1/2}$

Therefore, referring to (18), we must evaluate

$$\lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \zeta^2} d\eta'$$

Since the only discontinuity in

$$\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')$$

occurs at $\eta' = \eta$ we can write, using (19)

$$\lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{\zeta f_n(\eta')K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \zeta^2} d\eta'$$

$$= \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{\eta-\delta}^{\eta+\delta} \frac{\zeta f_n(\eta')K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \zeta^2} d\eta'$$

$$+ \int_{-1}^{\eta-\delta} \frac{f_n(\eta')K_n(0, \eta, \eta')}{(\eta' - \eta)^2} d\eta' + \int_{\eta+\delta}^{1} \frac{f_n(\eta')K_n(0, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \qquad (26)$$

The only term in $K_n(0, \eta, \eta')$ which causes trouble is the term

$$\varphi_1(\phi_{L.E.}) \left[\frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)}\right]^{1/2}$$

and if δ is small enough we can approximate $f_n(\eta')$ by $f_n(\eta)$

Therefore we must evaluate the following expressions

$$\int_0^{\delta} \frac{\zeta\sqrt{s}}{s^2 + \zeta^2} ds = \sqrt{\zeta}\int_0^{\delta/\zeta} \frac{\sqrt{s}}{s^2 + 1} ds = \sqrt{\zeta}\left\{\int_0^{\infty} \frac{\sqrt{s}}{s^2 + 1} ds - \int_{\delta/\zeta}^{\infty} \frac{\sqrt{s}}{s^2 + 1} ds\right\} = \sqrt{\zeta}\left\{\frac{\pi}{\sqrt{2}} - \int_{\delta/\zeta}^{\infty} \frac{\sqrt{s}}{s^2 + 1} ds\right\}$$

$$= \sqrt{\zeta}\left\{\frac{\pi}{\sqrt{2}} - \int_{\delta/\zeta}^{\infty} s^{-3/2}\left[1 - \frac{1}{s^2} + \frac{1}{s^4} - \frac{1}{s^6} \cdots\right] ds\right\} = \sqrt{\zeta}\left\{\frac{\pi}{\sqrt{2}} - 2\left[\frac{\zeta}{\delta}\right]^{1/2} + \frac{2}{5}\left[\frac{\zeta}{\delta}\right]^{5/2} - \frac{2}{9}\left[\frac{\zeta}{\delta}\right]^{9/2} \cdots\right\}$$

and,

$$2 \frac{\partial}{\partial \xi} \int_0^{\delta} \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \left\{ \frac{\pi}{\sqrt{2\xi}} + \frac{2}{\sqrt{\delta}} \left[-2 + \frac{6}{5} \frac{\xi^2}{\delta^2} - \frac{10}{9} \frac{\xi^4}{\delta^4} \cdots \right] \right\}$$

$$\therefore \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_0^{\delta} \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \infty \qquad (26a)$$

Subappendices E and F consider the case where $g(\xi, \eta, \eta')$ is considered to be a function of ξ, as in (3), but the expression (26a) is still infinite.

If tan ϕ is not continuous at some $\eta = \eta'$ then, from (23) and (18) or (26), it can be seen that the integral of

$$\frac{K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2}$$

will give a logarithmic infinity. If tan ϕ is continuous the first term of (23) will give an odd function of $(\eta' - \eta)$ which will give a finite value in the Cauchy principle value sense. Therefore kinks or cranks in the planform cannot be permitted because discontinuities in the angles of the constant percent chord lines will result. In fact subappendix G of this appendix shows that a logarithmic infinity occurs if the leading edge kink is considered as the limit of a hyperbola whose radius of curvature goes to zero. If the term in (18) involving tan ϕ is neglected for small $(\eta' - \eta)$, this has the effect of rounding the constant percent chord lines.

Now let $\eta' = \cos \theta'$ and

$$f_n(\eta') = \sum_{m=1}^{M} f_{nm} \bar{S}_m(\theta) \qquad (27)$$

where

$$\bar{S}_m(\theta') = \frac{2}{M+1}\sum_{\mu=1}^{M} \sin\mu\,\theta_m \sin\mu\,\theta' = \begin{cases} 1 & \theta' = \theta_m \\ \dfrac{1}{M+1}\,\dfrac{(-1)^{m+1}\sin\theta_m \sin(M+1)\,\theta'}{\cos\theta' - \cos\theta_m} & \theta' \neq \theta_m \end{cases}$$

$$\theta_m = \frac{m\,\pi}{M+1} \qquad \text{discrete points}$$

Now the first part of (18) may be performed in closed form

$$\frac{1}{2\pi}\fint_{-1}^{1} \frac{\dfrac{\partial f_n(\eta')}{\partial\eta'}}{\partial\eta_\nu - \eta'}\, d\,\eta' = \sum_{m=1}^{M} f_{nm}\, b_{\nu m} \tag{28}$$

where

$$b_{\nu m} = \begin{cases} \dfrac{M+1}{4}\,\dfrac{1}{\sin\theta_\nu} & \theta_\nu = \theta_m \\ \dfrac{1}{M+1}\,\dfrac{1-(-1)^{\nu+m}}{2}\,\dfrac{\sin\theta_m}{\left[\cos\theta_\nu - \cos\theta_m\right]^2} & \theta_\nu \neq \theta_m \end{cases} \tag{29}$$

Referring to (18), (19), (26), and (27) we define

$$A_{np\nu\nu} \equiv \frac{1}{2\pi} \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{\zeta K_n(\xi_p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2 + \zeta^2} \, d\eta' \qquad p = 0$$

$$= \frac{1}{2\pi} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{K_n(\xi_p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2} \, d\eta' \qquad p \neq 0 \tag{30}$$

$$B_{np\nu m} \quad \frac{1}{2\pi} \int_{\eta_\nu + \delta}^{1} \frac{K_n(\xi_p, \eta_\nu, \eta') \, \bar{S}_m \left[\cos^{-1} \eta'\right]}{(\eta_\nu - \eta')^2} \, d\eta' \tag{31}$$

and letting

$$\eta' = \frac{1 + \eta_\nu + \delta}{2} + \frac{1 - \eta_\nu \ \delta}{2} \cos\theta = \eta_1(\theta)$$

$$= \frac{1 - \eta_\nu - \delta}{4} \int_0^{\pi} \frac{K_n\left[\xi_p, \eta_\nu, \eta_1(\theta)\right] \bar{S}_m\left[\cos^{-1} \eta_1(\theta)\right]}{\left[\eta_\nu - \eta_1(\theta)\right]^2} \sin\theta \, d\theta$$

$$= \frac{1 - \eta_\nu - \delta}{4(M_1 + 1)} \sum_{i=1}^{M_1} \frac{K_n\left[\xi_p, \eta_\nu, \eta_1(\theta_i)\right] \bar{S}_m\left[\cos^{-1} \eta_1(\theta_i)\right]}{\left[\eta_\nu - \eta_1(\theta_i)\right]^2} \sin\theta_i$$

where

$$\theta_i = \frac{i\pi}{M_1 + 1} \qquad i = 1, 2, \ldots, M_1$$

$$C_{np\nu m} \equiv \frac{1}{2\pi}\int_{-1}^{\eta_\nu-\delta} \frac{K_n(\xi_p, \eta_\nu, \eta')\,\bar{S}_m\left[\cos^{-1}\ \eta'\right]}{(\eta_\nu - \eta')^2}\,d\,\eta'$$

and letting

$$\eta' = \eta_2(\theta) = \frac{-1+\eta_\nu - \delta}{2} + \frac{1+\eta_\nu - \delta}{2}\cos\theta$$

$$C_{np\nu m} = \frac{1-\eta_\nu - \delta}{4\pi}\int_0^{\pi} \frac{K_n\left[\xi_p, \eta_\nu, \eta_2(\theta)\right]\bar{S}_m(\theta)\ \sin\theta}{\left[\eta_\nu - \eta_2(\theta)\right]^2}\,d\,\theta$$

$$= \frac{1-\eta_\nu - \delta}{4(M_2+1)}\sum_{i=1}^{M_2} \frac{K_n\left[\xi_p, \eta_\nu, \eta_2(\theta_i)\right]\bar{S}_m(\theta_i)\ \sin\ \theta_i}{\left[\eta_\nu - \eta_2(\theta_i)\right]^2} \qquad \theta_i = \frac{i\pi}{M_2+1} \tag{32}$$

Therefore referring to (18), (19), (26), (28), (29), (30), (31), (32)

$$\alpha(\xi_p, \eta_\nu) = \sum_{n=0}^{N}\sum_{m=1}^{M} f_{nm}\left[H_{np\nu\nu}b_{\nu m} + \delta_{\nu m}A_{np\nu\nu} + B_{np\nu m} + C_{np\nu m}\right]$$

which is a set of linear equations to be solved for f_{nm}

$$p = 0,1,\ \ldots,\ N \qquad\qquad M \times (N+1)$$

$$\nu = 1,2,\ \ldots,\ M$$

$$H_{np\nu\nu} = H_n(\xi_p, \eta_\nu, \eta_\nu)$$

all of the H_n's and K_n's are computed numerically

Consider

$$\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)\, F(\eta')}{(\eta' - \eta)^2 + z^2}\, d\eta'$$

$$= \lim_{z \to 0} \left\{ F(\eta) \int_{-1}^{1} \frac{(\eta' - \eta)}{(\eta' - \eta)^2 + z^2}\, d\eta' + \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2}\, d\eta' \right\}$$

$$= \lim_{z \to 0} \left\{ \frac{1}{2} F(\eta) \log \left[(\eta' - \eta)^2 + z^2 \right] \right\} \Bigg|_{-1}^{1} + \int_{-1}^{1} \frac{(\eta' - \eta)\, [F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2}\, d\eta'$$

$$= F(\eta) \left\{ \log \frac{1-\eta}{\epsilon} + \log \frac{\epsilon}{1+\eta} \right\} + \lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2}\, d\eta'$$

If $F(\eta')$ is differentiable at $\eta' = \eta$ then $F(\eta') - F(\eta) = 0(\eta' - \eta)$ and we can write

$$\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)\, [F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2}\, d\eta'$$

$$= \lim_{\epsilon \to 0} \left\{ \int_{\eta+\epsilon}^{1} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)}\, d\eta' + \int_{-1}^{\eta-\epsilon} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)}\, d\eta' \right\} \tag{A1}$$

Therefore

$$\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta'-\eta)F(\eta')}{(\eta'-\eta)^2 + z^2} \, d\eta' = -\!\!\!\!\!\!\int_{-1}^{1} \frac{F(\eta')}{(\eta'-\eta)} \, d\eta'$$

$$= \lim_{\epsilon \to 0} \left\{ \int_{\eta+\epsilon}^{1} \frac{F(\eta')}{(\eta'-\eta)} \, d\eta' + \int_{-1}^{\eta-\epsilon} \frac{F(\eta')}{(\eta'-\eta)} \, d\eta' \right\}$$

$$\tilde{H}_n(p,q) = \tilde{H}_n(p_o, 0) + \oint \nabla \tilde{H}_n(p, q) \cdot d\bar{\ell}$$

$$= \tilde{H}_n(p_o, 0) + \oint \left| \frac{\partial \tilde{H}_n(p, q)}{\partial p} dp + \frac{\partial \tilde{H}_n(p, q)}{\partial q} dq \right| \tag{B1}$$

where C is a contour integral in the p, q plane from (p_o, 0 to (p, q)

$$\tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi'$$

$$\frac{\partial \tilde{H}_n(p, q)}{\partial p} = \int_0^1 k_n(\xi') \frac{q^2}{\left[(p - \xi')^2 + q^2\right]^{3/2}} d\xi'$$

if

$$p \neq 0, 1$$

we can write

$$h_n(\xi') = h_n(p) - h'_n(p)(p - \xi') + \frac{1}{2!} h''_n(p)(p - \xi')^2 + r_n(\xi')(p - \xi')^3 \tag{B2}$$

and since

$$\int \frac{x}{r^3} = -\frac{1}{r} \qquad \int \frac{x^2}{r^3}\,dx = \frac{xr}{2} - \frac{a^2}{2}\log\,(x+r)$$

and

$$\int \frac{a^2}{r^3} = \frac{x}{r}$$

where $r = \sqrt{x^2 + a^2}$

$$\frac{\partial}{\partial} \tilde{H}_n(p,q) = -\,h_n(p)\,\frac{(p-\xi')}{\sqrt{(p-\xi')^2 + q^2}}\Bigg|_{\xi'=0}^{\xi'=1} + 0(q^2)$$

or

$$\lim_{q \to 0} \frac{\partial}{\partial p} \tilde{H}_n(p,q) = 2\,h_n\,(p) \qquad p \neq 0,1 \tag{B3}$$

$$\frac{\partial}{\partial q} \tilde{H}_n(p,q) = -\,q \int_0^1 h_n(\xi')\,\frac{(p-\xi')}{\left[(p-\xi')^2 + q^2\right]^{3/2}}\,d\,\xi'$$

now write $(p \neq 0,1)$ $\quad h_n(\xi') = h_n(p) - h_n'\,(p)(p-\xi') + r_n(\xi')(p-\xi')^2$

Then as $q \to 0$

$$q \int_0^1 \frac{(p - \xi')}{\left[(p - \xi')^2 + q^2\right]^{3/2}} \, d\xi' = \frac{q}{\left[(p - \xi')^2 + q^2\right]^{3/2}} \Bigg|_0^1 = O(q)$$

and

$$q \int_0^1 \frac{r_n(\xi')(p - \xi')^3}{\left[(p - \xi')^2 + q^2\right]^{3/2}} \, d\xi' = O(q)$$

but

$$q \int_0^1 \frac{(p - \xi')^2}{\left[(p - \xi')^2 + q^2\right]^{3/2}} \, d\xi' = q \left\{ \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} + \log\left[(\xi' - p) + \sqrt{(p - \xi')^2 + q^2}\right] \right\} \Bigg|_0^1$$

$$= q \left\{ - \frac{(1 - p)}{\sqrt{(1 - p)^2 + q^2}} - \frac{p}{\sqrt{p^2 + q^2}} + \log\left[(1 - p) + \sqrt{(1 - p)^2 + q^2}\right] - \log\left[-p + \sqrt{p^2 + q^2}\right] \right\}$$

$$= O(q) - q \log\left[-p + \sqrt{p^2 + q^2}\right]$$

$$= O(q) - q \log\left[-1 + \sqrt{1 + \frac{p^2}{q^2}}\right]$$

$$= O(q) - 2 q \log |q|$$

Therefore as $q \to 0$

$$\frac{\partial}{\partial q} \tilde{H}_n (p,q) = - 2 h'_n (p) q \ln |q| + 0(q) \tag{B4}$$

and since $h_n(p)$ and $h'_n(p)$ are slowly varying for $p \neq 0,1$ referring to (B1) and using (B3) and (B4) for $(p-p_o) << 1$ and $q << 1$

$$\oint \frac{\partial}{\partial p} \tilde{H}_n (p,q)\, d = \int_{p_o}^{p} \frac{\partial}{\partial p} \tilde{H}_n(p,q)\, dp = 2 h_n(p_o)(p-p_o)$$

and

$$\oint \frac{\partial}{\partial q} \tilde{H}_n (p,q)\, dq = - 2 h'_n(p_o) \int_0^q q \ln q\, dq = - h'_n (p_o) q^2 \ln q$$

or to lowest order in $p-p_o$ and q (for $p_o \neq 0,1$)

$$\tilde{H}_n(p,q) = \tilde{H}_n(p_o,0) + 2 h_n(p_o)(p-p_o) - h'_n(p_o)\, q^2 \ln q \tag{B5}$$

$$\tilde{H}_n(p,q) = \int_0^1 h_n(\xi')\left\{1 + \frac{(p-\xi')}{\sqrt{(p-\xi')^2 + q^2}}\right\} d\xi'$$

Leading Edge Let

$$p = -\delta^2 \sin\tilde{\phi}$$

$$q = \delta^2 \cos\tilde{\phi}$$

or

$$\delta^2 = \sqrt{p^2 + q^2}$$

$$\tilde{\phi} = \tan^{-1}\frac{-p}{q}$$

and introduce a change of variables

$$\xi' = \sin^2\sigma$$

$$d\xi' = 2\sin\sigma\cos\sigma\, d\sigma$$

$$h_n(\sin^2\sigma) = h_n\left[\frac{1}{2}(1 - \cos 2\sigma)\right] = \hat{h}_n(2\sigma) = \frac{2}{\pi}\frac{\cos 2n\sigma + \cos 2(n+1)\sigma}{\sin 2\sigma}$$

$$= \frac{2}{\pi}\frac{\cos\left([2n+1] - 1\right)\sigma + \cos\left([2n+1] + 1\right)\sigma}{2\sin\sigma\cos\sigma} = \frac{2}{\pi}\frac{\cos(2n+1)\sigma}{\sin\sigma}$$

$$\left\{1 + \frac{p - \xi'}{\sqrt{(p-\xi')^2 + q^2}}\right\} = 1 + \frac{-(\delta^2\sin\tilde{\phi} + \sin^2\sigma)}{\sqrt{\sin^4\sigma + 2\delta^2\sin\tilde{\phi}\sin^2\sigma + \delta^4}}$$

and

$$\tilde{H}_n(p,q) = \hat{H}_n(\delta,\tilde{\phi})$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\left\{1 + \frac{-(\delta^2\sin\tilde{\phi} + \sin^2\sigma)}{\sqrt{\sin^4\sigma + 2\delta^2\sin\tilde{\phi}\sin^2\sigma + \delta^4}}\right\}\cos(2n+1)\sigma\cos\sigma\, d\sigma$$

$$\frac{\partial\hat{H}_n}{\partial\delta} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\frac{-2\sin\tilde{\phi}\cos[(2n+1)\sigma]\cos\sigma\, d\sigma}{\sqrt{\sin^4\sigma + 2\delta^2\sin\tilde{\phi}\sin^2\sigma + \delta^4}}\, d\sigma$$

$$+ \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\frac{2\delta\left[+\sin\tilde{\phi}\sin^2\sigma + \delta^2\right](\delta^2\sin\tilde{\phi} + \sin^2\sigma)\cos[(2n+1)\sigma]\cos\sigma\, d\sigma}{\left[\sin^4\sigma + 2\delta^2\sin\tilde{\phi}\sin^2\sigma + \delta^4\right]^{3/2}}$$

$$= \frac{8\delta^3\cos^2\tilde{\phi}}{\pi}\int_0^{\frac{\pi}{2}}\frac{\cos[(2n+1)\sigma]\cos\sigma\sin^2\sigma}{\left[\sin^4\sigma + 2\delta^2\sin\tilde{\phi}\sin^2\sigma + \delta^4\right]^{3/2}}\, d\sigma$$

as $\delta \to 0$ the only portion of the integrand which is important is in the region of $\sigma = 0$

C-4

Therefore let $x = \sin\sigma \quad dx = \cos\sigma \, d\sigma$ and expand

$$\cos\left[(2n+1)\,\sigma\right] = \cos\left[(2n+1)\left(x - \frac{x^3}{6}\right)\right]$$

$$= 1 - \frac{1}{2}(2n+1)^2\, x^2 = 1 - \frac{x^2}{2}\left(1 + 4n + 4n^2\right) + O(x^4)$$

Therefore to first order in x^2

$$\frac{\partial}{\partial\delta}\hat{H}_n(\delta,\tilde{\phi}) = \frac{8\,\delta^3 \cos^2\tilde{\phi}}{\pi}\int_0^1 \frac{\left[1 - \frac{x^2}{2}\left(1 + 4n + 4n^2\right)\right] x^2}{\left[x^4 + 2\delta^2 x^2 \sin\tilde{\phi} + \delta^4\right]^{3/2}}\, dx$$

$$= \frac{8\cos^2\tilde{\phi}}{\pi}\left\{\int_0^{\frac{1}{\delta}} \frac{x^2\, dx}{\left[x^4 + 2x^2\sin\tilde{\phi} + 1\right]^{3/2}} - \frac{\delta^2}{2}\int_0^{\frac{1}{\delta}} \frac{\left(1 + 4n + 4n^2\right) x^4\, dx}{\left[x^4 + 2x^2\sin\tilde{\phi} + 1\right]^{3/2}}\right\}$$

Since

$$\int_0^{\frac{1}{\delta}} \frac{x^2\, dx}{\left[x^4 + 2x^2\sin\tilde{\phi} + 1\right]^{3/2}} = \int_0^{\infty} \frac{x^2\, dx}{\left[x^4 + 2x^2\sin\tilde{\phi} + 1\right]^{3/2}} - \int_{\frac{1}{\delta}}^{\infty} \frac{dx}{x^4}$$

$$= \int_0^{\infty} \frac{x^2\, dx}{\left[x^4 + 2x^2\sin\tilde{\phi} + 1\right]^{3/2}} + O(\delta^3)$$

Near $\delta = 0$ we can say

$$\frac{\partial}{\partial \delta} \hat{H}_n(\delta, \tilde{\phi}) = \frac{8 \cos^2 \tilde{\phi}}{\pi} \int_0^\infty \frac{x^2 \left[1 - \frac{\delta^2 x^2}{2} (1 + 4n + 4n^2)\right]}{\left[x^4 + 2x^2 \sin \tilde{\phi} + 1\right]^{3/2}} dx + 0(\delta^3)$$

integrating from 0 to δ we get

$$\hat{H}_n(\delta, \tilde{\phi}) = \hat{H}_n(0, \tilde{\phi}) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi})\delta - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \delta^3 \right\} \tag{C1}$$

or

$$\tilde{H}_n(p,q) = \tilde{H}_n(0,0)$$

$$+ \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi}) \left(p^2 + q^2\right)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \left(p^2 + q^2\right)^{3/4} \right\} \tag{C2}$$

where

$$\tilde{\phi} = \tan^{-1} \frac{(-p)}{q}$$

and where

$$I_1(\tilde{\phi}) = \int_0^\infty \frac{x^2 \, dx}{\left[x^4 + 2x^2 \sin \tilde{\phi} + 1\right]^{3/2}} = \frac{1}{\cos \tilde{\phi}} \frac{d}{d\tilde{\phi}} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \tag{C3}$$

$$I_2(\tilde{\phi}) = \int_0^{\infty} \frac{x^4\, dx}{\left[x^4 + 2x^2 \sin\tilde{\phi} + 1\right]^{3/2}}$$

$$= -\sin\tilde{\phi}\; I_1(\tilde{\phi}) + \int_0^{\infty} \frac{(x^4 + x^2 \sin\tilde{\phi})}{\left[x^4 + 2x^2 \sin\tilde{\phi} + 1\right]^{3/2}}\, dx \qquad \text{(C4)}$$

$$u = x$$

$$du = dx$$

$$dv = \frac{(x^3 + x \sin\tilde{\phi})\, dx}{\left[x^4 + 2x^2 \sin\tilde{\phi} + 1\right]^{3/2}}$$

$$v = \frac{-\frac{1}{2}}{\sqrt{x^4 + 2x^2 \sin\tilde{\phi} + 1}}$$

Therefore

$$I_2(\tilde{\phi}) = -\sin\tilde{\phi}\; I_1(\tilde{\phi}) + \frac{1}{2} \int_0^{\infty} \frac{dx}{\sqrt{x^4 + 2x^2 \sin\tilde{\phi} + 1}} \qquad \text{(C5)}$$

$$\tilde{H}_n(p,\ q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi' \qquad \text{(D1)}$$

Trailing Edge, **let**

$$p = 1 - \delta^2 \sin\theta$$

$$q = \delta^2 \cos\theta$$

or

$$\delta^2 = \sqrt{(1 - p)^2 + q^2}$$

$$\theta = \tan^{-1} \frac{(1 - p)}{q} \qquad \text{(D2)}$$

and change variables in (D1)

$$\xi' = \sin^2\sigma$$

$$d\xi' = 2 \sin\sigma \cos\sigma \; d\sigma$$

$$h_n\ (\sin^2\sigma) = \frac{2}{\pi} \frac{\cos\ (2n + 1)\ \sigma}{\sin\sigma}$$

(see leading edge expansion)

$$\tilde{\Pi}_n\,(p,q) = \hat{\Pi}_n\,(\delta,\theta) = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{\cos^2\sigma - \delta^2 \sin\theta}{\sqrt{(\cos^2\sigma - \delta^2\sin\theta)^2 + \delta^4\cos^2\theta}} \right\} \cos\,(2n+1)\sigma\ \cos\sigma\ d\sigma$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{\sin^2\sigma - \delta^2\sin\theta}{\sqrt{\sin^4\sigma - 2\delta^2\sin^2\sigma\ \sin\theta + \delta^4}} \right\} \cos\left[(2n+1)\ \left(\frac{\pi}{2} - \sigma\right)\right]\ \sin\sigma\ d\sigma$$

Now

$$\cos\left[(2n+1)\ \left(\frac{\pi}{2} - \sigma\right)\right] = \cos\left[(2n+1)\ \frac{\pi}{2}\right]\cos\left[(2n+1)\,\sigma\right] + \sin\left[(2n+1)\ \frac{\pi}{2}\right]\sin\left[(2n+1)\,\sigma\right]$$

$$= \sin\left[(2n+1)\ \frac{\pi}{2}\right]\sin\left[(2n+1)\,\sigma\right] = (-1)^{n-1}\sin\left[(2n+1)\,\sigma\right]$$

The differentiation of the integrand with respect to δ may be performed easily if it is compared with the leading edge case.

$$\frac{\partial}{\partial\delta}\hat{H}_n\,(\delta,\theta)\Big| = (-1)^n\,\frac{8\delta^3\cos^2\theta}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin\left[(2n+1)\sigma\right]\,\sin^3\sigma\ d\sigma}{\left[\sin^4\sigma - 2\delta^2\sin\theta\ \sin^2\sigma + \delta^4\right]^{3/2}}$$

Changing variables of integration once again

$$x = \sin\sigma$$

$$dx = \cos\sigma\ d\sigma$$

$$\sin\left[(2n+1)\,\sigma\right] = (1+2n)x + O(x^3)$$

and

$$\frac{\partial}{\partial \delta} H_n(\delta,\theta) = (-1)^n (1+2n) \frac{8\delta^3 \cos^2\theta}{\pi} \int_0^{\frac{1}{\delta}} \frac{x^4 \, dx}{\left[x^4 - 2\delta^2 x^2 \sin\theta + \delta^4\right]^{3/2}}$$

$$= (-1)^n (1+2n) \frac{8\delta^2 \cos^2\theta}{\pi} \int_0^{\infty} \frac{x^4 \, dx}{\left[x^4 - 2 x^2 \sin\theta + 1\right]^{3/2}} + O(\delta^4)$$

$$= (-1)^n (1+2n) \frac{8\delta^2 \cos^2\theta}{\pi} I_1 (\theta + \pi) \tag{D3}$$

where

$$I_1 (\theta) = \int_0^{\infty} \frac{x^4 \, dx}{[x^4 + 2 x^2 \sin\theta + 1]^{3/2}} \tag{D4}$$

and integrating from 0 to δ we get

$$\widetilde{H}_n(p, q) = \widetilde{H}_n(1, q) + (-1)^n (1+2n) \frac{8 \cos^2\theta \; I_1 (\theta + \pi)}{3\pi} \left[(1-p)^2 + q^2\right]^{3/4} \tag{D5}$$

where

$$\theta = \tan^{-1} \frac{(1-p)}{q} \tag{D6}$$

Subappendix E

Suppose that k(x', y') may be written (as before)

$$\frac{c(\eta')}{b} \, k\left[x'(\xi', \eta'), y(\eta')\right] = \sum_{n=o}^{N} f_n(\eta') \, h_n(\xi')$$

where

$$h_n(\xi') = \frac{1}{\pi}\left[\frac{1 - \xi'}{\xi'}\right]^{1/2} \left[\frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'}\right]$$

then

$$\hat{h}_n(\Psi) = h_n\left[\frac{1}{2}(1 - \cos \Psi)\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos n\Psi + \cos (n + 1)\Psi}{\sin \Psi}\right] = \frac{2}{\pi} \frac{\cos\left(n + \frac{1}{2}\right)\Psi}{\sin \frac{\Psi}{2}}$$

Then with

$$\xi = \frac{x - x_v \frac{2y}{b}}{c \frac{2y}{b}}$$

$$\eta = \frac{2y}{b}$$

$$\zeta = \frac{2z}{b}$$

and from (3), (2) and (7) with ζ kept in g

$$\Phi(x, y, z) = \phi(\xi, \eta, \zeta) = \frac{b}{4\pi} \int_{-1}^{1} \frac{\zeta \hat{g}(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} d\eta'$$

$$\alpha(\xi, \eta) = \lim_{\zeta \to 0} \frac{-1}{2\pi} \frac{\partial}{\partial \zeta} \int_{-1}^{1} \frac{g(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} d\eta'$$

where

$$g(\xi, \eta, \eta', \zeta) = \sum_{n=0}^{N} f_n(\eta') \hat{H}_n(\xi, \eta, \eta', \zeta)$$

and

$$\hat{\hat{H}}_{n(\xi,\eta,\eta',\zeta)}$$

$$= \int_0^1 h_n(\xi') \left\{ 1 + \frac{\dfrac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[\dfrac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi'\right]^2 + \left[\dfrac{(\eta - \eta')\frac{1}{2}b}{c(\eta')}\right]^2 + \left[\dfrac{\zeta \frac{1}{2} b}{c(\eta')}\right]^2}} \right\} d\xi'$$

We can also define

$$\tilde{\tilde{H}}_n(p,q,\hat{\zeta}) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2 + \hat{\zeta}^2}} \right\} d\xi'$$

For small $\eta' - \eta$

$$p - p_o = - \frac{\frac{1}{2}b\,(\eta' - \eta)}{c(\eta)} \tan\phi$$

$$\phi = \text{slope of constant percent chord lines}$$

$$q = \frac{\frac{1}{2}b\,(\eta' - \eta)}{c(\eta')} \quad \frac{\frac{1}{2}b\,(\eta' - \eta)}{c(\eta)}$$

$$c(\eta') = c(\eta) + \frac{\partial}{\partial \eta'} c(\eta') \Big|_{\eta' = \eta} (\eta' - \eta)$$

$$\hat{\zeta} = \frac{\zeta\left(\frac{1}{2}b\right)}{c(\eta')} = \frac{z}{c(\eta')} = \frac{z}{c(\eta)}$$

$$\zeta = \frac{z}{\frac{1}{2}b}$$

<u>Leading Edge</u>, $P_o = 0$, $\phi = \phi_{LE}$ = leading edge sweep

$$p^2 + q^2 + \hat{\zeta}^2 = \left[\frac{\frac{1}{2}b}{c(\eta)}\right]^2 \left\{(\eta' - \eta)^2 \quad [1 + \tan^2\phi] + \left(\frac{z}{\frac{1}{2}b}\right)^2\right\}$$

$$= \frac{1}{\cos^2\phi} \left[\frac{\frac{1}{2}b}{c(\eta)}\right]^2 \left\{(\eta' - \eta)^2 + \zeta^2 \cos^2\phi\right\}$$

Therefore at the leading edge since $\tilde{\phi} = \tan^{-1}\left(\frac{-p}{q}\right) = \begin{cases} \phi_{LE} & \eta' > \eta \\ \phi_{LE} + \pi & \eta' < \eta \end{cases}$

$$\hat{K}_n(o, \eta, \eta', \zeta) = \frac{8(1 - \sin^2\phi_{LE}\sin^2\psi)}{\pi}\left\{I_1(\tilde{\phi}, \psi)\,[p^2 + q^2 + \hat{\zeta}^2]^{\frac{1}{4}} - \frac{1}{6}(1 + 4n + 4n^2)I_2(\tilde{\phi}, \psi)\,[p^2 + q^2 + \hat{\zeta}^2]^{\frac{3}{4}}\right\}$$

$$= \frac{8(1 - \sin^2\phi\ \sin^2\psi)}{\pi}\left\{I_1(\tilde{\phi}, \psi)\frac{1}{\cos^2\phi}\left[\frac{\frac{1}{2}b}{c(\eta)}\right]^{\frac{1}{2}}\left[(\eta' - \eta)^2 + \zeta^2\cos^2\phi\right]^{\frac{1}{4}}\right.$$

$$\left. - \frac{1}{6}(1 + 4n + 4n^2)\,I_2(\tilde{\phi}, \psi)\left[\frac{\frac{1}{2}b}{c(\eta)}\right]^{\frac{3}{2}}\left[(\eta' - \eta)^2 + \zeta^2\cos^2\phi\right]^{\frac{1}{4}}\right\}$$

$$\sin \psi = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \zeta^2}} = \frac{|\eta' - \eta|}{\sqrt{(\eta' - \eta)^2 + \zeta^2 \cos^2 \phi}} = \frac{|u|}{\sqrt{u^2 + \cos^2 \phi}}$$

where

$$u = \frac{\eta' - \eta}{\zeta}$$

letting $s = \eta' - \eta$ we must evaluate an expression of the form

$$\lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \int_{-\delta}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) \, I_1 (\tilde{\phi}, \psi) \, \zeta \left[s^2 + \zeta^2 \cos^2 \phi\right]^{\frac{1}{4}}}{s^2 + \zeta^2} \, ds \qquad \tilde{\phi} = \begin{matrix} \phi_{LE} & s > 0 \\ \phi_{LE} + \pi & s < 0 \end{matrix}$$

$$= \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \left\{ \sqrt{\zeta} \int_0^{\frac{\delta}{\zeta}} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \hat{\psi}) \left[I_1 (\phi_{LE}, \hat{\psi}) + I, (\phi_{LE} + \pi, \hat{\psi})\right] \left[u^2 + \cos^2 \phi\right]^{\frac{1}{4}}}{u^2 + 1} \, du \right\}$$

$$= \lim_{\zeta \to 0} \frac{\partial}{\partial \zeta} \left\{ \sqrt{\zeta} \int_0^{\infty} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \hat{\psi}) \left[I_1 (\phi_{LE}, \psi) + I_1 (\phi_{LE} + \pi, \hat{\psi})\right] \left[u^2 + \cos^2 \phi\right]^{\frac{1}{4}}}{u^2 + 1} \, du \right.$$

$$\left. - \sqrt{\zeta} \int_{\frac{\delta}{\zeta}}^{\infty} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \psi) \left[I_1 (\phi_{LE}, \hat{\psi}) + I_1 (\phi_{LE} + \pi, \hat{\psi})\right] \left[u^2 + \cos^2 \phi\right]^{\frac{1}{4}}}{u^2 + 1} \, du \right.$$

where

$$\sin \hat{\psi} = \frac{u}{\sqrt{u^2 + \cos^2 \phi}} \qquad \text{(no } \xi \text{ dependence!)}$$

and

$$I_1 (\phi, \psi) = \int_0^{\infty} \frac{x^2 \, dx}{\left[x^4 + 2x^2 \sin \phi \sin \psi + 1\right]^{3/2}}$$

The second integral goes to zero like $\sqrt{\xi}$ and the first is independent of ξ. Therefore

$$\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \sim \frac{1}{\sqrt{\xi}} \to \infty$$

$$\tilde{\tilde{H}}_n (P, q, \zeta) = \int_0^1 h_n (\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + \zeta^2}} \right\} d\xi'$$

Leading edge, let

$$p = -\delta^2 \sin \hat{\phi} \sin \psi$$

$$q = \delta^2 \cos \hat{\phi} \sin \psi$$

$$\zeta = \delta^2 \cos \psi$$

$$\delta^2 = \sqrt{p^2 + q^2 + \zeta^2}$$

$$\hat{\phi} = \tan^{-1} \frac{-p}{q}$$

$$\psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\zeta}$$

introduce a change of variables in the integral

$$\xi' = \sin^2 \sigma$$

$$d\xi' = 2 \sin \sigma \cos \sigma \, d\sigma$$

$$h_n\ (\sin^2\sigma) \;=\; h_n\left[\frac{1}{2}\ (1\ -\ \cos\ 2\sigma)\right] \;=\; \hat{h}_n\ (2\sigma) \;=\; \frac{2}{\pi}\ \frac{\cos\ 2n\ \sigma\ +\ \cos\ 2(n\ +\ 1)\sigma}{\sin\ 2\sigma}$$

$$= \frac{2}{\pi}\ \frac{\cos\left[(2n\ +\ 1)\ -\ 1\right]\sigma\ +\ \cos\left[(2n\ +\ 1)\ +\ 1\right]\sigma}{2\ \sin\ \sigma\ \cos\ \sigma} \;=\; \frac{2}{\pi}\ \frac{\cos\ (2n\ +\ 1)\sigma}{\sin\ \sigma}$$

and

$$\left\{1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + \zeta^2}}\right\} = \left\{1 + \frac{-\left[\delta^2 \sin\hat{\phi}\sin\psi + \sin^2\sigma\right]}{\sqrt{\sin^4\sigma + 2\delta^2\sin\hat{\phi}\sin\psi\sin^2\sigma + \delta^4}}\right\}$$

$$\tilde{H}_n\ (p, q, \zeta) = \hat{H}_n\ (\delta, \hat{\phi}, \psi) = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\left\{1 + \frac{-\left[\delta^2\sin\hat{\phi}\sin\psi + \sin^2\sigma\right]}{\sqrt{\sin^4\sigma + 2\delta^2\sin\hat{\phi}\sin\psi\sin^2\sigma + \delta^4}}\right\}\cos\left[(2n+1)\sigma\right]\cos\sigma\ d\sigma$$

$$\frac{\partial\hat{H}_n\ (\delta, \hat{\phi}, \psi)}{\partial\delta} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\frac{-2\delta\ \sin\hat{\phi}\sin\psi\ \cos\left[(2n+1)\sigma\right]\cos\sigma\ d\sigma}{\sqrt{\sin^4\sigma + 2\delta^2\sin\hat{\phi}\sin\psi\ \sin^2\sigma + \delta^4}}$$

$$+\frac{4}{\pi}\int_0^{\frac{\pi}{2}}\frac{2\delta\left[\sin\hat{\phi}\sin\psi\ \sin^2\sigma + \delta^2\right](\delta^2\sin\hat{\phi}\sin\psi + \sin^2\sigma)\cos\left[(2n+1)\sigma\right]\cos\sigma\ d\sigma}{\left[\sin^4\sigma + 2\delta^2\sin\hat{\phi}\sin\psi\ \sin^2\sigma + \delta^4\right]^{3/2}}$$

$$= \frac{8\delta^3\left[1 - \sin^2\hat{\phi}\sin^2\psi\right]}{\pi}\int_0^{\frac{\pi}{2}}\frac{\cos\left[(2n+1)\sigma\right]\cos\sigma\sin^2\sigma\ d\sigma}{\left[\sin^4\sigma + 2\delta^2\sin\hat{\phi}\sin\psi\ \sin^2\sigma + \delta^4\right]^{3/2}}$$

as $\delta \to o$ the only portion of the integrand which is important is in the region of $\sigma = o$.

Therefore let $x = \sin\ \sigma$, $dx = \cos\sigma\ d\sigma$ and expand

$$\cos\left[(2n+1)\ \sigma\right] = \cos\left[(2n+1)\left(x - \frac{x^3}{6}\right)\right] = 1 - \frac{1}{2}(2n+1)^2\ x^2$$

$$1 - \frac{x^2}{2}(1 + 4n + 4n^2) + O(x^4)$$

Therefore to first order in x^2

$$\frac{\partial}{\partial\delta}\ \hat{H}_n(\delta, \hat{\phi}, \psi) = \frac{8\,\delta^3\left[1 - \sin^2\hat{\phi}\ \sin^2\ \psi\right]}{\pi}\int_0^1 \frac{\left[1 - \frac{x^2}{2}(1 + 4n + 4n^2)\right]x^2\ dx}{\left[x^4 + 2\,\delta^2\ x^2\ \sin\hat{\phi}\ \sin\ \psi + \delta^4\right]^{3/2}}$$

$$= \frac{8\left[1 - \sin^2\hat{\phi}\ \sin^2\ \psi\right]}{\pi}\int_0^{\frac{1}{\delta}} \frac{x^2 - \frac{1}{2}\delta^2\ x^4\ (1 + 4n + 4n^2)}{\left[x^4 + 2\ x^2\ \sin\hat{\phi}\ \sin\ \psi + 1\right]^{3/2}}\ dx$$

$$= \frac{8\left[1 - \sin^2\hat{\phi}\ \sin^2\ \psi\right]}{\pi}\int_0^{\infty} \frac{x^2 - \frac{1}{2}\delta^2\ x^4\ (1 + 4n + 4n^2)}{\left[x^4 + 2\ x^2\ \sin\hat{\phi}\ \sin\ \psi + 1\right]^{3/2}}\ dx + O(\delta^3)$$

Integrating from 0 to δ we get

$$\hat{H}_n(\delta, \hat{\phi}, \psi) = \hat{H}_n(o, \hat{\phi}, \psi) + \frac{8\ (1 - \sin^2\hat{\phi}\ \sin^2\ \psi)}{\pi}\left|\ I_1(\hat{\phi}, \psi)\ \delta - \frac{1}{6}(1 + 4n + 4n^2)\ I_2(\hat{\phi}, \psi)\ \delta^3\right| \qquad \text{(C1)}$$

and

$$\tilde{\tilde{\Pi}}_n (p, q, \zeta) = \Pi_n (o, o, o) + \frac{8 (1 - \sin^2 \hat{\phi} \sin^2 \psi)}{\pi} \left| I_1 (\hat{\phi}, \psi) \left[p^2 + q^2 + \hat{\zeta}^2\right]^{\frac{1}{4}} - \frac{1}{6} (1 + 4n + 4n^2) I_2 (\hat{\phi}, \psi) (p^2 + q^2 + \hat{\zeta}^2)^{\frac{3}{4}} \right| \tag{C2}$$

where

$$\hat{\phi} = \tan^{-1} \frac{-p}{q}$$

$$\psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\hat{\zeta}}$$

$$I_1 (\phi, \psi) = \int_o^\infty \frac{x^2 \, dx}{\left[x^4 + 2 x^2 \sin \hat{\phi} \sin \psi + 1\right]^{3/2}} \tag{C3}$$

$$I_2 (\phi, \psi) = \int_o^\infty \frac{x^4 \, dx}{\left[x^4 + 2 x^2 \sin \hat{\phi} \sin \psi + 1\right]^{3/2}} \tag{C4}$$

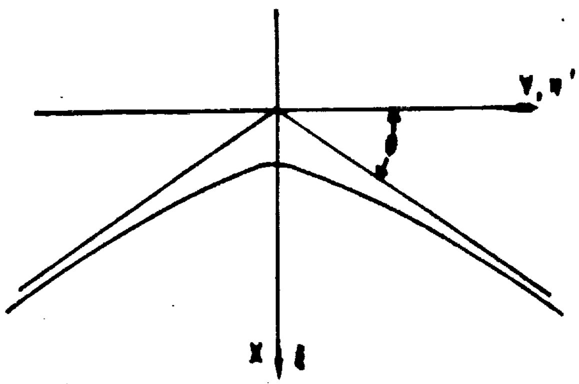
$$\sin \psi = \frac{\tan \psi}{\sqrt{1 + \tan^2 \psi}} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \hat{\zeta}^2}} = \frac{\sqrt{\frac{p^2 + q^2}{\zeta^2}}}{\sqrt{\frac{p^2 + q^2}{\zeta^2} + 1}}$$

$$\sin \phi = \frac{-p}{\sqrt{p^2 + q^2}}$$

$$\sin \phi \sin \psi = \frac{-p}{\sqrt{p^2 + q^2 + \hat{\zeta}^2}} = \frac{\frac{-p}{\zeta}}{\sqrt{\frac{p^2 + q^2}{\hat{\zeta}^2} + 1}}$$

$$\tilde{H}_n(p,q) = \tilde{H}_n(p_o,0) + 2\,h_n(p_o)(p - p_o) - h_n'(p_o)\,q^2 \ln|q|$$

Hyperbolic leading and trailing edges, constant chord.



$$\left[X_v(\eta')\right]^2 = c\,\tan^2\theta\,\left[\eta'^2 + \rho^2\tan^2\theta\right]$$

$$X_v(\eta') = c\,\tan\theta\,\sqrt{\eta'^2 + \rho^2\tan^2\theta}$$

$$\eta' >> 1 \qquad \frac{1}{c}\frac{dX_v}{d\eta} = \tan\theta$$

Curvature at $\eta' = 0$ $\quad \left.\frac{1}{c}\frac{d^2X_v}{d\eta'^2}\right|_{\eta'=0} = \frac{1}{\rho}$

From (21) at $\eta' = 0$

$$p - p_o = \frac{X_v(0) - X_v(\eta')}{c}$$

or

$$p - p_o = -\tan\theta\left[\sqrt{\eta'^2 + \rho^2\tan^2\theta} - \rho\tan\theta\right]$$

Therefore from (15) and (20), for small η'

$$H_n(\xi,0,\eta') = H_n(\xi,0,0) - 2h_n(\xi)\tan\theta\left[\sqrt{\eta'^2+\rho^2\tan^2\theta} - \rho\tan\theta\right]$$

$$+ h'_n(\xi)\,\eta'^2\ln|\eta'|$$

Therefore referring to (17) and (18) we must evaluate

$$\lim_{\zeta\to 0}\frac{\partial}{\partial\zeta}\int_0^\delta \frac{\zeta\left[\sqrt{s^2+a^2}-a\right]}{s^2+\zeta^2}\,ds$$

$$\int_0^\delta \frac{\zeta\left[\sqrt{s^2+a^2}-a\right]}{s^2+\zeta^2}\,ds$$

$$= \zeta\left\{\log\left[s+\sqrt{s^2+a^2}\right] + \frac{\sqrt{a^2-\zeta^2}}{\zeta}\tan^{-1}\frac{s\sqrt{a^2-\zeta^2}}{\zeta\sqrt{s^2+a^2}} - \frac{a}{\zeta}\tan^{-1}\frac{s}{\zeta}\right\}\Bigg|_0^\delta$$

$$= \zeta\left\{\log\left[\delta+\sqrt{\delta^2+a^2}\right] + \frac{\sqrt{a^2-\zeta^2}}{\zeta}\tan^{-1}\frac{\delta\sqrt{a^2-\zeta^2}}{\zeta\sqrt{\delta^2+a^2}} - \frac{a}{\zeta}\tan^{-1}\frac{\delta}{\zeta} - \log a\right\}$$

and

$$\lim_{a \to 0} \lim_{\zeta \to 0} \int_0^{\delta} \frac{\zeta\left[\sqrt{s^2 + a^2} - a\right]}{s^2 + \zeta^2} ds = \infty .$$

and since $a = \rho \tan \theta$, there is a logarithmic infinity as the radius of curvature goes to zero and the leading edge becomes kinked.

APPENDIX F

POTENTIAL FORM DRAG

The section load can be obtained by means of the Blasius theorem as follows;

$$F_X - i\,F_Y = \frac{1}{2}\rho i \int_C \left(\frac{dw}{dz}\right)^2 dz \tag{1}$$

where

$$\left(\frac{dw}{dz}\right)^2 = u^2 - 2uvi + (iv)^2 = u^2 - v^2 - 2uvi \tag{2}$$

Therefore;

$$F_x - iF_y = \frac{1}{2}\rho \int_C \left[2uv + i\,(u^2 - v^2)\right][dx + idy] \tag{3}$$

then

$$F_x = \frac{1}{2}\rho \int_C \left[2uvdx - (u^2 - v^2)\,dy\right] \tag{4}$$

$$F_y = -\frac{1}{2}\rho \int_C \left[2uvdy + (u^2 - v^2)\,dx\right] \tag{5}$$

Since all of the singularities are on the chordal plane, for the panels, equations (4) and (5) reduce to;

$$F_x = -\rho \sum_{i=1}^{N_i} \left(u_{u_i} v_{u_i} - u_{L_i} v_{L_i} \right) \Delta x_i \tag{6}$$

$$F_y = \rho/_2 \sum_{i=1}^{N_i} \left[\left(u_{u_i}^2 - v_{u_i}^2 \right) - \left(u_{L_i}^2 - v_{L_i}^2 \right) \right] \Delta x_i \tag{7}$$

where the subscripts u and L indicate upper and lower surfaces, respectively.

For the two-dimensional lifting case;

$$u_{u_i} = \frac{1}{2} \gamma_i + V_\infty \cos \alpha \tag{8}$$

$$u_{L_i} = -\frac{1}{2} \gamma_i + V_\infty \cos \alpha \tag{9}$$

$$v_{u_i} = v_{L_i} = -\frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Gamma_K}{x_i - x_K} + V_\infty \sin \alpha \tag{10}$$

Therefore,

$$F_x = -\rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \sin \alpha + \frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_K} \frac{\Gamma_i \Gamma_K}{x_i - x_K} \tag{11}$$

Since

$$\frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_i} \frac{\Gamma_i \Gamma_k}{x_i - x_K} = 0 \tag{12}$$

$$F_x = -\rho V_\infty \sin\alpha \sum_{i=1}^{N_i} \Gamma_i = -L \sin\alpha \tag{13}$$

where L is the lift and $\Gamma = \gamma \Delta x$ is the local vortex strength.

Also; from equations (7), (8), (9), and (10)

$$F_y = \rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \cos\alpha = L \cos\alpha \tag{14}$$

which demonstrates that the discrete vortex lattice always gives the correct chord force F_x and normal force F_y, provided the correct circulations is obtained.

Similarly, for the two-dimensional thickness case;

$$u_{u_i} = u_{L_i} = \frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Sigma_K}{x_i - x_K} + V_\infty \tag{15}$$

$$v_{u_i} = V_\infty \left(\frac{dz_t}{dx}\right)_i \tag{16}$$

$$v_{L_i} = - V_\infty \left(\frac{dz_t}{dx}\right)_i \tag{17}$$

and

$$\Sigma_i = 2 V_\infty \left(\frac{dz_t}{dx}\right)_i \Delta x_i \tag{18}$$

Therefore;

$$F_x = - \frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_i} \frac{\Sigma_i \ \Sigma_K}{x_i - x_K} - \rho V_\infty \sum_{i=1}^{N_i} \Sigma_i = 0 \tag{19}$$

since

$$\sum_{i=1}^{N_i} \Sigma_i = 0$$

if the airfoil is closed.

Also; $F_y = 0$ from equation (7), (15), (16), and (17). This demonstrates that the discrete source lattice also gives the correct chord force F_x and normal force F_y.

The above equations can be generalized to compute the section potential form drag on a finite wing due to lift and thickness by evaluating the component of force in the free stream direction and by using the three dimensional influence equations.

The section potential form drag due to lift is computed as follows;

$$d_{L_i} = -\rho \sum_{i=1}^{N_i} \Gamma_i w_i \tag{20}$$

where i is summed over the section of the panel and w_i is the total velocity normal to the panel chordal surface at the quarter chord of the i^{th} subpanel. The section induced drag coefficient

$$\frac{C_{d_{L_i}} C}{C_{AVG.}}$$

is then given by;

$$\frac{C_{d_{L_i}} C}{C_{AVG.}} = -\frac{2AR}{b} \sum_{i=1}^{N_i} \left(\frac{\Gamma_i}{V_\infty}\right)\left(\frac{w_i}{V_\infty}\right) \tag{21}$$

If w_i is computed at the three-quarter chord of the i^{th} subpanel, instead of the quarter chord of the i^{th} subpanel as is done in equation (21), the section zero percent suction drag coefficient

$$\frac{C_{d_{T=0}} C}{C_{AVG.}}$$

is obtained. The section leading edge thrust coefficient $(C_T\ C)/(C_{AVG.})$ is equal to;

$$\frac{C_T\ C}{C_{AVG.}} = \frac{C_{d_{T=0}} C}{C_{AVG.}} - \frac{C_{d_{L_i}} C}{C_{AVG.}} \tag{22}$$

The section potential form drag due to thickness is computed by a similar procedure.

$$d_{T_i} = -\rho \sum_{i=1}^{2N_i} \Sigma_i u_i - \rho V_\infty \sum_{i=1}^{2N_i} \left(\frac{V_{x_i}}{V_\infty}\right) \Sigma_i \qquad (23)$$

where i is summed over the section for both the quarter and three-quarter chord stations of each subpanel. Σ_i and u_i are the source strength and total velocity in the free stream direction, respectively, at either the quarter or three-quarter chord point of the subpanel. The section induced drag coefficient

$$\frac{C_{T_i} C}{C_{AVG.}}$$

is then given by;

$$\frac{C_{d_{T_i}} C}{C_{AVG.}} = -\frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left(\frac{\Sigma_i}{V_\infty}\right)\left(\frac{U_i}{V_\infty}\right) - \frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left(\frac{V_{x_i}}{V_\infty}\right)\left(\frac{\Sigma_i}{V_\infty}\right) \qquad (24)$$

For the special case where the chordal surfaces of the panels are planar and parallel to each other the integral of the section induced drag

$$\frac{C_{d_{L_i}} C}{C_{AVG.}}$$

over the span can be shown to be identical with the value of induced drag as computed in the far field, provided all of the lifting elements (bound vortices) are parallel and the lateral widths of the horseshoe vortices are equal for the complete system.

The total drag of a wing as computed in the near field is given by;

$$C_{D_i} = -\frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left(\frac{w}{V_\infty}\right)_{jk} \left(\frac{\Gamma}{V_\infty}\right)_j \Delta\eta_j \tag{25}$$

where

$$\left(\frac{w}{V_\infty}\right)_{jK} = \frac{\left(\frac{\Gamma}{V_\infty}\right)_K}{4\pi} \left\{ \frac{\beta^2 Y_{jK} + X_{jK}T + (T^2 + \beta^2)\, y_\nu}{\left(X_{jK} - TY_{jK}\right)\sqrt{\left(X_{jK} + Ty_\nu\right)^2 + \beta^2\left(Y_{jK} + y_\nu\right)^2}} \right.$$

$$- \frac{\beta^2 Y_{jK} + X_{jK}T - (T^2 + \beta^2)\, y_\nu}{(X_{jK} - TY_{jK})\sqrt{\left(X_{jK} - Ty_\nu\right)^2 + \beta^2\left(Y_{jK} - y_\nu\right)^2}}$$

$$+ \frac{1}{(Y_{jK} + y_\nu)}\left[1 + \frac{X_{jK} + Ty_\nu}{\sqrt{\left(X_{jK} + Ty_\nu\right)^2 + \beta^2\left(Y_{jK} + y_\nu\right)}}\right]$$

$$\left. - \frac{1}{(Y_{jK} - y_\nu)}\left[1 + \frac{X_{jK} - Ty_\nu}{\sqrt{\left(X_{jK} - Ty_\nu\right)^2 + \beta^2\left(Y_{jK} - y_\nu\right)}}\right] \right\} \tag{26}$$

and

$$X_{jK} = X_j - X_K$$

$$Y_{jK} = (Y_j - Y_K)$$

(X_K, Y_K) is the influencing point

(X_j, Y_j) is the point being influenced

y_ν is half of the spanwise lattice spacing

T is the tangent of the vortex line sweep

β^2 is $1 - M_\infty^2$

Equation (26) can be divided into two parts; that due to the near field stagger of the lifting element and that due to the limit of integration at infinity.

Therefore;

$$\left(\frac{w}{V_\infty}\right)_{jK} = \frac{\left(\frac{\Gamma}{V_\infty}\right)_K}{4\pi}\left[E_{S_{jk}} + E_{\infty_{jK}}\right] \tag{27}$$

The contribution to equation (26) from the near field limits of integration or lifting element stagger is given by $E_{S_{jK}}$,

$$E_{S_{jK}} = \left\{ \frac{\beta^2 Y_{jK} + TX_{jK} + (T^2+\beta^2)\, y_\nu}{(X_{jK} - TY_{jK})\sqrt{\left(X_{jK} + Ty_\nu\right)^2 + \beta^2\left(Y_{jK} + y_\nu\right)^2}} - \frac{\beta^2 Y_{jK} + TX_{jK} - (T^2+\beta^2)\, y_\nu}{(X_{jK} - TY_{jK})\sqrt{\left(X_{jK} - Ty_\nu\right)^2 + \beta^2\left(Y_{jK} - y_\nu\right)^2}} \right.$$
$$\left. + \frac{X_{jK} + Ty_\nu}{(Y_{jK} + y_\nu)\sqrt{\left(X_{jK} + Ty_\nu\right)^2 + \beta^2\left(Y_{jK} + y_\nu\right)^2}} - \frac{X_{jK} - Ty_\nu}{(Y_{jK} - y_\nu)\sqrt{\left(X_{jK} - Ty_\nu\right)^2 + \beta^2\left(Y_{jK} - y_\nu\right)^2}} \right\} \tag{28}$$

That due to the limits of integration at infinity is given by $E_{\infty\, jK}$

$$E_{\infty\, jK} = \left\{ \frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu} \right\} \tag{29}$$

The contribution to the total drag C_{D_i}, given by equation (25), from $E_{S_{jK}}$ is seen to be exactly zero for all planar wings and loadings provided both Γ and y_ν are constant everywhere on the wing. This is due to the fact that there is no contribution to the drag from $E_{S_{jK}}$ when $X_{jj} = Y_{jj} = 0$ or when $X_{KK} = Y_{KK} = 0$ due to taking the Cauchy principal value. Also, when $j \neq K$ the drag from $E_{S_{jK}}$ is zero because the mutual interference drag due to the stagger is zero. This is seen by interchanging the influencing point and the point being influenced and observing that $E_{S_{jK}} = -E_{S_{Kj}}$.

Therefore;

$$C_{D_i} = \frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left[\frac{\left(\frac{\Gamma}{V_\infty}\right)_K}{4\pi} E_{\infty\, jK} \right] \left(\frac{\Gamma}{V_\infty}\right)_j \Delta\eta_j \tag{30}$$

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_K \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu} \right) \Delta\eta_j \tag{31}$$

This equation is identical to that obtained from the standard Trefftz plane analysis. From reference (47);

$$D_i = -\frac{\rho V_\infty^2}{2} \iint_{S_{WAKE}} \phi \frac{\partial \phi}{\partial N} dS \tag{32}$$

where ϕ is the velocity potential in the far field.

In the case of a planar wing the vorticity trace in the Trefftz plane can be replaced by a slit and equation (32) replaced by;

$$D_i = -\frac{\rho V_\infty^2}{2} \int_{-b/2}^{b/2} \Delta \phi(Y) \frac{\partial \phi}{\partial N}(Y)\, dY \qquad (33)$$

where

$$\Delta \phi (Y) = \frac{K}{V_\infty} (Y) \qquad (34)$$

$$\frac{\partial \phi}{\partial N}(Y) = \mp \frac{1}{2\pi} \int_{-b/2}^{b/2} \frac{\frac{d}{dY_1}\frac{K}{V_\infty}(Y_1)\, dY_1}{(Y - Y_1)} \qquad (35)$$

and $K/V_\infty(Y)$ is the total circulation at a given lateral station. Therefore,

$$\frac{K}{V_\infty} = \sum_{i=1}^{N_i} \left(\frac{\Gamma}{V_\infty}\right)_i \qquad (36)$$

and N_i is the number of vortices per chord. Also;

$$\frac{\partial \phi}{\partial Y}(Y) = \mp \frac{1}{2\pi} \sum_{n=1}^{N_N} \left(\frac{K}{V_\infty}\right)_n \left(\frac{1}{Y - Y_n + y_\nu} - \frac{1}{Y - Y_n - y_\nu}\right) \qquad (37)$$

where N_n is the number of vortices per span.

After substituting equations (34), (35), (36), and (37) into equation (33) and letting $N = N_i \times N_n$;

$$D_i = \frac{\rho V_\infty^2}{4\pi} \sum_{j=1}^{N} \sum_{K=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_K \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{1}{Y_j - Y_K + y_\nu} - \frac{1}{Y_j - Y_K - y_\nu}\right) \Delta Y_j \qquad (38)$$

Since $\Delta Y_j = b/2 \, \Delta \eta_j$

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_K \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu}\right) \Delta \eta_j \qquad (39)$$

which is the same as equation (31).

The far field calculation of induced drag for a complete configuration composed of lifting bodies and thick lifting panels is done by representing the wake, from all of the bodies and panels, by an equivalent horseshoe vortex system where the bound segment of the horseshoe vortex is tangent to the trace of the wake in the Trefftz plane and the trailing legs are in the free stream direction. The section drag associated with the equivalent system, which in general is not equal to the actual configuration section induced drag is given by;

$$\vec{d}_j = \rho \, \vec{W}_j \times \vec{\Gamma}_j \, \Delta S_j \qquad (40)$$

where

$$\vec{W}_j = V_j \hat{J} + W_j \hat{K} \qquad (41)$$

and

$$\vec{\Gamma}_j = \Gamma_j \, (T_{Y_j} \hat{J} + T_{Z_j} \hat{K}) \qquad (42)$$

where T_{Y_j} and T_{Z_j} are components of the unit vector tangent to the trace of the wake at the j^{th} section.

$$V_j = \sum_{K=1}^{N} \frac{K_k}{4\pi} \left(E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k} \right) \quad (43)$$

$$W_j = \sum_{K=1}^{N} \frac{K_k}{4\pi} \left(E_{V_{jk}} T_{Z_k} + E_{W_{jk}} N_{Y_k} \right) \quad (44)$$

where T_{Y_k} and T_{Z_k} are the components of the unit vector tangent to the trace of the wake at K^{th} station. Also, N_{Y_k} and N_{Z_k} are components of the unit vector normal to the trace of the wake at the K^{th} section. N is the total number of sections along the trace of the wake for the complete configuration.

$$E_{V_{jk}} = \frac{\overline{Z}_{jk}}{\overline{Z}_{jk}^{\,2} + (\overline{Y}_{jk} + \frac{1}{2}\Delta S_k)^2} - \frac{\overline{Z}_{jk}}{\overline{Z}_{jk}^{\,2} + (\overline{Y}_{jk} - \frac{1}{2}\Delta S_k)^2} \quad (45)$$

$$E_{W_{jk}} = \frac{\overline{Y}_{jk} - \frac{1}{2}\Delta S_k}{\overline{Z}_{jk}^{\,2} + (\overline{Y}_{jk} - \frac{1}{2}\Delta S_k)^2} - \frac{\overline{Y}_{jk} + \frac{1}{2}\Delta S_k}{\overline{Z}_{jk}^{\,2} + (\overline{Y}_{jk} + \frac{1}{2}\Delta S_k)^2} \quad (46)$$

where

$$\overline{Y}_{jk} = (Y_k - Y_j) N_{Z_k} - (Z_k - Z_j) N_{Y_k} \quad (47)$$

$$\overline{Z}_{jk} = -(Y_k - Y_j) T_{Z_k} + (Z_k - Z_j) T_{Y_k} \quad (48)$$

and the indices j and k refer to the section being influenced and the influencing section, respectively.

therefore;

$$d_j = \rho(V_i \Gamma_j T_{Z_j} - W_j \Gamma_j T_{Y_j})\, \Delta S_j \tag{49}$$

the total configuration induced drag is then given by;

$$D_i = \frac{\rho}{4\pi} \sum_{j=1}^{N} \sum_{k=1}^{N} \Gamma_j \Gamma_k \Big((E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k})\, T_{Z_j} - (E_{V_{jk}} T_{Z_k} = E_{W_{jk}} N_{Z_k})\, T_{Y_j} \Big) \Delta S_j \tag{50}$$

therefore;

$$C_{D_i} = \frac{AR}{2\pi b^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{\Gamma}{V_\infty}\right)_k \Big((E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k})\, T_{Z_j} - (E_{V_{jk}} T_{Z_k} + E_{W_{jk}} N_{Z_k})\, T_{Y_j} \Big) \Delta S_j \tag{51}$$

NOTE; For the planar wing equation (51) reduces to

$$C_{D_i} = \frac{AR}{2\pi b^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{\Gamma}{V_\infty}\right)_k E_{W_{jk}}\, \Delta S_j \tag{52}$$

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{k=1}^{N} \left(\frac{\Gamma}{V_\infty}\right)_k \left(\frac{\Gamma}{V_\infty}\right)_j \left(\frac{1}{Y_{jk} + y_\nu} - \frac{1}{Y_{jk} - y_\nu} \right) \Delta\eta_j \tag{53}$$

which is the same as equations (31) and (30). This demonstrates that the same induced drag is obtained whether the equality of work and kinetic-energy increment, the equivalent far field horseshoe system, or the near field horseshoe vortices with the Kutta-Joukowsky theorem is used.

APPENDIX G

COMPUTER PROGRAM LISTING

```
      PRO7RAM DERIV(INPUT=201, OUTPUT     , TAPE5=INPUT, TAPE6=OUTPUT,
     1  TAPE18     , TAPE19     , TAPE20=1002, TAPE21=1002, TAPE23
     2  TAPE24     , TAPE10=1002, TAPE11     , TAPE12     )
      COMMON DA(5100)
      COMMON/PANATT/DUM1(   230)
      COMMON/NUMBER/ DUM2(135)
      COMMON/PANEL /DUM3(    34)
      COMMON/BODY/DUM4(31550)
      COMMON/CONPTS/ DUM5(7920)
      COMMON/SCRAT/ DUM7(25000)
      COMMON/INDEX /DUM10(7)
      COMMON /PANINF/ PANSYM(10), DUM11(600), PANREF(10), PCHORD(10)
      COMMON/CONTRV/ DUM13(240)
      COMMON/SLOPE/DUM14(2000)
      COMMON     /COMPRS/   BETAM
      CALL FTNBIN(1,0,0)
      DO 1 I=1,5100
    1 DA(I)=0.0
      OVL=3HOVL
      CALL  ATTACH(DUM1(1),DUM1(2),DUM1(3),DUM1(4),123)
      CALL OVERLAY(OVL,1,0)
      CALL OVERLAY(OVL,2,0)
      CALL OVERLAY(OVL,3,0)
      CALL OVERLAY(OVL,4,0)
      CALL OVERLAY(OVL,5,0)
      CALL OVERLAY(OVL,6,0)
      CALL OVERLAY(OVL,7,0)
      END
```

```
      SUBROUTINE ATTACH(XV,YV,ZV,NR,KODE)
C   FOR KODE = -1, THIS SUBROUTINE IS SETTING XATT FROM BODY INPUTS.
C   FOR KODE =  0, THIS SUBROUTINE IS SETTING XATT FROM PANEL INPUTS.
C   FOR KODE = 1, THIS SUBROUTINE SETS XV,YV,ZV,FROM XATT ARRAY.
      DIMENSION XV(NR,1), YV(NR,1), ZV(NR,1)
      COMMON DA(5000)
     1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2   ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/PANATT/ NATT(30),XATT(200)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      EQUIVALENCE (DA(2),PANS)
      DATA L/0/,L1/0/,L2/0/
      IF(KODE.EQ.123) RETURN
      NPANS=PANS
      NB1=NBVV+1
      IF(KODE) 1,2,200
1     NSA=NB
      GO TO 3
2     NSA=NPAN+NBODS
3     CONTINUE
      DO 100 I=1,NPANS
      I1=(I-1)*3+2
      IF(NATT(I1).EQ.NSA) GO TO 5
      GO TO 100
5     I2=NATT(I1+1)
      GO TO 70
10    L=L+1
      XATT(L)=-NATT(I1)
      XATT(L+1)= I2
      L=L+2
      XATT(L)=NB1
      NA=(I2-1)*LTDIV+1
```

```
      IF(KODE.EQ.0) GO TO 50
      DO 20 K=1,NB1
      L=L+1
      XATT(L)=XV(K,NA)
      XATT(L+1)=YV(K,NA)
      L=L+2
   20 XATT(L)=ZV(K,NA)
      GO TO 65
   50 NA=NATT(I1)
      DO 60 K=1,NB1
      L=L+1
      XATT(L)=XV(NA,K)
      XATT(L+1)=YV(NA,K)
      L=L+2
   60 XATT(L)=ZV(NA,K)
   65 L1= NATT(I1)
      L2= I2
      GO TO 100
   70 IF(L1.EQ.NATT(I1).AND.L2.EQ.I2) GO TO 100
      GO TO 10
  100 CONTINUE
      WRITE(6,101)(XATT(I),I=1,L)
  101 FORMAT(27H0IN SUB. ATTACH, XATT ARRAY/(1P8E13.4))
      RETURN
C    HERE, SUB. ATTACH FINDS WHICH SET OF X,Y,Z ARE FOR THE ATTACH LINE.
  200 AN1=NATTCH
      AN2=NTRATT
      DO 300 I=1,200
      IF(XATT(I).EQ.-AN1.AND.XATT(I+1).EQ.AN2) GO TO 201
      GO TO 300
  201 J=I+2
      NB1=XATT(J)+0.01
      DO 205 K=1,NB1
      J=J+1
      XV(K,1)=XATT(J)
      YV(K,1)=XATT(J+1)
      J=J+2
  205 ZV(K,1)=XATT(J)
      MP3=NB1
```

```
      WRITE(6,206)(XV(K,1),YV(K,1),ZV(K,1),K=1,NP1)
  206 FORMAT(43H0IN SUB. ATTACH, XT,YT,ZT, FOR ATTACH. LINE/(1P3E13.4))
      RETURN
  300 CONTINUE
      WRITE(6,305) NPAN
  305 FORMAT(35H0NO ATTACHMENT LINE FOUND FOR PANEL,I2,13HIN SUB ATTACH)
      RETURN
      END
```

```
      SUBROUTINE DECRD(DATA,IUNIT)
      DIMENSION DATA(1),ADATA(5),IDATA(17),IIDATA(8)
      DATA IBLANK/10H          /
   15 READ(IUNIT,16)IIDATA
   16 FORMAT(8A10)
C     IF (EOF(IUNIT).EQ. 0) GOTO 19
      IF(EOF, IUNIT) 199, 19
  199 CONTINUE
      IUNIT = -IUNIT
      RETURN
   19 DECODE(72,17,IIDATA)IADD,ADATA
   17 FORMAT(I12,5G12.0)
      DECODE(80,18,IIDATA)IDATA
   18 FORMAT(12X,17A4)
      J=IADD
      IF(IADD) 22,40,24
   22 J=-J
   24 DO 30 I=1,5
      L=3*I
      K=L-2
      DO 26 M=K,L
      IF(IDATA(M)-IBLANK)28,26,28
   26 CONTINUE
      GO TO 30
   28 DATA(J)=ADATA(I)
   30 J=J+1
      IF(IADD)100,40,15
   40 WRITE(6,50)IADD,IDATA
   50 FORMAT(17H0DECRD ER. CARD=(,1I12,17A4,2H).)
      CALL EXIT
  100 RETURN
      END
```

```
      SUBROUTINE CODIM(XI,YI,NI,T,ANS,NA)
C
C****            A CONTROLLED DEVIATION ITERPOLATION METHOD
C
      DIMENSION   XI(1)      ,YI(1)      ,T(1)       ,ANS(1)
C
C
      XK=1.0
      N=NI
      DO 910 IE=1,NA
      X=T(IE)
  100 IF(N-2)110,120,200
  110 Y = YI(N)
      GO TO 900
  120 Y = (YI(2)-YI(1))/(XI(2)-XI(1))* (X-XI(1)) +YI(1)
      GO TO 900
  200 J = 1
  210 IF(XI(J)-X)230,220,250
  220 Y =YI(J)
      GO TO 900
  230 J = J+1
      IF(J-N)210,210,250
  250 IF(J-2)120,155,260
  155 J = 3
      JJ = 1
      GO TO 285
  260 IF(J-N)280,265,270
  265 J = N-1
      JJ = 2
      GO TO 285
  270 Y= (YI(N)-YI(N-1))/(XI(N)-XI(N-1))* (X-XI(N-1))+YI(N-1)
      GO TO 900
  280 JJ = 3
  285 IF(N-3)290,290,295
  290 J = 3
  295 K = J-1
      M = K-1
      L = J+1
      A1 = X-XI(M)
```

```
      A2 = X-XI(K)
      A3 = X-XI(J)
      AL = (X-XI(K))/(XI(J)-XI(K))
      S = AL*YI(J)+(1.0-AL)*YI(K)
      C1= A3*A2/((XI(M)-XI(K))*(XI(M)-XI(J)))
      C2= A1*A3/((XI(K)-XI(M))*(XI(K)-XI(J)))
      C3= A2*A1/((XI(J)-XI(M))*(XI(J)-XI(K)))
      P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)
      IF(N-3)305,305,310
  305 P2 = P1
      GO TO 315
  310 A4 = X-XI(L)
      C4= A4*A3/((XI(K)-XI(J))*(XI(K)-XI(L)))
      C5= A2*A4/((XI(J)-XI(K))*(XI(J)-XI(L)))
      C6= A3*A2/((XI(L)-XI(K))*(XI(L)-XI(J)))
      P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)
  315 GO TO (320,330,350),JJ
  320 P2 = P1
      AL = (X-XI(1))/(XI(2)-XI(1))
      S =  AL*YI(2)+ (1.0-AL)*YI(1)
      P1=  S + XK*(P2-S)
      GO TO 350
  330 P1 = P2
      AL = (X-XI(N-1))/(XI(N)-XI(N-1))
      S  = AL*YI(N) +(1.0-AL)*YI(N-1)
      P2 = S+ XK*(P1-S)
  350 E1 =  ABS(P1-S)
      E2 =  ABS(P2-S)
      IF(E1+E2)400,400,410
  400 Y = S
      GO TO 900
  410 BT =  (E1*AL)/(E1*AL+(1.0-AL)*E2)
      Y  = BT*P2+(1.0-BT)*P1
  900 ANS(IE)=Y
  910 CONTINUE
      RETURN
      END
```

```
FUN3TION COSD(X)
Y=0.017453293*X
COSD=COS(Y)
RETURN
END
```

```
      SUBROUTINE DATAWR(DA)
      DIMENSION DA(1)
      DO 200 I=1,1000
      ID=(I-1)*5
      LDATA=0
      DO 100 J=1,5
      IJ=ID+J
      IF(DA(IJ).NE.0.0) LDATA=1
  100 CONTINUE
      IF(LDATA.EQ.0) GO TO 200
      ID1=ID+1
      WRITE(6,150) ID1,IJ,(DA(K),K=ID1,IJ)
  150 FORMAT(5H0DA(  ,I4,13H )  THRU DA(  ,I4,4H ) =1P5E15.6)
  200 CONTINUE
      WRITE(6,250)
  250 FORMAT(52H0ALL VALUES OF DA NOT GIVEN ABOVE ARE EQUAL TO ZERO.)
      RETURN
      END


      SUBROUTINE NORM(X,Y,Z)
      D=SQRT(X**2 + Y**2 + Z**2)
      X=X/D
      Y=Y/D
      Z=Z/D
      RETURN
      END
```

```
      FUNCTION DOT(X1,Y1,Z1,X2,Y2,Z2)                                 0  2690
      DOT=X1*X2+Y1*Y2+Z1*Z2                                           0  2700
      RETURN                                                          0  2710
      END                                                             0  2720

      FUNCTION ARCOS(X)                                               0  2730
      ARCOS=ACOS(X)                                                   0  2740
      RETURN                                                          0  2750
      END                                                             0  2760

      FUNCTION COTAN(X)                                               0  2770
      COTAN=COS(X)/SIN(X)                                             0  2780
      RETURN                                                          0  2790
      END                                                             0  2800
```

```
      FUN3TION   CODIM1 (X,XI,YI,N,XK)
C
C     CALLING SEQUENCE......
C          X     INDEPENDENT VARIABLE....ABSCISSA REQUESTED
C          XI    ARRAY OF GIVEN ABSCISSAS
C          YI    ARRAY OF GIVEN ORDINATES
C          N     NUMBER OF GIVEN POINTS DESCRIBING THE CURVE
C          XK    END INTERVAL INTERPOLATION CONTROL CONSTANT
C                    XK=0           STRAIGHT LINE INTERPOLATION
C                    XK=+1          FULL PARABOLIC INTERPOLATION
C                    XK BETWEEN     FUNCTION THAT LIES BETWEEN A STRAIGHT
C                       0 AND 1     LINE AND A PARABOLIC INTERPOLATION
C                    XK=-1          PROGRAM WILL COMPUTE END INTERPOLATION
C                                   CONTROL CONSTANT
C
C
C
      DIMENSION  XI(1),  YI(1)
C
      W = X
      N1 = N
C
C     DETERMINE THE NUMBER OF POINTS GIVEN
      IF (N1-2)100,200,300
C
C     ONE POINT GIVEN
  100 IF (XI(1)-W)130,175,130
  130 WRITE          ( 6,150)
  150 FORMAT (99H- ONLY ONE POINT WAS GIVEN FOR ARRAY XI IN CODIM1....TH
     1E ABSCISSA ARGUMENT IS NOT THE SAME AS THE  /16H0 GIVEN ABSCISSA)
  165 CALL DUMP
C
  175 CODIM1 = YI(1)
      GO TO 1700
C
C     TWO POINT STRAIGHT LINE COMPUTATION
  200 N1 = 2
C     TEST IF ABSCISSAS ARE IDENTICAL
  225 IF (XI(N1-1)-XI(N1))250,275,250
```

```
C     STRAIGHT LINE COMPUTATION
  250 CODIM1 = (YI(N1)-YI(N1-1))/(XI(N1)-XI(N1-1))*(W-XI(N1-1))+YI(N1-1)
      GO TO 1700
C     ERROR....ABSCISSAS ARE IDENTICAL
  275 WRITE              ( 6,285)XI(N1)
  285 FORMAT ( 90H- A STRAIGHT LINE COMPUTATION IN CODIM1 WAS ATTEMPTED
     1BUT THE TWO ABSCISSAS ARE IDENTICAL /22H0 THE ABSCISSA VALUE =
     2E17.8)
      GO TO 165
C
C     THERE ARE MORE THAN TWO POINTS...TEST IF THEY ARE INCREASING OR
C     DECREASING ALGEBRAICALLY
  300 IF (XI(1)-XI(2))400,325,6
C
C     XI(1) EQUALS XI(2)....ERROR
  325 WRITE              ( 6,350)XI(1)
  350 FORMAT ( 88H- THE FIRST TWO ABSCISSAS IN ARRAY XI IN CODIM1 ARE TH
     1E SAME...THIS IS AN ERROR.  ABSC =E17.8)
      GO TO 165
C
C     ABSCISSA VALUES ARE INCREASING IN VALUE ALGEBRAICALLY
C     FIND NEXT LARGEST ABSCISSA AFTER W
  400 DO 450  J=1,N1
      IF (W-XI(J))800,475,450
  450 CONTINUE
C     W IS GREATER THAN XI(N)
      GO TO 225
C
C     W IS EQUAL TO XI(J)
C     CHECK IF XI(J) IS LAST GIVEN ABSCISSA
  475 IF (J-N1)500,575,575
C     CHECK IF XI(J) = XI(J+1), IF SO THE FUNCTION IS NOT SINGLE VALUED
C     AND THE ORDINATE OF XI(J) IS RETURNED AND A STATEMENT IS PRINTED
  500 IF (XI(J)-XI(J+1))575,525,575
  525 WRITE              ( 6,550)XI(J)
  550 FORMAT ( 43H-  THERE IS MORE THAN ONE ABSCISSA EQUAL TOE17.8/77H0
     1THE ORDINATE VALUE OF THE FIRST SUCH ABSCISSA HAS BEEN RETURNED.
     2 CODIM1.     )
  575 CODIM1 = YI(J)
```

```
      GO TO 1700
C
C     ABSCISSAS ARE DECREASING IN VALUE ALGEBRAICALLY
C     FIND NEXT SMALLEST ABSCISSA AFTER W
  600 DO 650  J=1,N1
      IF (XI(J)-W)800,475,650
  650 CONTINUE
C     W IS LESS THAN XI(N)
      GO TO 225
C
C     TEST IF W LIES BETWEEN XI(1) AND XI(2)
  800 IF (J-2)200,825,850
C
C     W LIES BETWEEN XI(1) AND XI(2)
  825 J = 3
      JJ = 1
      GO TO 925
C
C     W OCCURS AFTER XI(2).....TEST IF W IS BETWEEN XI(N-1) AND XI(N)
  850 IF (J-N1)900,875,225
C
C     W LIES BETWEEN XI(N-1) AND XI(N)
  875 J = N1-1
      JJ = 2
      GO TO 925
C
C     W LIES BETWEEN XI(2) AND XI(N-1)
  900 JJ = 3
C
C     SETUP SUBSCRIPTS
  925 K = J-1
      M = K-1
      L = J+1
      XIM = XI(M)
      XIK = XI(K)
      XIJ = XI(J)
      XIL = XI(L)
C
C     TEST IF N=3....IF SO ALTER M AND JT SO THAT DO LOOPS 1040 AND
```

```
C       1120 TEST ONLY 3 POINTS VICE 4
        IF (NI-3)970,940,970
  940 IF (JJ-2)950,960,970
  950 JT = 2
        GO TO 1000
  960 M = 1
  970 JT = J
C
C       TEST IF ABSCISSA VALUES ARE INCREASING OR DECREASING ALGEBRAICALLY
 1000 IF (XI(1)-XI(2))1020, 325,1100
C
C       TEST IF ABSCISSA VALUES ARE ALL INCREASING ALGEBRAICALLY
 1020 DO 1040  IB=M,JT
        IF (XI(IB)-XI(IB+1))1040,1060,1060
 1040 CONTINUE
        GO TO 1175
 1060 IB1 = IB+1
        WRITE           ( 6,1070)IB,XI(IB),IB1,XI(IB1)
 1070 FORMAT ( 79H- ALL ABSCISAS IN CODIM1 ARE NOT EITHER ALL INCREASING
     1/DECREASING ALGEBRAICALLY /10H0 ABSCISSAI7,2H =E17.8,10X, 8HABSCIS
     2SAI7,2H =E17.8)
        GO TO 165
C
C       TEST IF ABSCISSA VALUES ARE ALL DECREASING ALGEBRAICALLY
 1100 DO 1120 IB=M,JT
        IF (XI(IB)-XI(IB+1))1060,1060,1120
 1120 CONTINUE
C
 1175 A1 = W-XIM
        A2 = W-XIK
        A3 = W-XIJ
        A4 = W-XIL
        AL = (W-XIK)/(XIJ-XIK)
        C1 = A3*A2/((XIM-XIK)*(XIM-XIJ))
        C2 = A1*A3/((XIK-XIM)*(XIK-XIJ))
        C3 = A2*A1/((XIJ-XIM)*(XIJ-XIK))
        C4 = A4*A3/((XIK-XIJ)*(XIK-XIL))
        C5 = A2*A4/((XIJ-XIK)*(XIJ-XIL))
        C6 = A3*A2/((XIL-XIK)*(XIL-XIJ))
```

```
      S = AL*YI(J)+(1.-AL)*YI(K)
      P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)
      P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)
      GO TO (1200,1400,1500),JJ
C
C     W LIES BETWEEN XI(1) AND XI(2)
 1200 P2 = P1
      IF (XK)1230,1220,1220
 1220 XE = XK
      GO TO 1260
C     COMPUTE XK
 1230 SLOPE1=   ABS((YI(K)-YI(M))/(XIK-XIM))
      SLOPE2 =   ABS((YI(K)-YI(J))/(XIK-XIJ))
      XE = 1.-( ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))
C
 1260 AL = (W-XI(1))/(XI(2)-XI(1))
      S = AL*YI(2)+(1.0-AL)*YI(1)
      P1 = S+XE*(P2-S)
      GO TO 1500
C
C     W LIES BETWEEN XI(N-1) AND XI(N)
 1400 P1 = P2
      IF (XK)1430,1420,1420
 1420 XE = XK
      GO TO 1460
 1430 SLOPE1 =   ABS((YI(J)-YI(L))/(XIJ-XIL))
      SLOPE2 =   ABS((YI(J)-YI(K))/(XIJ-XIK))
      XE = 1.-( ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))
C
 1460 AL = (W-XIJ)/(XIL-XIJ)
      S = AL*YI(L)+(1.-AL)*YI(J)
      P2 = S+XE*(P1-S)
C
 1500 E1 =   ABS(P1-S)
      E2 =   ABS(P2-S)
      IF (E1+E2)1530,1530,1560
 1530 CODIM1 = S
      GO TO 1700
C
```

```
1560 BT = (E1*AL)/(E1*AL+(1.-AL)*E2)
     CODIM1 = BT*P2+(1.-BT)*P1
C
1700 RETURN
     END
```

```
      PROGRAM BODYGM
      COMMON DA(5000)
     1   ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2    ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1    ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2    ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3    ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/ XV1(151,31),YV(151,31),ZV(151,31)
      EQUIVALENCE(DA(1),BODIES),(DA(15),BODYNO)
      EQUIVALENCE(DA(10), ALPHA),    (DA(11),BETA)
      COMMON      /COMPRS/   BETAM
      EQUIVALENCE(DA(3),FMACH)
C***********************************
C***********************************
      COMMON/PANATT/NATT(30),XATT(200)
      COMMON/SCRAT/ ATACH(50)
      CALL NARDAP
      REWIND 21
      READ(21) ATACH
      DO 5 I=1,10
      K=(I-1)*5
      J=(I-1)*3
      NATT(J+1)=ATACH(K+1)
      NATT(J+2)=ATACH(K+2)
    5 NATT(J+3)=ATACH(K+3)
      WRITE(6,205) NATT
  205 FORMAT(*ONATT*/(3I5))
C***********************************
C***********************************
      DO 1 I=1,14
    1 DA(I)=0.0
      MTAPE=19
      NTAPE=20
C   UNIT 18 - VORTEX COORDINATES
      REWIND 18
C   UNITS 19,20 - AX,AY,AZ MATRICES  (COMPUTED IN SUBROUTINE INFL)
      REWIND 19
      REWIND 20
```

```
C    UNIT  21 - COMPLETE INFLUENCE MATRIX *A* (COMPUTED IN SUB. MATA)
        REWIND 21
C    UNIT 23 - PHI,THETA MATRIX.
        REWIND 23
C    REWIND UNIT 12.   UNIT 12 WILL HAVE DATA USED FOR FORCES.
        REWIND 12
        CALL DECRD(DA)
        NBODS=BODIES
        ALPHA=ALPHA/57.29577951
        BETA=BETA/57.29577951
        BETAM = SQRT(1.0-FMACH**2)
        IF(NBODS.EQ.0) GO TO 105
C   NCT IS COUNTER FOR CONTROL POINTS.  NCT IS INCREMENTED IN SUB. VPTS.
        NCT=0
        NCTV=0
        KCON=0
        KL=0
        KLT=0
        DO 100 I=1,NBODS
        DO 10 J=15,3419
 10     DA(J)=0.0
        CALL DECRD(DA)
        NB=BODYNO
        CALL SETDAT
        CALL XYZVB
        CALL MATL(KL)
        CALL MATLT(KLT)
        CALL VPTS
        CALL ATTACH(XV1,YV,ZV,151,-1)
        NSEG=TSEG(I)+LSEG(I)
        KCON=KCON+NSEG
        IF(NSEG.EQ.0) GO TO 100
        CALL MATPT
 100    CONTINUE
C
C    VORTEX POINTS (INCLUDING DIVISION PTS.) ARE ON UNIT 18. (ALL BODIES)
C    UNIT 23 HAS PHI,THETA MATRIX FOR ALL BODIES.
C    ALNGTH HAS L FOR ALL BODIES
C
```

```
105   CONTINUE                                              1   0790
      END                                                   1   0800
```

```
      SUBROUTINE MATPT
C     COMPUTE PHI,THETA CONSTRAINT TRANSFORMATION MATRIX.
C
      COMMON DA(5000)
     1   ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2    ,LNDIV,LTDIV,LNPTS,LTPTS
      EQUIVALENCE (DA(17),CHORD)
     1   ,(DA(3140),CSLN),(DA(3190),CSLT),(DA(3210),CFLN),(DA(3250),CFLT)
     2   ,(DA(41),XS),(DA(34),FUNCLN),(DA(35),FUNCLT),(DA(36),SEGLN)
     3   ,(DA(37),SEGLT),(DA(27),XBO)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1    ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2    ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3    ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/SCRAT/ XVV(200),THVV(151,31)
     1    ,XVOR(100)
      DIMENSION CFLN(40),CFLT(40),CSLN(50),CSLT(20),ANORM(3),XS(49)
      DIMENSION PHTH(5000)
      COMMON/BODY/ ARRAY(20000)
      EQUIVALENCE(ARRAY(15001),PHTH)
C    DIVLDA IS NO. LONG. DIV. GIVEN IN DATA.
      EQUIVALENCE (DIVLDA,DA(32))
      LNFUNC=FUNCLN
      LTFUNC=FUNCLT
      LNSEG=SEGLN
      LTSEG=SEGLT
      PI=3.1415926
C   DETERMINE IF Y,Z AND NORMALS ARE REQUIRED FOR CONSTRAINT FUNCTIONS.
C   SET KODE=1 IF THEY ARE REQUIRED.
      KODE=0
      DO 1 I=1,2
      IF(CFLN(I).NE.2.0) GO TO 1
      KODE=1
      GO TO 3
    1 CONTINUE
      DO 2 I=1,3
      IF(CFLT(I).NE.2.0.AND.CFLT(I).NE.3.0) GO TO 2
      KODE=1
      GO TO 3
```

```
2     CONTINUE
3     CONTINUE
      IF(LTFUNC) 10,10,11
10    JTLIM=NTVV
      GO TO 12
11    JTLIM=LTFUNC
12    IF(LNFUNC)13,13,14
13    JNLIM=NBVV
      GO TO 15
14    JNLIM=LNFUNC
15    CONTINUE
      KFUNC=0
      IF(LNFUNC.NE.0) GO TO 25
      IF(LTFUNC.EQ.0) GO TO 25
      KFUNC=1
      JTS=JTLIM
      JTLIM=JNLIM
      JNLIM=JTS
25    CONTINUE
      ITAPE=0
      DO 5000 JTT=1,JTLIM
      JT=JTT
      IF(KFUNC.NE.1) GO TO 26
      JN=JTT
26    DO 5000 JNN=1,JNLIM
      ITAPE=ITAPE+1
      IF(KFUNC.EQ.0) GO TO 261
      JT=JNN
      GO TO 27
261   JN=JNN
27    IF(LTFUNC.EQ.0) GO TO 28
      LTCF=CFLT(JT)
28    IF(LNFUNC.EQ.0) GO TO 29
      LNCF=CFLN (JN)
29    CONTINUE
      KC=0
      DO 1000 J=1,NTVV
      DO 1000 I=1,NBVV
      KC=KC+1
```

```
      IF(LNCF.EQ.1) GO TO 75
      IF(LNCF.EQ.0) GO TO 75
      DO 50 II=1,LNSEG
      LNCS=CSLN(II)
      IF(I.LE.LNCS) GO TO 55
50    CONTINUE
55    IF(II.NE.1) GO TO 60
      X0=XS(1)-XB0
58    PHI0=ARCOS(1.0-2.0*X0/CHORD)
      GO TO 65
60    II1=CSLN(II-1)
      IX1=II1
      IX2=IX1+1
      X0=0.75*XVOR(IX2)-0.25*XVOR(IX1)
      X0=X0-XB0
      GO TO 58
65    II1=CSLN(II)
      IF(II1.LT.NBVV) GO TO 66
      XF=CHORD
      GO TO 68
66    IX1=II1
      IX2=IX1+1
      XF=0.75*XVOR(IX2)-0.25*XVOR(IX1)
      XF=XF-XB0
      IF(XF.GT.CHORD) XF=CHORD
68    PHIF=ARCOS(1.0-2.0*XF/CHORD)
      X=XVOR(I)
      X=X-XB0
      PHI =ARCOS(1.0-2.0*X /CHORD)
      PRAT=PI*(PHI-PHI0)/(PHIF-PHI0)
75    IF(LTCF.EQ.1) GO TO 85
      IF(LTCF.EQ.0) GO TO 85
      DO 80 JJ=1,LTSEG
      LTCS=CSLT(JJ)
      IF(J.LE.LTCS) GO TO 81
80    CONTINUE
81    JX1=(J-1)*LTDIV+1
      JX2=J*LTDIV+1
      I1=(I-1)*DIVLDA+1
```

```
      THETA=0.5*(THVV(I1,JX1)+THVV(I1,JX2))
      IF(JJ.NE.1) GO TO 82
      THETO=THVV(I1,1)
      GO TO 83
   82 JX1=(CSLT(JJ-1))*LTDIV+1.01
      THETO=THVV(I1,JX1)
   83 JX2=LTCS*LTDIV+1
      THETF=THVV(I1,JX2)
      TRAT=PI*(THETA-THETO)/(THETF-THETO)
   85 CONTINUE
      IF(KODE.EQ.0) GO TO 100
      CALL VYZN(J,I,Y,Z,ANORM)
  100 CONTINUE
      LATF=1
      LONF=1
C  IF LATF=0, DO NOT COMPUTE GT.
C  IF LONF=0, DO NOT COMPUTE GN.
      IF(LTFUNC.NE.0) GO TO 105
      LATF=0
      IF(JT.NE.J) GO TO 30
      GT=1.0
      GO TO 105
   30 GT=0.0
      LONF=0
  105 CONTINUE
      IF(LNFUNC.NE.0) GO TO 106
      LONF=0
      IF(JN.NE.I) GO TO 251
      GN=1.0
      GO TO 106
  251 GN=0.0
      LATF=0
  106 CONTINUE
      IF(LATF.EQ.0) GO TO 300
      GO TO(110,120,130,140,150,160,170,180,190,200),LTCF
  110 GT=1.0
      GO TO 300
  120 GT=ANORM(3)/SQRT(ANORM(3)**2+ANORM(2)**2)
      GO TO 300
```

```
130   GT=ANORM(2)/SQRT(ANORM(3)**2+ANORM(2)**2)
      GO TO 300
140   GT=SIN(TRAT)
      GO TO 300
150   GT=COS(TRAT)
      GO TO 300
160   GT=TRAT
      GO TO 300
170   GT=SIN(2.0*TRAT)
      GO TO 300
180   GT=COS(2.0*TRAT)
      GO TO 300
190   GT=TRAT**2
      GO TO 300
200   GT=0.0
300   CONTINUE
      IF(LONF.EQ.0) GO TO 500
      GO TO (310,320,330,340,350,360,370,380,390,400
     1 ,403,404,405,406,407,408,409,410,411,412,413,414,415),LNCF
310   GN=1.0
      GO TO 500
320   T1=Y*ANORM(1)/ANORM(2)+Z*ANORM(1)/ANORM(3)
      GN=1.0/SQRT(1.0+(T1**2)/(Y**2+Z**2))
      GO TO 500
330   GN=COTAN(0.5*PHI)
      GO TO 500
340   GN=COTAN(0.5*(PI-PHI))
      GO TO 500
350   GN=SIN(PRAT)
      GO TO 500
360   GN=COS(PRAT)
      GO TO 500
370   GN=(X-X0)/(XF-X0)
      GO TO 500
380   GN=SIN(2.0*PRAT)
      GO TO 500
390   GN=COS(2.0*PRAT)
      GO TO 500
400   GN=((X-X0)/(XF-X0))**2
```

```
      GO TO 500
403   GN=SIN(3.0*PRAT)
      GO TO 500
404   GN=SIN(4.0*PRAT)
      GO TO 500
405   GN=SIN(5.0*PRAT)
      GO TO 500
406   GN=SIN(6.0*PRAT)
      GO TO 500
407   GN=SIN(7.0*PRAT)
      GO TO 500
408   GN=SIN(8.0*PRAT)
      GO TO 500
409   GN=SIN(9.0*PRAT)
      GO TO 500
410   GN=SIN(10.0*PRAT)
      GO TO 500
411   GN=SIN(11.0*PRAT)
      GO TO 500
412   GN=SIN(12.0*PRAT)
      GO TO 500
413   GN=SIN(13.0*PRAT)
      GO TO 500
414   GN=SIN(14.0*PRAT)
      GO TO 500
415   GN=SIN(15.0*PRAT)
500   CONTINUE
1000  PHTH(KC)=GN*GT
5000  WRITE(23)(PHTH(K1),K1=1,KC)
      RETURN
      END
```

```
      SUBROUTINE VPTS
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2 ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      DIMENSION XAREA(5000),YAREA(5000),ZAREA(5000)
     1     ,  XPC(5000),  YPC(5000),  ZPC(5000),   XTL(100)
      EQUIVALENCE(XAREA,XPC,D(15001)),(YAREA,YPC,D(20001))
     1          ,(ZAREA,ZPC,D(25001)),(XTL,D(14100))
     *          ,(CHORD,DA(17)),        (BREF,DA(16))
     2      ,(DA(7),XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
      COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)
     2 ,TMX(1320),  TMY(1320),   TMZ(1320)
     3 ,TTX(1320),  TTY(1320),   TTZ(1320)
     4 ,DUMB(5000)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1 ,XN(1320),    YN(1320),    ZN(1320)
      DIMENSION D(31000)
      EQUIVALENCE (D,XV(1,1))
      EQUIVALENCE (DA(2800),FLNC),(DA(3100),FLTC)
      DIMENSION FLNC(150),FLTC(40)
      COMMON/SCRAT/ XVV(200),DUMTHV(4681),XVOR(100),XCON(100)
      DIMENSION T11(300),T21(300),T31(300)
     1        ,T12(300),T22(300),T32(300)
      EQUIVALENCE(XN(1),T11),(XN(301),T21),(XN(601),T31)
     1          ,(YN(1),T12),(YN(301),T22),(YN(601),T32)
      EQUIVALENCE(BODYWR(1),XV(1,1))
      DIMENSION BODYWR(31000)
C    31000 WORDS OF /BODY/ WILL BE WRITTEN ON UNIT 18.
C
      IF(FLNC(1).NE.0.0) GO TO 2
      DO 1 I=1,LNPTS
    1 FLNC(I)=I
    2 IF(FLTC(1).NE.0.0) GO TO 4
      DO 3 J=1,LTPTS
    3 FLTC(J)=J
```

```
4     CONTINUE
      NB1=NBVV+1
      SUBAX=0.0
      SUBAY=0.0
      SUBAZ=0.0
      K=0
      DO 50 J=1,NTV
      J1=J+1
      DO 50 I=1,NBV
      I1=I+1
      K=K+1
      X31=XV (I1,J1)-XV (I,J)
      Y31=YV(I1,J1)-YV(I,J)
      Z31=ZV(I1,J1)-ZV(I,J)
      X24=XV (I,J1)-XV (I1,J)
      Y24=YV(I,J1) -YV(I1,J)
      Z24=ZV(I,J1) -ZV(I1,J)
      XAREA(K) = (Y31*Z24-Z31*Y24)*0.5
      YAREA(K) = (Z31*X24-X31*Z24)*0.5
      ZAREA(K) = (X31*Y24-Y31*X24)*0.5
      SUBAX=SUBAX+XAREA(K)
      SUBAY=SUBAY+YAREA(K)
50    SUBAZ=SUBAZ+ZAREA(K)
      WRITE(6,55)  SUBAX,SUBAY,SUBAZ
55    FORMAT(1H0/(1P8E15.6))
      DO 100 I=1,NBVV
      K1=(I-1)*LNDIV+1
      K2=K1+LNDIV
100   XVOR(I)=0.75*XVV(K1)+0.25*XVV(K2)
      WRITE(6,55)(XVOR(I),I=1,NBVV)
      N3=NBV+1
      N2=NTV+1
      DO 300 I=1,LNPTS
      KC=FLNC(I)
      KC=(KC-1)*LNDIV+1
300   XCON(I)= 0.25*XVV(KC)+0.75*XVV(KC+LNDIV)
C
C   WRITE AREAS ON UNIT 12 FOR USE IN CALCULATING FORCES.
C
```

```
      WRITE(12) XAREA,YAREA,ZAREA
      K=0
      DO 60 J=1,NTV
      J1=J+1
      DO 60 I=1,NBV
      I1=I+1
      K=K+1
      XPC(K)=0.25*(XV(I,J) + XV(I1,J) + XV(I,J1) + XV(I1,J1))
      YPC(K)=0.25*(YV(I,J) + YV(I1,J) + YV(I,J1) + YV(I1,J1))
   60 ZPC(K)=0.25*(ZV(I,J) + ZV(I1,J) + ZV(I,J1) + ZV(I1,J1))
      DO 65 K=1,NB1
      K1=(K-1)*LNDIV+1
   65 XTL(K)=XVV(K1)
C
C    WRITE SUB-PANEL CENTERS AND X FOR PANEL EDGES ON UNIT 12 FOR FORCES.
C    ALSO WRITE XCON=X COORDS FOR CONTROL POINTS AND OTHER NEEDED TERMS.
      WRITE(12) XPC,YPC,ZPC,XTL,XCON,FLNC,FLTC,NBV,NTV,LNPTS,LTPTS
     1          ,LNDIV,LTDIV,NBVV,NTVV,CHORD,XCG,YCG,ZCG,ALPHA,BREF
      WRITE(6,55)(XCON(I),I=1,LNPTS)
      NLAT1=LTDIV/2 + 1
      JS=NCT
      DO 500 J=1,LTPTS
      K1=LTDIV*(FLTC(J)-1.0)+NLAT1
      K2=K1+1
      DO 400 I1=1,NBV
      I2=I1+1
      XQ(I1)=0.25*(XV(I2,K1)+XV(I1,K1)+XV(I2,K2)+XV(I1,K2))
      T11(I1)=XV(I2,K1)-XV(I1,K1)+XV(I2,K2)-XV(I1,K2)
      T21(I1)=YV(I2,K1)-YV(I1,K1)+YV(I2,K2)-YV(I1,K2)
  400 T31(I1)=ZV(I2,K1)-ZV(I1,K1)+ZV(I2,K2)-ZV(I1,K2)
      CALL INTER(XQ,T11,NBV,XCON,T12,LNPTS)
      CALL INTER(XQ,T21,NBV,XCON,T22,LNPTS)
      CALL INTER(XQ,T31,NBV,XCON,T32,LNPTS)
      DO 410 I=1,LNPTS
      DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
      T12(I)=T12(I)/DENOM
      T22(I)=T22(I)/DENOM
      T32(I)=T32(I)/DENOM
      JSI=JS+I
```

```
      TMX(JSI)=T12(I)
      TMY(JSI)=T22(I)
  410 TMZ(JSI)=T32(I)
      DO 450 I1=1,NBV
      I2=I1+1
      T11(I1)=XV(I1,K2)-XV(I1,K1)+XV(I2,K2)-XV(I2,K1)
      T21(I1)=YV(I1,K2)-YV(I1,K1)+YV(I2,K2)-YV(I2,K1)
  450 T31(I1)=ZV(I1,K2)-ZV(I1,K1)+ZV(I2,K2)-ZV(I2,K1)
      CALL INTER(XQ,T11,NBV,XCON,T12,LNPTS)
      CALL INTER(XQ,T21,NBV,XCON,T22,LNPTS)
      CALL INTER(XQ,T31,NBV,XCON,T32,LNPTS)
      DO 460 I=1,LNPTS
      DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
      T12(I)=T12(I)/DENOM
      T22(I)=T22(I)/DENOM
      T32(I)=T32(I)/DENOM
      JSI=JS+I
      TTX(JSI)=T12(I)
      TTY(JSI)=T22(I)
  460 TTZ(JSI)=T32(I)
      JS=JS+LNPTS
  500 CONTINUE
      JS=NCT
      DO 600 K=1,LTPTS
      K1=LTDIV*(FLTC(K)-1.0)+NLAT1
      K2=K1+1
      CALL INTER(XV(1,K1),YV(1,K1),N3,XCON,T11,LNPTS)
      CALL INTER(XV(1,K2),YV(1,K2),N3,XCON,T12,LNPTS)
      CALL INTER(XV(1,K1),ZV(1,K1),N3,XCON,T21,LNPTS)
      CALL INTER(XV(1,K2),ZV(1,K2),N3,XCON,T22,LNPTS)
      DO 550 I=1,LNPTS
      JS=JS+1
      XQ(JS)=XCON(I)
      YQ(JS)=0.5*(T11(I)+T12(I))
  550 ZQ(JS)=0.5*(T21(I)+T22(I))
  600 CONTINUE
      DO 250 J=1,N2
      CALL INTER(XV(1,J),YV(1,J),N3,XVOR,T11,NBVV)
      CALL INTER(XV(1,J),ZV(1,J),N3,XVOR,T12,NBVV)
```

```
      DO 250 I=1,NBVV
      YV(I,J)=T11(I)
      ZV(I,J)=T12(I)
  250 CONTINUE
      DO 650 I=1,NBVV
      DO 650 J=1,N2
  650 XV(I,J)=XVOR(I)
      DO 700 J=1,N2
      XV(NB1,J)=XVV(N3)
      YV(NB1,J)=YV(N3,J)
  700 ZV(NB1,J)=ZV(N3,J)
      DO 800 J=1,LTPTS
      DO 800 I=1,LNPTS
      NCT=NCT+1
      XN1=TMY(NCT)*TTZ(NCT)-TMZ(NCT)*TTY(NCT)
      YN1=TMZ(NCT)*TTX(NCT)-TMX(NCT)*TTZ(NCT)
      ZN1=TMX(NCT)*TTY(NCT)-TMY(NCT)*TTX(NCT)
      DENOM=XN1**2+YN1**2+ZN1**2
      XN(NCT)=XN1/DENOM
      YN(NCT)=YN1/DENOM
      ZN(NCT)=ZN1/DENOM
  800 CONTINUE
      WRITE(18) BODYWR
      WRITE(6,55) (XQ(I),I=1,NCT)
      WRITE(6,55) (YQ(I),I=1,NCT)
      WRITE(6,55) (ZQ(I),I=1,NCT)
      WRITE(6,55) (XN(I),I=1,NCT)
      WRITE(6,55) (YN(I),I=1,NCT)
      WRITE(6,55) (ZN(I),I=1,NCT)
C
      RETURN
      END
```

```
      SUBROUTINE MATLT(K)
C
C   BLNGTH GIVE LATERAL LENGTHS OF PANELS ............
C    FIRST LONGITUDINALLY -- THEN LATERALLY.
C
      COMMON DA(5000)
     1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2    ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/BODY/ ARRAY(31000)
      DIMENSION XV1(151,31),YV(151,31),ZV(151,31),BLNGTH(5000)
      EQUIVALENCE(BLNGTH,ARRAY(15001)),(XV1,ARRAY(1))
     1           ,(YV,ARRAY(4682)),(ZV,ARRAY(9363))
      REAL L2
      LN=LNDIV/2+1
      DO 100 J=1,NTVV
      JT=(J-1)*LTDIV
      DO 100 I=1,NBVV
      II1=(I-1)*LNDIV+LN
      II2=II1+1
      L2=0.0
      DO 50 JJ=1,LTDIV
      JJ1=JT+JJ
      JJ2=JJ1+1
      X1=0.5*(XV1(II1,JJ1)+XV1(II2,JJ1))
      X2=0.5*(XV1(II1,JJ2)+XV1(II2,JJ2))
      DX=X2-X1
      Y1=0.5*(YV(II1,JJ1)+YV(II2,JJ1))
      Y2=0.5*(YV(II1,JJ2)+YV(II2,JJ2))
      DY=Y2-Y1
      Z1=0.5*(ZV(II1,JJ1)+ZV(II2,JJ1))
      Z2=0.5*(ZV(II1,JJ2)+ZV(II2,JJ2))
      DZ=Z2-Z1
   50 L2=SQRT(DX**2+DY**2+DZ**2)+L2
      K=K+1
  100 BLNGTH(K)=L2
      WRITE(6,200)(BLNGTH(I),I=1,K)
  200 FORMAT(*0BLNGTH*/(1P8E15.6))
      WRITE(18) BLNGTH
      RETURN
```

1 5340

END

```
      SUBROUTINE MATL(K)
      COMMON DA(5000)
     1   ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2    ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1    ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2    ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3    ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/ ARRAY(31000)
      DIMENSION XV1(151,31),YV(151,31),ZV(151,31),ALNGTH(5000)
      EQUIVALENCE(ALNGTH,ARRAY(15001)),(XV1,ARRAY(1))
     1            ,(YV,ARRAY(4682)),(ZV,ARRAY(9363))
      DO 10 J=1,NTVV
      JJ1=(J-1)*LTDIV+LTDIV/2 + 1
      JJ2=JJ1+1
      DO 10 I=1,NBVV
      K=K+1
      VL=0.0
      DO 4 II=1,LNDIV
      II1=LNDIV*(I-1)+II
      II2=II1+1
      X1=0.5*(XV1(II1,JJ1)+XV1(II1,JJ2))
      X2=0.5*(XV1(II2,JJ1)+XV1(II2,JJ2))
      DX=X2-X1
      Y1=0.5*(YV(II1,JJ1)+YV(II1,JJ2))
      Y2=0.5*(YV(II2,JJ1)+YV(II2,JJ2))
      DY=Y2-Y1
      Z1=0.5*(ZV(II1,JJ1)+ZV(II1,JJ2))
      Z2=0.5*(ZV(II2,JJ1)+ZV(II2,JJ2))
      DZ=Z2-Z1
    4 VL=SQRT(DX**2+DY**2+DZ**2)+VL
   10 ALNGTH(K)=VL
      WRITE(6,200)(ALNGTH(I),I=1,K)
  200 FORMAT(*0ALNGTH*/(1P8E15.6))
      WRITE(18) ALNGTH
      RETURN
      END
```

```
      SUBROUTINE XYZVB
C
      COMMON DA(5000)
     1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      EQUIVALENCE(DA(3),FMACH)
      COMMON/COMPRS/ BETAM
      EQUIVALENCE (DA(17),CHORD),                  (DA(33),DIVLAT)
     1 ,(DA(20),BODTAB),(DA(19),SYM),(DA(2151),XTHV),(DA(131),TH)
     2  ,(DA(800),R), (DA(1850),XV), (DA(41),XS), (DA(86),XTH)
     3 ,(DA(1600), CPTS),(DA(1601),XMF),(DA(1650),YMF),(DA(1700),ZMF)
     4 ,(DA(1750),YCAM) ,(DA(1800),ZCAM),(DA(2200),THVS),(DA(22),CAMBI)
      EQUIVALENCE (DA(21),BMULT)
      COMMON/SCRAT/ XVV(200),THVV(151,31)
      DIMENSION THV(31,151),   THSS(151,31),   RS(31,31),    S(151,31)
     1         , ST(31,31),      RR(151,31),   YCM(200),     ZCM(200)
     2         ,YCP(200),        ZCP(200),   SINYC(200),   COSYC(200)
     3        ,SINZC(200),     COSZC(200),     SLAT(100),    TH1(200)
     4        ,XTHV(150),        TH(669),        R(800),      XS(49)
     5        , XTH(49),        XMF(49),       YMF(49),      ZMF(49)
     6        ,YCAM(50),      THVS(800),       XV(150),     ZCAM(50)
      DIMENSION        D(25000),  THETA(200),       Y1(50),       Z1(50)
     1         ,     YM(200),        ZM(200),     YC(200),      ZC(200)
     2         ,     YY(151,31),     ZZ(151,31),    Y(151,31),    Z(151,31)
     3         ,     XM(200),        XP(200)
      EQUIVALENCE(D,ARRAY)
      EQUIVALENCE(D(1),THV),     (D(4682),THSS),     (D(9363),RS)
     1          ,(D(10324),S),   (D(15005),ST),      (D(15966),RR)
     2          ,(D(20647),YCM),(D(20847),ZCM),      (D(21047),YCP)
     3          ,(D(21247),ZCP),(D(21447),SINYC),    (D(21647),COSYC)
     4        ,(D(21847),SINZC),(D(22047),COSZC),    (D(22247),SLAT)
     5        ,(D(22347),TH1),   (D(22547),THETA),   (D(22747),Y1)
     6        ,(D(22797),Z1),    (D(22847),YM),      (D(23047),ZM)
     7        ,(D(23247),YC),    (D(23447),ZC)
     8        ,(D(23647),XM),    (D(23847),XP)
```

```
      COMMON/BODY/ ARRAY(31000)
      EQUIVALENCE(XV1,ARRAY(1)), (YV,ARRAY(4682)),(ZV,ARRAY(9363))
     1           ,(XBOO,ARRAY(26964)),(YBOO,ARRAY(26969))
     2           ,(ZBOO,ARRAY(26974))
      DIMENSION XV1(151,31),YV(151,31),ZV(151,31),XBOO(5),YBOO(5)
     1          ,ZBOO(5)
      COMMON/CONPTS/XQ(1320),YQ(1320),ZQ(1320)
      EQUIVALENCE (YY(1,1),THSS(1,1)),(ZZ(1,1),RS(1,1))
     1 ,(Y(1,1),YV(1,1)),(Z(1,1),ZV(1,1))
C
      MCPTS=CPTS
C  TEST FOR (R,THETA) INPUT OR (Y,Z) INPUT
C
      IF(BODTAB.NE.0.) GO TO 110
C  R,THETA INPUT
C
C  SET UP THV ARRAY
C    TEMPORARY CHANGE.......   SET GRIDL=1.
C    THIS IS USED FOR EVEN DELTA THETA VORTEX INPUT,
C       AND ONLY ONE SET OF THETA'S CREATED IN SUBROUTINE SETDAT.
      GRIDL=1.0
C
 56   N1=NTVV+1
      LOC=0
      DO 65 I=1,NXTHV
      DO 60 J=1,N1
      LOC=LOC+1
 58   TH1(J)=THVS(LOC)
 60   CONTINUE
      IF(GRIDL.GT.0.)LOC=0
 65   CALL FILLDV(TH1,THV(1,I),N1,DIVLAT)
      WRITE(6,500)(TH1(J),J=1,N1)
 400  FORMAT(1H1/(1P8E15.6))
 500  FORMAT(*0XYZVB*/(5X,1P6E15.6))
      IF(BODTAB.EQ.1.0) GO TO 150
C
C  A ROW OF THV IS FOR A TRAILING VORTEX.
C
C  NOW INTERPOLATE IN THE THV ARRAY TO GET THVV AT XVV STATIONS.
```

```
C  ALSO INTERPOLATE FOR THSS'S AT XS'S.
C
      N2=NTV+1
      N3=NBV+1
      WRITE(6,500)(XTHV(J),J=1,NXTHV)
      WRITE(6,500) (XV(I),I=1,NXTHV)
      WRITE(6,500)(XVV(I),I=1,N3)
      WRITE(6,500)(XS(I),I=1,NX)
      DO 70 I=1,N2
      DO 68 J=1,NXTHV
   68 TH1(J)=THV(I,J)
      CALL CODIM(XV,TH1,NXTHV,XVV,THVV(1,I),N3)
   70 CALL CODIM(XV,TH1,NXTHV,XS,THSS(1,I),NX)
C
C
C  INTERPOLATE ON R VS. THETA AT EACH XS STATION TO
C   OBTAIN R'S AT  THETA'S.
C
      L=2
      IREGNO=1
      JSUB=1
      KSUB=1
      DO 90 I=1,NX
   71 IF(XTH(L)-XTH(L-1).GE.0.0) GO TO 72
      IF(XS(I).GT.XTH(L)) GO TO 80
      GO TO 75
   72 IF(XS(I).LT.XTH(L)) GO TO 80
   75 JSUB=JSUB+NTH(IREGNO)+1
      KSUB=KSUB+NTH(IREGNO)
      IREGNO=IREGNO+1
      L=L+1
      IF(IREGNO.LE.NXTH) GO TO 71
      WRITE(6,73)
   73 FORMAT(*0IREGNO GREATER THAN NXTH*)
      STOP
   80 DO 85 K=1,N2
   85 THETA(K)=THSS(I,K)
   90 CALL CODIM(TH(JSUB),R(KSUB),NTH(IREGNO),THETA,RS(1,I),N2)
C$$$$$$$$$$$,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
```

```
      WRITE(6,4000)((RS(I,J),J=1,NX),I=1,N2)
 4000 FORMAT(1H0/(1P10E12.4))
C
C  COLUMNS OF RS ARE R'S AT THSS'S.
C  EACH COLUMN CORRESPONDS TO ONE XS STATION.
C
C  ROWS OF RS ARE R VS. XS FOR SUCCESSIVE TRAILING VORTICES.
C  THERE ARE N2 ROWS AND NX COLUMNS IN RS.   (N2 * NX)
C
C  INTERPOLATE ON RS VS. XS CURVES DEVELOPED IN ABOVE STEP
C   TO OBTAIN R AT XVV STATIONS.
C
      DO 100 I=1,N2
      DO 95 J=1,NX
   95 THETA(J)=RS(I,J)
  100 CALL INTER(XS,       THETA,NX,XVV,      RR(1,I),N3)
C
C  ARRAYS THVV(I,J) AND RR(I,J)   I=1,N3  J=1,N2
C   DESCRIBE LOCATION OF VORTEX POINTS.
C  ROWS CORRESPOND TO BOUND VORTEX LINES.
C  COLUMNS CORRESPOND TO TRAILING VORTEX LINES.
C
C
      GO TO 180
C
C  Y,Z INPUT  OR  DELTA-S INPUT
C
  110 CONTINUE
C  DEVELOP Z VS. S AND Y VS. S AT INPUT STATIONS.
      KSUB=1-NTH(1)
      JSUB=KSUB-1
      DO 120 I=1,NXTH
      KSUB=KSUB+NTH(I)
      JSUB=JSUB+NTH(I)+1
      N=NTH(I)
      IF(SYM.LT.0.) N=N+1
      Y(1,I)=TH(JSUB)
      Z(1,I)= R(KSUB)
      DELS=0.0
```

```
      S(1,I)=0.0
      DO 120 J=2,N
      IF(J.EQ.N.AND.SYM.LT.0.) GO TO 115
      K1=JSUB+J-1
      K2=KSUB+J-1
      Y(J,I)=TH(K1)
      Z(J,I)=R(K2)
      DELY=TH(K1)-TH(K1-1)
      DELZ= R(K2)- R(K2-1)
      DELS=SQRT(DELY**2+DELZ**2) + DELS
      S(J,I)=DELS
      GO TO 120
  115 S(J,I)=SQRT((Y(J-1,I)-Y(1,I))**2 + (Z(J-1,I)-Z(1,I))**2) + DELS
      Y(N,I)=Y(1,I)
      Z(N,I)=Z(1,I)
  120 CONTINUE
      IF(THVS(1).NE.0.) GO TO 56
C
C  Y,Z INPUT
C
      DO 130 I=1,NXTH
      N=NTH(I)
      CALL FILLDV(S(1,I),SLAT,N,DIVLAT)
      NLAT=DIVLAT*N + 1.01
      CALL CODIM(S(1,I),Y(1,I),N,SLAT,YY(1,I),NLAT)
  130 CALL CODIM(S(1,I),Z(1,I),N,SLAT,ZZ(1,I),NLAT)
C  A COLUMN OF YY (OR ZZ) ARE VALUES OF Y (OR Z) AT INPUT XS STATIONS.
C  NOW INTERPOLATE TO OBTAIN Y AND Z AT XVV'S.
C
  132 N3=NBV+1
      N2=NTV+1
      DO 140 I=1,N2
      DO 135 J=1,NXTH
      Y1(J)=YY(I,J)
  135 Z1(J)=ZZ(I,J)
      CALL INTER(XS,Y1,NXTH,XVV,YV(1,I),N3)
  140 CALL INTER(XS,Z1,NXTH,XVV,ZV(1,I),N3)
      GO TO 220
  150 CONTINUE
```

```
C
C  NOW A COLUMN OF THV CONTAINS S'S AT AN INPUT X-THETA STATION.
C  A ROW OF THV GIVES S'S FOR A TRAILING VORTEX (AT ALL INPUT X-THETA'S)
C
C  INTERPOLATE IN THE S VS. XTH PLOTS FOR S'S AT INPUT XS STATIONS.
C
      N2=NTV+1
      DO 160 I=1,N2
      DO 155 J=1,NXTH
  155 THETA(J)=THV(I,J)
  160 CALL INTER(XTH,THETA,NXTH,XS,ST(1,I),NX)
C
C  INTERPOLATE IN THE Y VS. S  AND Z VS. S ARRAYS AT XS STATIONS
C   FOR Y'S AND Z'S AT ST'S FOUND ABOVE.
C
      DO 170 I=1,NX
      DO 165 J=1,N2
  165 THETA(J)=ST(I,J)
      CALL CODIM(S(1,I),Y(1,I),N,THETA,YY(1,I),N2)
  170 CALL CODIM(S(1,I),Z(1,I),N,THETA,ZZ(1,I),N2)
      GO TO 132
  180 CONTINUE
C
C  COMPUTE  YY,ZZ FROM  THVV AND RR.
      DO 200 I=1,N3
      DO 200 J=1,N2
      THSS(I,J)=RR(I,J)*SIND(THVV(I,J))
  200 S   (I,J)=RR(I,J)*COSD(THVV(I,J))
C   THSS IS THE Y ARRAY.    S IS THE Z ARRAY.
C   THESE WILL BE MULTIPLIED BY THE MULT. FACTORS TO GET YV,ZV.
C
  220 CONTINUE
C
C  PERFORM MULTIPLICATION AND REVISION OF VORTEX POINT
C   COORDINATES DUE TO CAMBER
C
C  INTERPOLATE FOR MULTIPLICATION FACTORS AND CAMPER AT XVV'S.
      IF(MCPTS.NE.0) GO TO 223
      DO 222 I=1,N3
```

```
      YC(I)=0.0
      ZC(I)=0.0
      YM(I)=1.0
  222 ZM(I)=1.0
      GO TO 236
  223 CONTINUE
      CALL INTER(XMF,YMF,MCPTS,XVV,YM,N3)
      IF(BMULT.EQ.1.0) GO TO 230
      DO 225 I=1,N3
  225 ZM(I)=YM(I)
      GO TO 235
  230 CONTINUE
      CALL INTER(XMF,ZMF,MCPTS,XVV,ZM,N3)
  235 CONTINUE
      CALL INTER(XMF,YCAM,MCPTS,XVV,YC,N3)
      CALL INTER(XMF,ZCAM,MCPTS,XVV,ZC,N3)
  236 CONTINUE
C
C  FOR CAMBI=0, COMPUTE ANGLES IN X-Z AND X-Y PLANES WHICH CAMBER LINES
C   MAKE AT XVV STATIONS.
C
      IF(CAMBI.NE.0.)GO TO 280
      DX=CHORD*0.001
      DO 240 I=1,N3
      XM(I)=XVV(I)+DX
  240 XP(I)=XVV(I)-DX
      CALL INTER(XVV,YC,N3,XM,YCM,N3)
      CALL INTER(XVV,ZC,N3,XM,ZCM,N3)
      CALL INTER(XVV,YC,N3,XP,YCP,N3)
      CALL INTER(XVV,ZC,N3,XP,ZCP,N3)
      DO 260 I=1,N3
      DY=YCP(I)-YCM(I)
      DZ=ZCP(I)-ZCM(I)
      YCA=ATAN(DY/(2.*DX))
      ZCA=ATAN(DZ/(2.*DX))
      SINYC(I)=SIN(YCA)
      COSYC(I)=COS(YCA)
      SINZC(I)=SIN(ZCA)
  260 COSZC(I)=COS(ZCA)
```

```
      GO TO 290
  280 DO 285 I=1,N3
      SINYC(I)=0.0
      COSYC(I)=1.0
      SINZC(I)=0.0
  285 COSZC(I)=1.0
  290 DO 300 J=1,N2
      DO 300 I=1,N3
      Y2=YM(I)*THSS(I,J)
      Z2=ZM(I)*S(I,J)
      XV1(I,J)=XVV(I)-Y2*SINYC(I)*COSZC(I)-Z2*SINZC(I) + XBOO(NB)
      YV(I,J) =(YC(I)+Y2*COSYC(I) +YBOO(NB))*BETAM
  300 ZV(I,J) =(ZC(I)-Y2*SINYC(I)*SINZC(I)+Z2*COSZC(I)+ZBOO(NB))*BETAM
      WRITE(6,301) N2,N3
      WRITE(12)(YV(N3,J),ZV(N3,J),J=1,N2)
      WRITE(6,7003) (J,YV(N3,J),ZV(N3,J),J=1,N2)
 7003 FORMAT(*0BODY DRAG COORDS.*/(I5,2F15.5))
  301 FORMAT(*0XYZVB  N2,N3=*2I5)
      RETURN
      END
```

```
      SUBROUTINE SETDAT
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2   ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/ DUMMY(26963),XB00(5),YB00(5),ZB00(5)
      EQUIVALENCE (DA(40), FNX),(DA(85),FNXTH), (DA(30),FLONGV)
     1 ,(DA(31),FLATV),(DA(32),DIVLON),(DA(33),DIVLAT),(DA(86),XTH)
     2 ,(DA(41),XS),(DA(2150), FNXTHV),(DA(23),VORLN), (DA(24),VORLT)
     3 ,(DA(2151),XTHV),(DA(130),THN),(DA(1850),XV),(DA(17),CHORD)
     4   ,(DA(38),PTSLN),(DA(39),PTSLT)
     5  ,(DA(27),XB0),(DA(28),YB0),(DA(29),ZB0)
      EQUIVALENCE(DA(36),SEGLN),(DA(37),SEGLT)
     1  ,(DA(34),FUNCLN),(DA(35),FUNCLT)
      DIMENSION XTH(49),XS(49),XTHV(149),THN(670),XV(150)
      COMMON/SCRAT/ XVV(200)
      DIMENSION NPP(48)
      EQUIVALENCE(NPP(1),NVPTS(1))
      EQUIVALENCE(DA(19),SYM)
      EQUIVALENCE(DA(2200),THVS)
      DIMENSION THVS(799)
      NX=FNX
      NXTH=FNXTH
      IF(NXTH.EQ.0) NXTH=NX
C
      LOC=1
      DO 10 I=1,NXTH
      NTH(I)=THN(LOC)
   10 LOC=LOC+NTH(I)+1
      IF(DIVLON.EQ.0.0) DIVLON=1.0
      IF(DIVLAT.EQ.0.0) DIVLAT=1.0
      LNDIV=DIVLON
      LTDIV=DIVLAT
      LNDIVB(NB)=LNDIV
      LTDIVB(NB)=LTDIV
      LTPTS=PTSLT
```

```
      LNPTS=PTSLN
      LTC(NB)=LTPTS
      LNC(NB)=LNPTS
      LSEG(NB)=SEGLN
      TSEG(NB)=SEGLT
      LFUNC(NB)=FUNCLN
      TFUNC(NB)=FUNCLT
      SYMM(NB)=SYM
      NTVV=FLATV
C  NBVV EQUALS NO. OF LONGITUDINAL PANELS.
C  NTVV EQUALS NO. OF LATERAL PANELS.
      NBVV=FLONGV
      LNVOR=VORLN
      LTVOR=VORLT
      NXTHV=FNXTHV
      XBOO(NB)=XBO
      YBOO(NB)=YBO
      ZBOO(NB)=ZBO
C
C  SET UP LONGITUDINAL VORTEX GRID (XVV ARRAY)
      NB1=NBVV+1
      IF(LNVOR) 15,20,30
 15   DEL=CHORD/NBVV
      XV(1)=0.0
      GO TO 24
 20   DEL=3.1415926/NBVV
      XV(1)=0.0
 24   DO 26 I=2,NB1
 26   XV(I)=XV(I-1)+DEL
C
 30   CONTINUE
C  FOR LNVOR=1, XV IS GIVEN IN INPUT
      NB1=NBVV+1
      IF(LNDIV.NE.1) GO TO 40
      DO 35 I=1,NB1
 35   XVV(I)=XV(I)
      GO TO 45
 40   CALL FILLDV(XV,XVV,NB1,DIVLON)
 45   CONTINUE
```

```
C
      NVL(NB)=NBVV
      NVT(NB)=NTVV
      NTV=NTVV*DIVLAT+0.01
      NBV=NBVV*DIVLON+0.01
      NLN(NB)=NBV+1
      NLT(NB)=NTV+1
      NVPTS(NB)=NLN(NB)*NLT(NB)
      NCPTS(NB)=LTPTS*LNPTS
      WRITE(6,400)(NPP(I),I=1,48)
  400 FORMAT(1H0,10(3X,I5))
      IF(LNVOR.NE.0) GO TO 50
      C2=0.5*CHORD
      N1=NBV+1
C  NBV IS TOTAL NO. OF LONGITUDINAL VORTICES.
      DO 48 I=1,N1
   48 XVV(I)=C2*(1.0-COS(XVV(I)))
   50 CONTINUE
C
C
C   SET-UP BODY LATERAL VORTEX PARAMETERS IF REQUIRED HERE.
       IF(LTVOR) 300,310,320
C   THETA'S SAME AS BODY DEFINITION STATIONS
C   FOR THIS OPTION THERE MUST BE (NTVV+1) THETA'S (THN'S) GIVEN AT X'S.
  300 THETAI=0
      N1=NTVV+1
      M1=0
      M2=1
      DO 301 I=1,N1
      M1=M1+1
      M2=M2+1
      THVS(M1)=THN(M2)
      IF(I.EQ.N1) M2=M2+1
  301 CONTINUE
      GO TO 320
C   THIS OPTION IS FOR EVEN DELTA THETA'S.   DETERMINE(NTVV+1) THVS'S.
  310 THETAI=1.0
      N1=NTVV+1
      IF(SYM) 311,312,311
```

```
311   TRANGE=360.0
      GO TO 313
312   TRANGE=180.0
313   DTH=TRANGE/NTVV
      DO 314 I=1,N1
314   THVS(I)=(I-1)*DTH
C    FOR LTVOR = 1, THVS ARE GIVEN IN DATA.
320   CONTINUE
      IF(NXTHV.EQ.0) NXTHV=FLONGV+1.01
      IF(LNVOR.NE.0) GO TO 58
      DO 54 I=  1,NXTHV
54    XV(I)=C2*(1.0- COS(XV(I)))
58    IF(FNXTH.NE.0) GO TO 65
      DO 60 I=1,NXTH
60    XTH(I)=XS(I)
65    CONTINUE
C
      CALL DATAWR(DA)
      RETURN
      END
```

```
      SUBROUTINE VYZN(J,I,Y,Z,AN)
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2  ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON /BODY/ A(31000)
      DIMENSION XV(151,31),YV(151,31),ZV(151,31)
      EQUIVALENCE(XV,A(1)),(YV,A(4682)),(ZV,A(9363))
      DIMENSION AN(1)
      J1=LTDIV*(J-1)+LTDIV/2 + 1
      J2=J1+1
      IF(I.NE.1) GO TO 100
      X3=XV (2,J1)
      X4=XV (2,J2)
      X1=2.0*XV (1,J1)-X3
      X2=2.0*XV (1,J2)-X4
      Y3=YV(2,J1)
      Y4=YV(2,J2)
      Y1=2.0*YV(1,J1)-Y3
      Y2=2.0*YV(1,J2)-Y4
      Z3=ZV(2,J1)
      Z4=ZV(2,J2)
      Z1=2.0*ZV(1,J1)-Z3
      Z2=2.0*ZV(1,J2)-Z4
      GO TO 200
100   I1=I-1
      I2=I1+2
      X1=XV (I1,J1)
      X2=XV (I1,J2)
      X3=XV (I2,J1)
      X4=XV (I2,J2)
      Y1=YV(I1,J1)
      Y2=YV(I1,J2)
      Y3=YV(I2,J1)
      Y4=YV(I2,J2)
      Z1=ZV(I1,J1)
      Z2=ZV(I1,J2)
      Z3=ZV(I2,J1)
      Z4=ZV(I2,J2)
200   Y=0.25*(Y1+Y2+Y3+Y4)
```

```
Z=0.25*(Z1+Z2+Z3+Z4)
TX=X3-X1+X4-X2
TY=Y3-Y1+Y4-Y2
TZ=Z3-Z1+Z4-Z2
D=SQRT(TX**2+TY**2+TZ**2)
T1=TX/D
T2=TY/D
T3=TZ/D
TX=X2-X1+X4-X3
TY=Y2-Y1+Y4-Y3
TZ=Z2-Z1+Z4-Z3
D=SQRT(TX**2+TY**2+TZ**2)
AN(1)=(T2*TZ-T3*TY)/D
AN(2)=(T3*TX-T1*TZ)/D
AN(3)=(T1*TY-T2*TX)/D
RETURN
END
```

```
      SUBROUTINE INTER (X,Y,N,X1,Y1,N1)
      DIMENSION X(1),Y(1),X1(1),Y1(1)
      IF(X(2)-X(1).GT.0.0) GO TO 20
      IM=1
      XMIN=X1(1)
      DO 5 I=2,N1
      IF(XMIN-X1(I).LE.0.0) GO TO 6
      XMIN=X1(I)
      IM=I
    5 CONTINUE
    6 NU=IM
      IM=1
      XMIN=X(1)
      DO 10 I=2,N
      IF(XMIN-X(I).LE.0.0) GO TO 11
      XMIN=X(I)
      IM=I
   10 CONTINUE
   11 CONTINUE
      CALL REVERS(X,IM)
      CALL REVERS(Y,IM)
      CALL REVERS(X1,NU)
      CALL CODIM(X,Y,IM,X1,Y1,NU)
      CALL REVERS(X,IM)
      CALL REVERS(Y,IM)
      CALL REVERS(X1,NU)
      CALL REVERS(Y1,NU)
      WRITE(6,100)(X(I),Y(I),I=1,IM),(X1(I),Y1(I),I=1,NU)
  100 FORMAT('0SUB. INTER'/(1P8E16.6))
      CALL CODIM(X(IM),Y(IM),N-IM+1,    X1(NU+1),Y1(NU+1),N1-NU)
      NX=N-IM+1
      NY=N1-NU
      WRITE(6,100)(X(IM-1+I),Y(IM-1+I),I=1,NX),(X1(NU+I),Y1(NU+I)
     1  ,I=1,NY)
      RETURN
   20 CALL CODIM(X,Y,N,X1,Y1,N1)
      RETURN
      END
```

```
      SUBROUTINE FILLDV(X,Y,N,DIV)
      DIMENSION X(1),Y(1)
      N1=N-1
      NDIV=DIV+0.01
      DO 10 I=1,N1
      DEL=(X(I+1)-X(I))/DIV
      II=(I-1)*NDIV
      DO 10 J=1,NDIV
      K=II+J
   10 Y(K)=X(I)+(J-1)*DEL
      Y(K+1)=X(N)
      RETURN
      END
```

```
      SUBROUTINE REVERS(X,N)
      DIMENSION X(1)
      NL=N/2
      DO 100 I=1,NL
      XHOLD=X(I)
      N1=N+1-I
      X(I)=X(N1)
  100 X(N1)=XHOLD
      RETURN
      END
```

```
      PROGRAM PANELGM
      COMMON DA(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5  ,BDUMMY(12130),XYZN(3000)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,ANATT
     3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/SCRAT/ DUMS(6000)
      EQUIVALENCE(DA(2),PANS)
      COMMON /PANINF/ PANSYM(10), DUM11(400), PANREF(10), PCHORD(10)
      EQUIVALENCE (DA(3421),PREF), (DA(3422),PCH)
      IF(PANS.EQ.0.0) GO TO 2
      NPANS=PANS
      DO 1 I=1,NPANS
      DO 10 J=3420,5000
   10 DA(J)=0.0
      CALL DECRD(DA)
      CALL PANDAT
      PANSYM(I)=IPSYM
      PCHORD(I) = PCH
      PANREF(I) = PREF
      CALL PANEL1
      CALL ATTACH(XVR,YVR,ZVR,10,0)
    1 CONTINUE
    2 CONTINUE
      END
```

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```
      SUBROUTINE PANEL1
C
C  THIS SUBROUTINE STARTS GEOM. OF A PANEL.
COMPUTES XJ,YJ,ZJ  POINTS.(COORDINATES OF JUNCTURE)
COMPUTES XR,YR,ZR POINTS.(COORDINATES OF OUTER EDGE OF ROOT OR PANEL
C  HAVING NO ROOT SECTION.
      COMMON DA(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1  ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2  ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4  ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5   ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6   ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
     7   ,XVS(100),YVS(100),ZVS(100)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/SCRAT/ XA(21),YA(21),ZA(21),SA(21),SJ(20)
     1  ,DUMMY(1300)
      COMMON     /COMPRS/   BETAM
      EQUIVALENCE(PERIM,DUMMY)
      DIMENSION PERIM(400)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      EQUIVALENCE(DA(3450),PERP),  (DA(4600),PXCV),  (DA(4680),CPLN)
     1           ,(DA(4720),CPLT),    (DA(4640),PETV)
      EQUIVALENCE(DA(3432),XPO),   (DA(3433),YPO),   (DA(3434),ZPO)
      DIMENSION PERP(150),PXCV(40),PETV(40),CPLN(40),CPLT(40)
      LOGICAL ROOT,OUTER
      NSPACE = 0
      DO 11 I=1,NPERPT
      IF(PERP(4*I).NE.0.0) GO TO 12
      NSPACE=NSPACE + 1
   11 CONTINUE
   12 WRITE(6,13) NSPACE
```

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```
13    FORMAT(*ONSPACE =*,I3)
      ROOT=.FALSE.
      OUTER=.FALSE.
      NBP1 =NBVVP+1
      IF(NSPACE.NE.0) ROOT=.TRUE.
      IF(.NOT.ROOT) OUTER=.TRUE.
      DO 1 I=1,NPERPT
      I1=NPERPT+1-I
      I2=I
      IF(PERP(4*I1).NE.0.0) GO TO 2
1     CONTINUE
2     NTIP=NPERPT+1-I2
      IF(OUTER) GO TO 3
      IF(PERP(4*(NTIP-1)).EQ.0.0) GO TO 3
      OUTER=.TRUE.
3     CONTINUE
      CALL PERGM(PERP,PERIM)
      DO 4 I=1,NTIP
      I1=4*(NPERPT-I)
      XTE(I)=PERIM(I1+1)
      YTE(I) = PERIM(I1+2) *BETAM
      ZTE(I) = PERIM(I1+3) *BETAM
      I4=4*(I-1)
      XLE(I)=PERIM(I4+1)
      YLE(I) = PERIM(I4+2) *BETAM
4     ZLE(I) = PERIM(I4+3) *BETAM
      YPO = YPO *BETAM
      ZPO = ZPO *BETAM
C
C    FIND MINIMUM XLE+XPO (ROOT) AND SET=XVS(100).
      XMLE=XLE(1)+XPO
      IF(NPAN.EQ.1) GO TO 100
      IF(XMLE.LT.XMLEH) GO TO 100
      GO TO 110
100   XMLEH=XMLE
110   XVS(100)=XMLEH
      WRITE(6,120) NPAN,XMLE,XVS(100)
120   FORMAT(30HOPANEL1... NPAN,XMLE,XVS(100)=I5,1P2E20.6)
C
```

```
C
      WRITE(6,200)(I,XLE(I),YLE(I),ZLE(I),I=1,NTIP)
      WRITE(6,200)(I,XTE(I),YTE(I),ZTE(I),I=1,NTIP)
200   FORMAT(1H ,I3,1P3E16.6)
      DO 401 I=1,NTIP
401   ETLE(I)=SQRT((YLE(I)-YLE(1))**2+(ZLE(I)-ZLE(1))**2)
     1   /SQRT((YLE(NTIP)-YLE(1))**2+(ZLE(NTIP)-ZLE(1))**2)
      DELS=0.0
      NTIP1=NTIP-1
      SLE(1)=0.0
      DO 5 I=1,NTIP1
      DY=YLE(I)-YLE(I+1)
      DZ=ZLE(I)-ZLE(I+1)
      DELS=DELS+SQRT(DY**2+DZ**2)
      SLE(I+1)=DELS
      IF(I.EQ.NSPACE) DSROOT=DELS
5     CONTINUE
      CHTIP=PERIM(4*NTIP)
      N4=4*NTIP
      XTIP=PERIM(N4-3)
      YTIP=PERIM(N4-2)
      ZTIP=PERIM(N4-1)
      IF(ROOT) GO TO 10
C  FOR NO ROOT SECTION.....
      XRLE=XTIP
      YRLE=YTIP
      ZRLE=ZTIP
      XRTE=PERIM(N4+1)
      YRTE=PERIM(N4+2)
      ZRTE=PERIM(N4+3)
      GO TO 20
C  FOR LIFTING PANEL WITH ROOT SECTION.....
10    NS4=4*NSPACE
      XRLE=PERIM(NS4+1)
      YRLE=PERIM(NS4+2)
      ZRLE=PERIM(NS4+3)
      N1=4*(NPERPT-NSPACE-1)
      XRTE=PERIM(N1+1)
      YRTE=PERIM(N1+2)
```

```
      ZRTE=PERIM(N1+3)
C  NLROOT EQUALS NO. OF TR. VORTICES IN ROOT SECTION.
      NLROOT=NSPACE+1
 20   XJLE=PERIM(1)
      YJLE=PERIM(2)
      ZJLE=PERIM(3)
      N4=4*NPERPT
      XJTE=PERIM(N4-3)
      YJTE=PERIM(N4-2)
      ZJTE=PERIM(N4-1)
      IF(ROOT) GO TO 30
C  FOR NO ROOT, DIVIDE JUNCTURE TR. VORTEX INTO BOUND VORTEX COORDINATES
      CX=XJTE-XJLE
      CY=YJTE-YJLE
      CZ=ZJTE-ZJLE
      XJ(1)=XJLE
      YJ(1)=YJLE
      ZJ(1)=ZJLE
      DO 25 I=2,NBVVP
      DPER= PXCV(I)-PXCV(I-1)
      XJ(I)=XJ(I-1)+CX*DPER
      YJ(I)=YJ(I-1)+CY*DPER
 25   ZJ(I)=ZJ(I-1)+CZ*DPER
      N1=NBVVP+1
      XJ(N1)=XJTE
      YJ(N1)=YJTE
      ZJ(N1)=ZJTE
      GO TO 50
 30   CONTINUE
C
C
C  COMPUTE XJ,YJ,ZJ ARRAYS FOR ROOT-BODY JUNCTURE.
      XJ(1)=XJLE
C  COMPUTE Y,Z ON TR. VORTEX FOR 20 X'S.
      NA=20
      DX=(XJTE-XJLE)/NA
      XA(1)=XJLE+XPO
      DO 45 I=1,NA
 45   XA(I+1)=XA(I)+DX
```

```
      NAP1=NA+1
      CALL CODIM(XVT,YVT,N1,XA,YA,NAP1)
      CALL CODIM(XVT,ZVT,N1,XA,ZA,NAP1)
      WRITE(6,451) (XA(I),YA(I),ZA(I),I=1,NAP1)
 451  FORMAT(*0XA,YA,ZA*/(1P6E20.5))
      SAI=0.0
      SA(1)=0.0
      DO 46 I=1,NA
      DY= YA(I+1)-YA(I)
      DZ= ZA(I+1)-ZA(I)
      SAI=SAI+SQRT(DX**2+DY**2+DZ**2)
 46   SA(I+1)=SAI
      SJ(1)=0.0
      DO 47 I=1,NBVVP
 47   SJ(I+1)=SJ(I)+(PXCV(I+1)-PXCV(I))*SAI
      CALL CODIM(SA,XA,NAP1,SJ,XJ,NBP1)
      CALL CODIM(SA,YA,NAP1,SJ,YJ,NBP1)
      CALL CODIM(SA,ZA,NAP1,SJ,ZJ,NBP1)
 50   CONTINUE
      DO 49 I=1,NBP1
      XJ(I)=XJ(I)-XPO
      YJ(I)=YJ(I)-YPO
 49   ZJ(I)=ZJ(I)-ZPO
C   COMPUTE XR,YR,ZR ARRAYS
      DX= XRTE-XRLE
      XR(1)=XRLE
      YR(1)=YRLE
      ZR(1)=ZRLE
      DO 55 I=1,NBVVP
      XR(I+1) = XRLE + DX*PXCV(I+1)
      YR(I+1)=YRLE
 55   ZR(I+1)=ZRLE
        WRITE(6,200)(I,XJ(I),YJ(I),ZJ(I),I=1,NBP1)
        WRITE(6,200)(I,XR(I),YR(I),ZR(I),I=1,NBP1)
C
      CALL PANEL2
      RETURN
      END
```

```
      SUBROUTINE PANDAT
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVP(7),LTDIVP(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON DA(5000)
      EQUIVALENCE(DA(3420),PN),    (DA(3426),PSYM),    (DA(3429),WCI)
      EQUIVALENCE(DA(3437),PNBVV),(DA(3438),PNTVV),   (DA(3439),PLNCF)
     2          ,(DA(3440),PLTCF),(DA(3441),PLNCP),   (DA(3442),PLTCP)
     3          ,(DA(3443),PTSPER),(DA(3444),ROOTSP), (DA(3445),ATTCH)
     4          ,(DA(3447),PCLN),  (DA(3448),PCLT),   (DA(3600),PANXC)
     5          ,(DA(3630),ETALF), (DA(4130),XCTHK),  (DA(4160),ETATH)
     6          ,(DA(4680),CPLN),  (DA(4720),CPLT)
      EQUIVALENCE (DA(3430),VLNI),    (DA(3431),VLTI)
     1            ,(DA(4600),PXCV),   (DA(4640),PETV)
      EQUIVALENCE (DA(3423),SPAN)
      COMMON/COMPRS/ BETAM
      DIMENSION PXCV(40), PETV(40)
      DIMENSION CPLN(40),CPLT(40)
      COMMON/PANATT/ NATT(30),XATT(200)
      COMMON/CONTRV/ LNN(3,40),LTT(3,40)
      DIMENSION NLOOK(50)
      EQUIVALENCE(NLOOK,NVPTS)
      WRITE(6,1) NLOOK
    1 FORMAT(*0PANDAT*/(1H ,10I10))
      CALL DATAWR(DA)
      SPAN=SPAN*BETAM
      NPAN=PN
      IPSYM=PSYM
      IWC=WCI
      NBVVP=PNBVV
      NTVVP=PNTVV
      NPP= NPAN +NBODS
      NVL(NPP) =NBVVP
      NVT(NPP) =NTVVP
```

```
      NLN(NPP) =NBVVP+1
      NLT(NPP) =NTVVP+1
      NVPTS(NPP) = NBVVP*NTVVP
      LNCFP=PLNCF
      LTCFP=PLTCF
      LFUNC(NPAN+1) = LNCFP
      TFUNC(NPAN+1) = LTCFP
      LNCPP=PLNCP
      LTCPP=PLTCP
      LTC(NPP) = LTCPP
      LNC(NPP) = LNCPP
      NCPTS(NPP) = LNCPP*LTCPP
      NPERPT=PTSPER
      NSPACE=ROOTSP
      NAP=(NPAN-1)*3+2
      NATTCH=NATT(NAP)
      NTRATT=NATT(NAP+1)
      IF(NATTCH.EQ.0) GO TO 101
      WRITE(6,100) NPAN,NATTCH,NTRATT
100   FORMAT(*0PANEL*,I3,* IS ATTACHED TO COMPONENT*,I3,*AT TRAILING VOR
     1TEX*,I2)
      GO TO 103
101   WRITE (6,102) NPAN,NATTCH
102   FORMAT(*0PANEL*,I3,* IS NOT ATTACHED.  NATTCH=*,I2)
103   CONTINUE
      NPRCLN=PCLN
      NPRCLT=PCLT
      NWCTXC=PANXC
      NWCTET=ETALF
      NTHXC=XCTHK
      NTHET=ETATH
      IF(CPLN(1).NE.0.0) GO TO 6
      DO 5 I=1,LNCPP
5     CPLN(I)=I
6     IF(CPLT(1).NE.0.0) GO TO 8
      DO 7 I=1,LTCPP
7     CPLT(I)=I
8     CONTINUE
C
```

```
C     SAVE CONTROL POINT LOCATIONS FOR USE IN VELOCITY CALCULATION.
      DO 15 I=1,LNCPP
 15   LNN(NPAN,I)=CPLN(I)
      DO 16 I=1,LTCPP
 16   LTT(NPAN,I)=CPLT(I)
C
      LNI=VLNI
      IF(LNI) 10,20,25
 10   PXCV(1)=0.0
      D=1.0/NBVVP
      DO 11 I=1,NBVVP
 11   PXCV(I+1)=PXCV(I)+D
      GO TO 25
 20   PXCV(1)=0.0
      D=180.0/NBVVP
      DO 21 I=1,NBVVP
 21   PXCV(I+1)=0.5*(1.0-COSF(D*I))
 25   CONTINUE
      NL1=NBVVP+1
      WRITE(6,26) (PXCV(I),I=1,NL1)
 26   FORMAT(*0PXCV ARRAY*/(1P6E15.6))
      LTI=VLTI
      IF(LTI) 40,30,55
 30   PETV(1)=0.0
      D=1.0/NTVVP
      DO 31 I=1,NTVVP
      IF(I-NTVVP) 311,312,311
 311  PETV(I+1)=PETV(I)+D
      GO TO 31
 312  PETV(I+1)=1.0
 31   CONTINUE
      GO TO 55
 40   PETV(1)=0.0
      D=180.0/NTVVP
      DO 41 I=1,NTVVP
      IF(I-NTVVP) 411,412,411
 411  PETV(I+1)=0.5*(1.0-COSF(D*I))
      GO TO 41
 412  PETV(I+1)=1.0
```

```
41    CONTINUE
55    NT1=NTVVP+1
      WRITE(6,42) (PETV(I),I=1,NT1)
42    FORMAT(*0PETV ARRAY*/(1P6E15.6))
      RETURN
      END
```

```
      SUBROUTINE PANEL2
C
C   THIS SUBROUTINE (CALLED BY PANEL1) COMPUTES VORTEX PTS. AND
C    CONTROL PTS. FOR ROOT AND OUTER PANEL.
C
      COMMON DA(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1  ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSURV(100),CHORD(100)
     2  ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4  ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5   ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6   ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
     7   ,XVS(100),YVS(100),ZVS(100)
      COMMON/PANEL/ NPAN,IPSYM,IVC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NNCTXC,NNCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
      DIMENSION XCR(1000),   YCR(1000),    ZCR(1000)
      EQUIVALENCE(XCR,XQ),   (YCR,YQ),   (ZCR,ZQ)
      COMMON/SCRAT/ XA(21),YA(21),ZA(21),SA(21),SJ(20)
     1  ,DUMMY(1300),ETC(50),ETV(50)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      EQUIVALENCE(DA(3450),PERIM),    (DA(4600),PXCV),   (DA(4650),CPLN)
     1          ,(DA(4720),CPLT),     (DA(4640),PETV)
      EQUIVALENCE(DA(3432),XP0),    (DA(3433),YP0),    (DA(3434),ZP0)
      DIMENSION PERIM(50),PXCV(40),PETV(40),CPLN(40),CPLT(40)
      DIMENSION DCX(20),DCY(20),DCZ(20)
      DIMENSION BODYAR(31000)
      EQUIVALENCE(XVR(1,1),BODYAR(1))
      DIMENSION STARAY(20),SLCF(50),SAVEC(6000)
      EQUIVALENCE(DA(4840),STARAY),(DA(4860),SLCF),(DA(3440),FMU)
     1          ,(DA(3430),FNU),(BODYAR(15001),SAVEC)
      LOGICAL ROOT,OUTER,KLT,KLN
      LOGICAL FIRST
```

```
      LOGICAL ROOTP,OUTERP
      NSPP(NPAN)=NSPACE
      ROOTP(NPAN)=ROOT
      OUTERP(NPAN)=OUTER
      NU=FNU
      NW=FNW
      LETV=0
      LETC=0
      NBP1=NBVVP+1
      NLROOT=NSPACE + 1
      IF(.NOT.ROOT) GO TO 65
C     COMPUTE ALL PANEL  POINTS BOUNDED BY JLE,RLE,RTE,JTE.
      N1=NSPACE-1
      DO 58 J=1,NBP1
      XVR(NLROOT,J)=XR(J)
      YVR(NLROOT,J)=YR(J)
      ZVR(NLROOT,J)=ZR(J)
      XVR(1,J)=XJ(J)
      YVR(1,J)=YJ(J)
   58 ZVR(1,J)=ZJ(J)
      DO 581 I=1,NLROOT
      XVR(I,1)=XLE(I)
      YVR(I,1)=YLE(I)
      ZVR(I,1)=ZLE(I)
      XVR(I,NBP1)=XTE(I)
      YVR(I,NBP1)=YTE(I)
  581 ZVR(I,NBP1)=ZTE(I)
      DO 60 J=2,NBVVP
      CSX=XR(J)-XJ(J)
      CSY=YR(J)-YJ(J)
      CSZ=ZR(J)-ZJ(J)
      CS2=CSX**2+CSY**2+CSZ**2
      PER=  PXCV(J)
      DENOM=0.0
      DO 59 I=1,NSPACE
      DXLE=XLE(I+1)-XLE(I)
      DYLE=YLE(I+1)-YLE(I)
      DZLE=ZLE(I+1)-ZLE(I)
      CALL NORM(DXLE,DYLE,DZLE)
```

```
      DXTE=XTE(I+1)-XTE(I)
      DYTE=YTE(I+1)-YTE(I)
      DZTE=ZTE(I+1)-ZTE(I)
      CALL NORM(DXTE,DYTE,DZTE)
      DCX(I)=DXLE+(DXTE-DXLE)*PER
      DCY(I)=DYLE+(DYTE-DYLE)*PER
      DCZ(I)=DZLE+(DZTE-DZLE)*PER
      CALL NORM(DCX(I),DCY(I),DCZ(I))
   59 DENOM=DENOM + DOT(DCX(I),DCY(I),DCZ(I),CSX,CSY,CSZ)
      DS=CS2/DENOM
      DO 60 I=1,N1
      XVR(I+1,J)=XVR(I,J)+DS*DCX(I)
      YVR(I+1,J)=YVR(I,J)+DS*DCY(I)
  200 FORMAT(1H ,I3,1P3E16.6)
   60 ZVR(I+1,J)=ZVR(I,J)+DS*DCZ(I)
      DO 618 I=2,NSPACE
      SAI=0.0
      SA(1)=0.0
      DO 601 J=1,NBVVP
      DX= XVR(I,J+1)-XVR(I,J)
      DY= YVR(I,J+1)-YVR(I,J)
      DZ=ZVR(I,J+1)-ZVR(I,J)
      SAI= SAI + SQRT(DX**2 + DY**2 + DZ**2)
  601 SA(J+1)=SAI
      DO 603 J=1,NBP1
      XA(J)=XVR(I,J)
      YA(J)=YVR(I,J)
  603 ZA(J)=ZVR(I,J)
      SJ(1)=0.0
      DO 604 J=1,NBVVP
  604 SJ(J+1)=SJ(J)+SAI*(PXCV(J+1)-PXCV(J))
      CALL CODIM(SA,XA,NBP1,SJ,DCX,NBVVP)
      CALL CODIM(SA,YA,NBP1,SJ,DCY,NBVVP)
      CALL CODIM(SA,ZA,NBP1,SJ,DCZ,NBVVP)
      DO 618 J=2,NBVVP
      XVR(I,J)=DCX(J)
      YVR(I,J)=DCY(J)
  618 ZVR(I,J)=DCZ(J)
C
```

```
65    CONTINUE
      DO 70 I=1,NTVVP
      IF(I.NE.1) GO TO 68
      IF(ROOT) GO TO 66
      CH = XJ(NBP1)-XJ(1)
      GO TO 70
66    CH=0.0
      DO 67 J=1,NBVVP
      DX=XVR(I,J)-XVR(I,J+1)
      DY=YVR(I,J)-YVR(I,J+1)
      DZ=ZVR(I,J)-ZVR(I,J+1)
67    CH=CH+SQRT(DX**2+DY**2+DZ**2)
      GO TO 70
68    IF(ROOT.AND.I.LE.NLROOT) GO TO 66
      CH=CHORD(I-1)+(PETV(I)-PETV(I-1))*(CHTIP-CHORD(I-1))/
     1  (1.0-PETV(I-1))
70    CHORD(I)=CH
      CHORD(NTVVP+1)=CHTIP
      NTV1=NTVVP+1
        WRITE(6,200)(I,CHORD(I),PETV(I),CHTIP,I=1,NTV1)
      NCO=0
      NCOO=0
      IF(NBODS.EQ.0.AND.NPAN.EQ.1) NCT=0
      NCP=NCT
      IF(NPAN.GT.1) NCP=MP1
      IF(.NOT.ROOT) GO TO 85
COMPUTE VORTEX AND CONTROL POINTS IN ROOT REGION.
      LT=1
      DO 80 I=1,NSPACE
      KLT=.FALSE.
      LTCP=CPLT(LT)
      IF(I.NE.LTCP) GO TO 71
      KLT=.TRUE.
      LT=LT+1
71    CONTINUE
      LN=1
      DO 75 J=1,NBVVP
      KLN=.FALSE.
      LNCP=CPLN(LN)
```


C-5

```
      IF(J.NE.LNCP) GO TO 72
      KLN=.TRUE.
      LN=LN+1
72    CONTINUE
      DX1=XVR(I,J+1)-XVR(I,J)
      DY1=YVR(I,J+1)-YVR(I,J)
      DZ1=ZVR(I,J+1)-ZVR(I,J)
      DX2=XVR(I+1,J+1)-XVR(I+1,J)
      DY2=YVR(I+1,J+1)-YVR(I+1,J)
      DZ2=ZVR(I+1,J+1)-ZVR(I+1,J)
      NCOO=NCOO+1
      PL1=SQRT(DX1**2+DY1**2+DZ1**2)
      PL2=SQRT(DX2**2+DY2**2+DZ2**2)
      PLL(NCOO)=0.5*(PL1+PL2)
      DX11=XVR(I+1,J)-XVR(I,J)
      DY11=YVR(I+1,J)-YVR(I,J)
      DZ11=ZVR(I+1,J)-ZVR(I,J)
      DX21=XVR(I+1,J+1)-XVR(I,J+1)
      DY21=YVR(I+1,J+1)-YVR(I,J+1)
      DZ21=ZVR(I+1,J+1)-ZVR(I,J+1)
      PL1=SQRT(DX11**2+DY11**2+DZ11**2)
      PL2=SQRT(DX21**2+DY21**2+DZ21**2)
      PLT(NCOO)=0.5*(PL1+PL2)
      T1=DX11/SQRT(DY11**2+DZ11**2)
      T2=DX21/SQRT(DY21**2+DZ21**2)
      FIRST=.FALSE.
      R1=0.25
722   NCO=NCO+1
      XSS(NCO)=XPO+0.5*(XVR(I,J)+R1*DX1+XVR(I+1,J)+R1*DX2)
      YSS(NCO)=YPO+0.5*(YVR(I,J)+R1*DY1+YVR(I+1,J)+R1*DY2)
      ZSS(NCO)=ZPO+0.5*(ZVR(I,J)+R1*DZ1+ZVR(I+1,J)+R1*DZ2)
      TS(NCO)=T1+R1*(T2-T1)
      DYS(NCO)=DY11+R1*(DY21-DY11)
      DZS(NCO)=DZ11+R1*(DZ21-DZ11)
      IF(FIRST) GO TO 724
      FIRST=.TRUE.
      R1=0.75
      GO TO 722
724   CONTINUE
```

```
      IF(.NOT.KLT) GO TO 73
      IF(.NOT.KLN) GO TO 73
      NCP=NCP+1
      XCR(NCP)=0.5*(XVR(I,J)+0.75*DX1+XVR(I+1,J)+0.75*DX2) +XP0
      YCR(NCP)=0.5*(YVR(I,J)+0.75*DY1+YVR(I+1,J)+0.75*DY2) +YP0
      ZCR(NCP)=0.5*(ZVR(I,J)+0.75*DZ1+ZVR(I+1,J)+0.75*DZ2) +ZP0
   73 CONTINUE
      XVR(I,J)=XVR(I,J)+0.25*DX1 +XP0
      YVR(I,J)=YVR(I,J)+0.25*DY1 +YP0
      ZVR(I,J)=ZVR(I,J)+0.25*DZ1 +ZP0
      IF(I.NE.NSPACE) GO TO 75
      XVR(I+1,J)=XVR(I+1,J)+0.25*DX2 +XP0
      YVR(I+1,J)=YVR(I+1,J)+0.25*DY2 +YP0
      ZVR(I+1,J)=ZVR(I+1,J)+0.25*DZ2 +ZP0
   75 CONTINUE
      LETV=LETV+1
      ETV(LETV)=0.5*(PETV(I+1)+PETV(I))
      IF(.NOT.KLT) GO TO 80
      LETC=LETC+1
      ETC(LETC)=ETV(LETV)
   80 CONTINUE
      DO 82 I=1,NLROOT
      XVR(I,NBP1)=XVR(I,NBP1)+XP0
      YVR(I,NBP1)=YVR(I,NBP1)+YP0
   82 ZVR(I,NBP1)=ZVR(I,NBP1)+ZP0
      NCR=NCOO
C
   85 IF(.NOT.OUTER) GO TO 120
      DO 90 I=1,NBVVP
   90 XCVO(I)=0.75*PXCV(I)+0.25*PXCV(I+1)
      XCVO(NBVVP+1)=PXCV(NBVVP+1)
      DO 92 I=1,LNCPP
      J=CPLN(I)
   92 XCCO(I)=0.25*PXCV(J)+0.75*PXCV(J+1)
C
   95 CONTINUE
      NTVOUT=NTVVP-NSPACE
COMPUTE VORTEX POINTS AND CONTROL POINTS FOR OUTER PANEL.
C  SPACE CPLT ARRAY OUT OF ROOT REGION.
```

```
      DO 96 I=1,LTCPP
      IC=I
      LTCP=CPLT(I)
      IF(LTCP.GT.NSPACE) GO TO 97
96    CONTINUE
97    CONTINUE
      DO 110 I=1,NTVOUT
      INS=I+NSPACE
      ET1=PETV(INS)
      ET2=PETV(INS+1)
      ET=0.5*(ET1+ET2)
      CH1=CHORD(INS)
      CH2=CHORD(INS+1)
      CHAVG=0.5*(CH1+CH2)
      DO 98 L=1,NTIP
      IF(ET.LE.ETLE(L+1).AND.ET.GT.ETLE(L)) GO TO 99
98    CONTINUE
99    DELET=ETLE(L+1)-ETLE(L)
      RATIO=(ET-ETLE(L))/DELET
      XLE1=XLE(L)+(XLE(L+1)-XLE(L))*RATIO
      YLE1=YLE(L)+(YLE(L+1)-YLE(L))*RATIO
      ZLE1=ZLE(L)+(ZLE(L+1)-ZLE(L))*RATIO
      R1=(ET1-ETLE(L))/DELET
      R2=(ET2-ETLE(L))/DELET
      DX=XLE(L+1)-XLE(L)
      DY=YLE(L+1)-YLE(L)
      DZ=ZLE(L+1)-ZLE(L)
      XL1=XLE(L)+DX*R1
      YL1=YLE(L)+DY*R1
      ZL1=ZLE(L)+DZ*R1
      XL2=XLE(L)+DX*R2
      YL2=YLE(L)+DY*R2
      ZL2=ZLE(L)+DZ*R2
      DX=XTE(L+1)-XTE(L)
      DY=YTE(L+1)-YTE(L)
      DZ=ZTE(L+1)-ZTE(L)
      XT1=XTE(L)+DX*R1
      YT1=YTE(L)+DY*R1
      ZT1=ZTE(L)+DZ*R1
```

```
      XT2=XTE(L)+DX*R2
      YT2=YTE(L)+DY*R2
      ZT2=ZTE(L)+DZ*R2
C
C   SAVE VORTEX POINTS ON TRAILING EDGE.  (THOSE THAT LIE OUTSIDE ROOT.)
      XVS(I)=XT1+XP0
      YVS(I)=YT1+YP0
      ZVS(I)=ZT1+ZP0
      IF(I.NE.NTVOUT) GO TO 990
      XVS(I+1)=XT2+XP0
      YVS(I+1)=YT2+YP0
      ZVS(I+1)=ZT2+ZP0
  990 CONTINUE
      DXL=XL2-XL1
      DYL=YL2-YL1
      DZL=ZL2-ZL1
      YSUBV(I)=0.5*SQRT(DYL**2+DZL**2)
      DXT=XT2-XT1
      DYT=YT2-YT1
      DZT=ZT2-ZT1
      TTLE=DXL/SQRT(DYL**2+DZL**2)
      TTTE=DXT/SQRT(DYT**2+DZT**2)
      DO 100 J=1,NBVVP
      NCOO=NCOO+1
      PD=PXCV(J+1)-PXCV(J)
      PLL(NCOO)=0.5*PD*(CH1+CH2)
      FIRST=.FALSE.
      XCTERM=XCVO(J)
      XVO(I,J)=XP0+XLE1+XCTERM*CHAVG
      YVO(I,J)=YP0+YLE1
      ZVO(I,J)=ZP0+ZLE1
  991 NCO=NCO+1
      IF(FIRST) GO TO 992
      XSS(NCO)=XVO(I,J)
      YSS(NCO)=YVO(I,J)
      ZSS(NCO)=ZVO(I,J)
      GO TO 993
  992 XSS(NCO)=XP0+XLE1+XCTERM*CHAVG
      YSS(NCO)=YP0+YLE1
```

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      ZSS(NCO)=ZP0+ZLE1
  993 DYS(NCO)=DYL
      DZS(NCO)=DZL
      TS(NCO)=TTLE+(TTTE-TTLE)*XCTERM
      IF(FIRST) GO TO 100
      FIRST=.TRUE.
      XCTERM=0.25*PXCV(J)+0.75*PXCV(J+1)
      GO TO 991
  100 CONTINUE
      LETV=LETV+1
      ETV(LETV)=ET
      LTCP=CPLT(IC)
      IF(I+NSPACE.NE.LTCP) GO TO 110
      IC=IC+1
      LETC=LETC+1
      ETC(LETC)=ET
      DO 105 I1=1,LNCPP
      NCP=NCP+1
      XCR(NCP)=XP0+XLE1+XCCO(I1)*CHAVG
      YCR(NCP)=YP0+YLE1
  105 ZCR(NCP)=ZP0+ZLE1
  110 CONTINUE
  120 CONTINUE
      MP1=NCP
C
C
      CALL PANFNC(NCT,ETV,ETC)
      NCT=NCP
C
      IF(NU+NW.EQ.0) GO TO 1111
CALL SUBROUTINES REQUIRED TO SET UP PANEL CONSTRAINT MATRIX.
      LOC1=(NPAN-1)*1000+1
      LOC2=LOC1+400
      LOC3=LOC2+400
C   ARRAY XA IS USED FOR SCRATCH.
      CALL TXOC(NU,XA,0,XA,0,NU,NPVVP,
     1   SAVEC(LOC1),SAVEC(LOC2),NSPACE,XA,XCVO,XCCO)
      CALL MATETA(SAVEC(LOC3),NU,NTVVP,STARAY,ETV,SLCF)
 1111 CONTINUE
```

```
WRITE(18) BODYWR
RETURN
END
```

```
      SUBROUTINE PERGM(PERIM,ARRAY)
      DIMENSION PERIM(1),ARRAY(1)
      COMMON/PANEL/ NPAN,IPSYM,IAC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NNCTXC,NNCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
      LOGICAL ROOT,OUTER
      COMMON DA(5000)
      EQUIVALENCE (DA(3432),XPO),   (DA(3433),YPO),   (DA(3434),ZPO)
      NP=NTIP
      N4=4*NTIP
      DO 10 I=1,N4
   10 ARRAY(I)=PERIM(I)
      N=2*NTIP-NPERPT
      DO 20 I=1,N
      NP=NP+1
      N1=4*(NTIP+I-1)+1
      N2=4*(NTIP-I)+1
      ARRAY(N1)=ARRAY(N2)+ARRAY(N2+3)
      ARRAY(N1+1)=ARRAY(N2+1)
      ARRAY(N1+2)=ARRAY(N2+2)
   20 ARRAY(N1+3)=0.0
      IF(ROOT) CALL ATTACH(XVT,YVT,ZVT,50,1)
      NNATT=MP3
      N=NTIP-N
      IF(N.EQ.0) GO TO 35
      DO 30 I=1,N
      NP=NP+1
      N1=N1+4
      N2=N4+(I-1)*4+1
      ARRAY(N1)=PERIM(N2)
      ARRAY(N1+1)=PERIM(N2+1)
      ARRAY(N1+2)=PERIM(N2+2)
      ARRAY(N1+3)=0.0
```

```
      IF(I.NE.N) GO TO 30
      IF(.NOT.ROOT) GO TO 30
      ARRAY(N1)=ARRAY(N1)+XPO
      CALL CODIM(XVT,YVT,NNATT,ARRAY(N1),ARRAY(N1+1),1)
      CALL CODIM(XVT,ZVT,NNATT,ARRAY(N1),ARRAY(N1+2),1)
      ARRAY(N1)=ARRAY(N1)-XPO
      ARRAY(N1+1)=ARRAY(N1+1)-YPO
      ARRAY(N1+2)=ARRAY(N1+2)-ZPO
30    CONTINUE
35    NPERPT = NP
      NP4=4*NP
      WRITE(6,100) NTIP
      WRITE(6,100) NSPACE
      WRITE(6,100) NPERPT,(I,ARRAY(I),I=1,NP4)
100   FORMAT(*0PANEL PERIMETER*/I4/(I4,1PE15.5))
      RETURN
      END
```

```
      SUBROUTINE MATETA( TETA, NW, NETA, STARAY, ETA, SLCF)
C     SUBROUTINE TO BUILD T(ETA) MATRIX
C
C     TETA   -  T(ETA MATRIX
C     NW     -  NUMBER OF SPECIAL LATERAL CONSTRAINT FUNCTIONS
C     NETA   -  NUMBER OF ETA STATIONS
C     STARAV -  LIST OF LATERAL CONSTRAINT FUNCTIONS
C     ETA    -  ARRAY OF ETAS
C     SLCF   -  LIST OF SPECIAL LATERAL CONSTRAINT FUNCTIONS
C
      DIMENSION  TETA(NETA, NW), STARAY(NW), ETA(NETA), SLCF(1)
      REAL M
      INTEGER PTR
      EQUIVALENCE (RI,ETAI), (RO,ETAO)
      DATA PIO2/1.5707 96326 79489/
      DO 20 INW=1, NW
      IF(STARAY(INW)-100.) 1, 21, 21
    1      DO 10 IETA=1, NETA
   10      TETA(IETA, INW) = SQRT( 1.C - ETA(IETA)*ETA(IETA)) * ETA(IETA
     1    )**STARAY(INW)
   20 CONTINUE
      RETURN
   21 NWI = INW
      DO 80 INW=NWI, NW
      PTR = STARAY(INW) -. 4779
C     GET INFO OUT OF SPECIAL LATERAL CONSTRAINT FUNCTION TABLE
      ETAB = SLCF(PTR+1)
      RI = SLCF(PTR+2)
      RO = SLCF(PTR+3)
      IF( SLCF(PTR))22,31,22
   22 THI = ACOS(ETAI)
      THO = ACOS(ETAO)
      DO 30 IETA=1, NETA
      TH = ACOS(ETA(IETA))
   30 TETA(IETA, INW) = M(TH, THI)  -  M(TH, THO)
      GOTO 80
   31 A21 = 1 - RO
      A5  = 1 / RI
      A9  = 1 - ETAB
```

```
      A35 = (A9 + RI)*A5
      A35B= ACOS(ETAB - RI)
      THB = ACOS(ETAB)
      IF(A21-ABS(ETAB))32,32,41
   32 A50 = A9 * A5 + 1.0
      DO 40 IETA=1, NETA
      TH = ACOS(ETA(IETA))
   40 TETA(IETA, INW) = A35 * P(TH,A35B)-A50       *P(TH,THB)
      GOTO 80
   41 A8  = RI+RO
      A11 = RI*RO
      A18 = A8/A11
      A22 = A21-ETAB
      A 51 = A18 * A 9
      A27B=ACOS(ETAB+RO)
      IF(RI-ABS(ETAB))42,42,51
   42 A50 = A22 * A5
      A26B= ACOS(ETAB-RO)
      DO 50 IETA=1, NETA
      TH = COS(ETA(IETA))
   50 TETA(IETA,INW) = A50    *P(TH,A27B)+A35*P(TH, A35B) - P(TH, THB)*A52
     11
      GOTO 80
   51 A 50 = A 22 * ( A 18 - A 5)
      A 52 = 1.0 - ETAB * A5
      DO 60 IETA=1, NETA
      TH = COS(ETA(IETA))
   60 TETA(IETA,INW)=P(TH,A27B)* A 50        + A52 * M(TH,PIO2) +
     1 A5*P(TH,PIO2)-P(TH,THB)* A51
   80 CONTINUE
      RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUPROUTINE    TXOC(    NU,THETAF,NF, THETAK, NK, NCOL, NXOC,
     1SCRTCH, SCRCH,NROOT,          THET,XCI, XCJ)
C     NU      -  NU FOR THIS PANEL
C     THETAF  -  ARRAY OF THETAFS
C     THETAK  -  ARRAY OF THETAKS
C     NF      -  NUMBER OF THETAF S
C     NK      -  NUMBER OF THETAK S
C     NCOL    -  NU+NF+NK
C     NXOC    -  NUMBER OF X OVER C S
C     SCRTCH  -  T(XOC) RESULT
C     SCRCH   -  SCRTCH ARRAY IF NROOT ' 0  MATRIX  FOR USE IN ROOT SECTI
C     NROOT    -  NUMBER OF ELEMENTS IN THE ROOT SECTION
C     THET    -  SCRTCH MATRIX THAT THETAS ARE PUT IN TO
C     XCI     -  ARRAY OF X OVER C S 1/4 PANEL
C     XCJ     -  ARRAY OF X OVER C S 3/4 PANEL
C
C
      DIMENSION                                THETAF(NF), THETAK(NK),
     1SCRTCH(NXOC,NCOL), SCRCH(NXOC,NCOL),                 THET(1)
     3, XCI(1), XCJ(1)
      CALL FILLNU(SCRTCH(1,1), NXOC, NXOC*NU, XCI, THET)
      CALL FILNFK(SCRTCH(1, NU+1), NF, NXOC, THETAF,.T., THET)
        CALL FILNFK(SCRTCH(1, NU+NF+1),NK, NXOC, THETAK, .F., THET)
      CALL EFIL(XCJ, XCI, NXOC, SCRCH)
       CALL GELG(SCRTCH,SCRCH, NXOC, NCOL, 1.0E-12, I)
      IF( I ) 11, 19, 11
   11 PRINT 16, I
   16 FORMAT(*0GELG ERROR*, I3)
   19 IF(NROOT.LT.1) RETURN
      DO 20 IUFK=1, NCOL
      TEMP = 0.0
      DO 20 IXOC=1, NXOC
      TEMP = TEMP + SCRTCH(IXOC, IUFK)
   20 SCRCH(IXOC, IUFK) = TEMP
      RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUBROUTINE PANFNC(KL,ETV,ETC)
C  IWC=WING-CONTOUR INDICATOR
C     =0 FOR LOCAL ANGLES OF ATTACK, ALPHA, GIVEN.
C     =1 FOR DEFLECTIONS, Z/C  GIVEN.
      DIMENSION ETV(1),ETC(1)
      COMMON DA(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1  ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2  ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4  ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5   ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6   ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1              ,XN(1320),YN(1320),ZN(1320)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      EQUIVALENCE(ALPHA,DA(3660)),  (TWIST,DA(4100)),  (XA,DA(3601))
     1          ,(ETAI,DA(3631)),  (FNXA,DA(3600)),  (FNETAI,DA(3630))
     2          ,(FNXC,DA(3437)),  (FNEC,DA(3442))
C   DUMS(1504) PRESERVES /SCRAT/ FROM PANEL2.
      COMMON/SCRAT/ DUMS(1504),ALP(400),TW(20)
     1   ,DYI(400),DZI(400),THK(820),XC(50),XCTHK(51)
     2   ,DUMX(101),DUMY(2020),DUMZ(101),XP(40),ZP(40),CH(21),CHC(20)
      DIMENSION ALPHA(440),TWIST(30),XA(29),ETAI(29)
      EQUIVALENCE(DA(4600),PXCV),  (DA(4131),XTHICK),  (DA(4130),FNXTHK)
     1          ,(DA(4161),ETATHK),(DA(4160),FETTHK),  (DA(4190),THICK)
      DIMENSION PXCV(40),XTHICK(29),ETATHK(29),THICK(410)
      EQUIVALENCE(DA(4640),PETV)
      COMMON     /COMPRS/   BETAM
      DIMENSION PETV(40)
      COMMON/SLOPE/SIGMAP(500),  DZDXT(500),DZDXC(500),TANPI(500)
      EQUIVALENCE(MP5,LSIGMA,LS)
      EQUIVALENCE(DA(7),XCG),(DA(8),YCG),(DA(9),ZCG)
     1  ,(DA(11),BETA),(DA(12),PSTAR),(DA(13),QSTAR),(DA(14),RSTAR)
      EQUIVALENCE(DA(4720),CPLT),(DA(4680),CPLN)
```

```
      DIMENSION CPLT(40),CPLN(40)
      EQUIVALENCE(B,DA(3423)),(REFA,DA(3421)),(CHRM,DA(3422))
     1  ,(SPCF,DA(4880))
      DIMENSION SPCF(40)
      COMMON/NUMBER/ NLOCK(50)
      IF(NPAN.EQ.1) MP5=0
C    MP5 WILL BE TOTAL NO. OF SOURCE POINTS.
C
      NXA=FNXA
      NETAI=FNETAI
      NXC=FNXC
      NEC=FNEC
      NXC2=2*NXC
      NXTHK=FNXTHK
      NETTHK=FETTHK
      WRITE(6,100)(ETV(I),I=1,4)
      WRITE(6,100)(ETC(I),I=1,4)
100   FORMAT(*0PANFNC*/(1P8E15.6))
      IN0=100
      IN1=IN0+1
      DO 1 I=1,IN1
1     DUMX(I)=(I-1)*0.01
      NT1=NTVVP+1
      K=0
      DX=0.01
      DO 10 I=1,LNCPP
      K=K+1
      XP(K)=XCCO(I)-DX
      K=K+1
10    XP(K)=XCCO(I)+DX
      IF(NXA.NE.0) GO TO 33
      N1=LNCPP*NEC
      DO 31 I=1,N1
31    ALP(I)=0.0
      DO 32 I=1,NEC
32    TW(I)=0.0
      GO TO 34
33    CONTINUE
      WRITE(6,100)(XP(I),I=1,20)
```

```
      DO 2 J=1,NETAI
      J1=(J-1)*NXA+1
      J2=(J-1)*IN1+1
      CALL CODIM(XA,ALPHA(J1),NXA,DUMX,DUMY(J2),IN1)
      IF(IWC.EQ.1) GO TO 2
      CALL QTFG(DUMX,DUMY(J2),DUMY(J2),IN1)
2     CONTINUE
      WRITE(6,100)(DUMY(I),I=1,202)
      CALL XLINE(PETV,CHORD,NT1     ,ETAI,CH,NETAI)
      CALL XLINE(PETV,CHORD,NT1     ,ETC,CHC,LTCPP)
      WRITE(6,100) (CHORD(J),J=1,NT1)
      WRITE(6,100)(CH(J),J=1,NETAI)
      WRITE(6,100)(CHC(J),J=1,LTCPP)
      WRITE(6,100)(ETAI(J),J=1,NETAI)
      WRITE(6,100)(DUMX(J),J=1,IN1)
      K1=0
      L=2
      DO 6 I=1,NEC
3     IF(ETC(I).GT.ETAI(L)) GO TO 5
      LM1=L-1
      D1=ETC(I)-ETAI(LM1)
      D2=ETAI(L)-ETAI(LM1)
      R=D1/D2
      L1=(LM1-1)*IN1
      L2=L1+IN1
      K=0
      DO 4 J=1,IN1
      L1=L1+1
      L2=L2+1
      ZZZ =    CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
      K=K+1
4     DUMZ(K) = ZZZ/CHC(I)
      LN2=2*LNCPP
      WRITE(6,100) CH(LM1),CH(L),R
      CALL CODIM(DUMX,DUMZ,IN1,XP,ZP,LN2)
      WRITE(6,100)(DUMZ(J),J=1,IN1)
      WRITE(6,101) LN2
      WRITE(6,100)(XP(J),ZP(J),J=1,LN2)
      DO 20 J=1,LNCPP
```

```
      K1=K1+1
      J2=2*J
20    ALP(K1)=(ZP(J2)-ZP(J2-1))*0.5/DX
      WRITE(6,100)(ALP(J),J=1,K1)
      GO TO 6
5     L=L+1
      GO TO 3
6     CONTINUE
      WRITE(6,100) (ETAI(I),TWIST(I),I=1,NETAI)
      CALL CODIM(ETAI,TWIST,NETAI,ETC,TW,NEC)
34    CONTINUE
      L=0
      DO 9 J=1,NXC
      L=L+1
      XCTHK(L)=PXCV(J)
      XC(L)=0.75*PXCV(J)+0.25*PXCV(J+1)
      L=L+1
      XCTHK(L)=0.5*(PXCV(J)+PXCV(J+1))
9     XC(L)=0.25*PXCV(J)+0.75*PXCV(J+1)
      XCTHK(L+1)=PXCV(NXC+1)
      CALL FINDF(XC,NXC2, ETV,NTVVP,DYS,XCCO,LNCPP,ETC,NEC,DYI)
      WRITE(6,100)(DYI(I),I=1,80)
      CALL FINDF(XC,NXC2, ETV,NTVVP,DZS,XCCO,LNCPP,ETC,NEC,DZI)
      WRITE(6,100)(DZI(I),I=1,80)
      CALL XLINE(PETV,CHORD,NT1,ETV,CHC,NTVVP)
      IF(NXTHK.NE.0) GO TO 8
      NT=NTVVP*NXC2
      DO 7 I=1,NT
      LSIGMA=LSIGMA+1
7     SIGMA(LSIGMA)=0.0
      GO TO 81
8     CONTINUE
      CALL XLINE(PETV,CHORD,NT1,ETATHK,CH,NETTHK)
      DO 102 J=1,NETTHK
      J1=(J-1)*NXTHK+1
      J2=(J-1)*IN1+1
102   CALL CODIM(XTHICK,THICK(J1),NXTHK,DUMX,D[illegible](J2),IN1)
      L=2
      DO 106 I=1,NTVVP
```

```
103   IF(ETV(I).GT.ETATHK(L)) GO TO 105
      LM1=L-1
      D1=ETV(I)-ETATHK(LM1)
      D2=ETATHK(L)-ETATHK(LM1)
      R=D1/D2
      L1=(LM1-1)*IN1
      L2=L1+IN1
      K=0
      DO 104 J=1,IN1
      L1=L1+1
      L2=L2+1
      TH = CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
      K=K+1
104   DUMZ(K) = TH/CHC(I)
      L3=(I-1)*(NXC2+1)+1
      CALL CODIM(DUMX,DUMZ,IN1,XCTHK,THK(L3),NXC2+1)
      GO TO 106
105   L=L+1
      GO TO 103
106   CONTINUE
      K1=0
      K=KL
      L2=1
      DO 110 I=1,NEC
      MT=(CPLT(I)-1.0)*(NXC2+1)+0.01
      DO 110 J=1,LNCPP
      M1=CPLN(J)*2.0+0.01
      M=MT+M1
      ST=(THK(M+1)-THK(M))/(XCTHK(M1+1)-XCTHK(M1))
      K=K+1
      TANP1(K) = 1.0 +TS(L2)**2 *BETAM**2
      L2=L2+2
      DZDXT(K)=ST
      K1=K1+1
      DZDXC(K)=ALP(K1)
110   CONTINUE
81    CONTINUE
      WRITE(6,100)(XC(I),I=1,NXC2)
      WRITE(6,100)(XCTHK(I),I=1,NXC2)
```

```
      WRITE(6,100)(XCCO(I),I=1,LNCPP)
      WRITE(6,100)(THK(I),I=1,30)
      K1=0
      K=KL
      DO 40 I=1,NEC
      TW1=TW(I)
      DO 40 J=1,LNCPP
      K=K+1
      K1=K1+1
      SQ1= SQRT(DYI(K1)**2 + DZI(K1)**2)
      YN(K) = -DZI(K1)/SQ1
      ZN(K) = +DYI(K1)/SQ1
C   XN = DZ/DX + TWIST , OR  ALPHA+TWIST
      TD=0.0
      XN(K)=(-ALP(K1)+TW1+TD)*BETAM
 101  FORMAT(1H ,3I5,1P5E15.5)
 40   CONTINUE
      IF(NXTHK.EQ.0.0) GO TO 23
C
C.....................................................................
C        TEMPORARY CBAR,BREF
      CBAR=1.0
      BREF=1.0
C
      L2=0
      DO 22 I=1,NTVVP
      DO 21 J=1,NXC2
      LS=LS+1
      L2=L2+1
      YY=YSS(L2)-YCG
      ZZ=ZSS(L2)-ZCG
      VX=1.0-2.0*(QSTAR*ZZ/CBAR - RSTAR*YY/BREF)
      PP = SQRT(1.0+TS(LS)**2)
      SIGMA(LS) = 2.0*CHC(I)*BETAM*(THK(L2+1)-THK(L2))* VX/PP
      AL=0.5*PLL((J+1)/2)
 21   SIGMAP(LS) = 0.5*BETAM*(THK(L2+1)+THK(L2))*VX*AL/(XC(J)*(1.0-XC(J)
     1)*PP)
 22   L2=L2+1
 23   CONTINUE
```

```
      WRITE(6,6001) (SIGMA(I),I=1,280)
 6001 FORMAT(*0SIGMA*/(1P10E13.4))
      WRITE(6,100)(SIGMAP(I),I=1,40)
      WRITE(6,6000) NLOOK
 6000 FORMAT(*0NLOOK IN PANFNC*/(10I5))
      CALL XLINE(ETLE,YLE,NTIP,PETV,DUMY,NT1)
      CALL XLINE(ETLE,ZLE,NTIP,PETV,DUMZ,NT1)
      BS=0.0
      DO 200 I=1,NTVVP
      DUMX(I)=SQRT((DUMY(I+1)-DUMY(I))**2+(DUMZ(I+1)-DUMZ(I))**2)
      BS=BS+DUMX(I)
  200 CONTINUE
      CALL XLINE(ETLE,YLE,NTIP,ETC,DUMY,LTCPP)
      CALL XLINE(ETLE,ZLE,NTIP,ETC,DUMZ,LTCPP)
      CALL XLINE(ETLE,XLE,NTIP,ETC,XP,LTCPP)
      WRITE(12) B,REFA,LTCPP,(ETC(I),I=1,50),LNCPP,XCCO,DYI,DZI,TW,CHC
     1    ,CHRM,(XP(I),DUMY(I),DUMZ(I),I=1,LTCPP),CPLT,SPCF,BS,DUMX
      RETURN
      END
```

```
              REAL FUNCTION  P(T,TS)
    1 S = SIN(.5*(TS-T))
      SD= SIN(.5*(TS+T))
      IF(S*SD) 4,3,4
    3 T = T + 1.0E-6
      GOTO 1
    4 C = COS(.5*(TS+T))
      CD= COS(.5*(TS-T))
      IF(C*CD) 8, 3, 8
    8 CTS = COS(TS)
      CT  = COS(T)
      CTSPCT = CTS + CT
      P = ((CTS-CT)*ALOG(ABS(S/SD))+    CTSPCT*CTSPCT*ALOG(ABS(C/CD))+
     1(4.0*TS*CTS-2.0*SIN(TS))*SIN(T)) /(6.2831 85307 17958*(1.0-CTS))
      IF(LEGVAR(P)) 3,9, 3
    9 RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      REAL FUNCTION M(TH,THS)
    1 S = SIN((THS-TH)*.5)
      SD= SIN((THS+TH)*.5)
      IF(S*SD) 3,2,3
    2 TH = TH + 1.0E-6
      GOTO 1
    3 C = COS((THS+TH)*.5)
      CD= COS((THS-TH)*.5)
      IF(C*CD) 4,2,4
    4 CTHS = COS(THS)
      CTH  = COS(TH)
      M= 3.1830 98861 83790E-1 *((CTHS -CTH)*ALOG(ABS(S/SD))+(CTHS+CTH)*
     1ALOG(ABS(C/CD))+2.0*THS*SIN(TH))
      IF(LEGVAR(M)) 2,5,2
    5 RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUBROUTINE GELG(R,A,M,N,EPS,IER)
C
C     ..................................................................
C
C        SUBROUTINE GELG
C
C        PURPOSE
C           TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.
C
C        USAGE
C           CALL GELG(R,A,M,N,EPS,IER)
C
C        DESCRIPTION OF PARAMETERS
C           R      - THE M BY N MATRIX OF RIGHT HAND SIDES.  (DESTROYED)
C                    ON RETURN R CONTAINS THE SOLUTION OF THE EQUATIONS.
C           A      - THE M BY M COEFFICIENT MATRIX.  (DESTROYED)
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
C           N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.
C           EPS    - AN INPUT CONSTANT WHICH IS USED AS RELATIVE
C                    TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C                    IER=0  - NO ERROR,
C                    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
C                             PIVOT ELEMENT AT ANY ELIMINATION STEP
C                             EQUAL TO 0,
C                    IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
C                             CANCE INDICATED AT ELIMINATION STEP K+1,
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX A.
C
C        REMARKS
C           INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
C           IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN
C           SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
C           THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
C           GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
C           ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -
C           INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
```

```
C           SCALED MATRIX A AND APPROPRIATE TOLERANCE
C           INTERPRETED THAT MATRIX A HAS THE RANK K.
C           GIVEN IN CASE M=1.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WI
C           COMPLETE PIVOTING.
C
C     ..................................................................
C
C
C
      DIMENSION A(1),R(1)
      IF(M)23,23,1
C
C     SEARCH FOR GREATEST ELEMENT IN MATRIX A
    1 IER=0
      PIV=0.
      MM=N*M
      NM=N*M
      DO 3 L=1,MM
      TB=ABS(A(L))
      IF(TB-PIV)3,3,2
    2 PIV=TB
      I=L
    3 CONTINUE
      TOL=EPS*PIV
C     A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).
C
C
C     START ELIMINATION LOOP
      LST=1
      DO 17 K=1,M
C
C     TEST ON SINGULARITY
      IF(PIV)23,23,4
```

```
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 PIVI=1./A(I)
      J=(I-1)/M
      I=I-J*M-K
      J=J+1-K
C     I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
C
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
      DO 8 L=K,NM,M
      LL=L+I
      TB=PIVI*R(LL)
      R(LL)=R(L)
    8 R(L)=TB
C
C     IS ELIMINATION TERMINATED
      IF(K-M)9,18,18
C
C     COLUMN INTERCHANGE IN MATRIX A
    9 LEND=LST+M-K
      IF(J)12,12,10
   10 II=J*M
      DO 11 L=LST,LEND
      TB=A(L)
      LL=L+II
      A(L)=A(LL)
   11 A(LL)=TB
C
C     ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
   12 DO 13 L=LST,MM,M
      LL=L+I
      TB=PIVI*A(LL)
      A(LL)=A(L)
   13 A(L)=TB
C
C     SAVE COLUMN INTERCHANGE INFORMATION
      A(LST)=J
C
```

```
C     ELEMENT REDUCTION AND NEXT PIVOT SEARCH
      PIV=0.
      LST=LST+1
      J=0
      DO 16 II=LST,LEND
      PIVI=-A(II)
      IST=II+M
      J=J+1
      DO 15 L=IST,MM,M
      LL=L-J
      A(L)=A(L)+PIVI*A(LL)
      TB=ABS(A(L))
      IF(TB-PIV)15,15,14
   14 PIV=TB
      I=L
   15 CONTINUE
      DO 16 L=K,NM,M
      LL=L+J
   16 R(LL)=R(LL)+PIVI*R(L)
   17 LST=LST+M
C     END OF ELIMINATION LOOP
C
C
C     BACK SUBSTITUTION AND BACK INTERCHANGE
   18 IF(M-1)23,22,19
   19 IST=MM+M
      LST=M+1
      DO 21 I=2,M
      II=LST-I
      IST=IST-LST
      L=IST-M
      L=A(L)+.5
      DO 21 J=II,NM,M
      TB=R(J)
      LL=J
      DO 20 K=IST,MM,M
      LL=LL+1
   20 TB=TB-A(K)*R(LL)
      K=J+L
```

```
      R(J)=R(K)
   21 R(K)=TB
   22 RETURN
C
C
C     ERROR RETURN
   23 IER=-1
      RETURN
      END
```

```
      SUBROUTINE QTFG(X,Y,Z,NDIM)
C
C
C     ..................................................................
C
C        SUBROUTINE QTFG
C
C        PURPOSE
C           TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN
C           GENERAL TABLE OF ARGUMENT AND FUNCTION VALUES.
C
C        USAGE
C           CALL QTFG (X,Y,Z,NDIM)
C
C        DESCRIPTION OF PARAMETERS
C           X      - THE INPUT VECTOR OF ARGUMENT VALUES.
C           Y      - THE INPUT VECTOR OF FUNCTION VALUES.
C           Z      - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE
C                    IDENTICAL WITH X OR Y.
C           NDIM   - THE DIMENSION OF VECTORS X,Y,Z.
C
C        REMARKS
C           NO ACTION IN CASE NDIM LESS THAN 1.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           BEGINNING WITH Z(1)=0, EVALUATION OF VECTOR Z IS DONE BY
C           MEANS OF TRAPEZOIDAL RULE (SECOND ORDER FORMULA).
C           FOR REFERENCE, SEE
C           F.B.HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS,
C           MCGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP.75.
C
C     ..................................................................
C
C
      DIMENSION X(1),Y(1),Z(1)
C
```

```
      SUM2=0.
      IF(NDIM-1)4,3,1
C
C     INTEGRATION LOOP
    1 DO 2 I=2,NDIM
      SUM1=SUM2
      SUM2=SUM2+.5*(X(I)-X(I-1))*(Y(I)+Y(I-1))
    2 Z(I-1)=SUM1
    3 Z(NDIM)=SUM2
    4 RETURN
      END
```

```
      SUBROUTINE EFIL( XCJ, XCI, NI,      E)
C
C     FILL E MATRIX
C
C     XCI   -  .25 PANEL
C     XCJ   -  .75 PANEL
C
      DIMENSION E(NI,NI), XCI(1), XCJ(1)
      NJJ=NI-1
      DO 10 J=1, NJJ
      DO 10 I=1, NI
   10 E(J,I) = 1.0/(XCJ(J)-XCI(I))
      DO 20 I=1, NI
   20 E(NI,I) = 1.0
      RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUBROUTINE FILNFK(W   , NF , NJ, THETA, FLAG, THET)
C     SUBROUTINE TO FILL THE NF OR NK  SECTION OF THE W MATRIX
C
C     W      -  NF MATRIX
C     NF     -  NUMBER OF COL IN W
C     NJ     -  NUMBER OF ROWS IN NFM
C     THETA  -  ARRAY OF UNIQUE THETA FSOR KS
C     FLAG   -  LOGICAL VARIABLE T(WORK ON NF) F(WORK ON NK)
C     THET   -  ARRAY OF THETAS FROM FILLNU
      DIMENSION W(NJ,NF), THETA(1), THET(1)
      LOGICAL FLAG
      DATA PI/ 3.1415 92653 58979/
      IF(NF.LE.0) RETURN
      I = 1
      NJJ = NJ - 1
      DO 400 M=1, NF
      TH = THETA(M)
      THMPI = TH - PI
      DO 300 J=1, NJJ
      IF((FLAG.AND.(THET (J).GE.TH)).OR.((.NOT.FLAG).AND.(THET (J).GT.TH
     1))) GOTO 221
      W(J,I) = THMPI
      GOTO 300
  221 W(J,I) = TH
  300 CONTINUE
      W(NJ,I) = .5 * SIN(TH)
  400 I=I+1
      RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUBROUTINE  FILLNU(NUM, NJ, NUE, XC, SCRTCH)                     2  14160
C     SUBROUTIN TO FILL THE NU SECTION OF THE W MATRIX                 2  14170
C                                                                      2  14180
C     NUM    -  NU MATRIX                                              2  14190
C     NJ     -  NUMBER OF ROWS OF NUM                                  2  14200
C     NUE    -  NUMBER OF ELEMENTS IN          NUM (NU*NJ)             2  14210
C     XC     -  ARRAY OF X OVER C S                                    2  14220
C     SCRTCH-  ARRAY OF THETAS                                         2  14230
C     THIS ROUTIN SHOULD BE CALLED ONCE(FOR THE ROUTINE WITH THE LARGEST2 14240
C     NU)                                                              2  14250
C                                                                      2  14260
      REAL NUM(NUE), XC(NJ) , SCRTCH(NJ)                               2  14270
      NU = NUE/NJ                                                      2  14280
      NJJ = NJ - 1                                                     2  14290
      DO 10  I=1, NJJ                                                  2  14300
      NUM(I) = 1.0                                                     2  14310
   10 SCRTCH(I) = A COS(1.0 - 2.0*XC(I))                               2  14320
      NUM(NJ) = .5                                                     2  14330
      IF( NUE .LE. NJ) RETURN                                          2  14340
      NVE = NUE -1                                                     2  14350
      RI =0.                                                           2  14360
      DO 30 J=NJ, NVE, NJ                                              2  14370
      RI = RI + 1.0                                                    2  14380
      DO 40 L=1, NJJ                                                   2  14390
   40 NUM(J+L) = -COS((RI ) * SCRTCH(L))                               2  14400
      IF(J-NJ)21,21,22                                                 2  14410
   21 NUM(J+NJ) = .25                                                  2  14420
      GOTO 30                                                          2  14430
   22 NUM(J+NJ) = .0                                                   2  14440
   30 CONTINUE                                                         2  14450
      RETURN                                                           2  14460
C     BLAINE D. GAITHER 9/72                                           2  14470
      END                                                              2  14480
```

```
      FUNCTION TANDEL(ETA, XOC)
C     COMMON STATEMENT THAT PUTS FLP(1,1) ON THE FLAPS
C     THIS FUNCTION RETURNS TAN(DELTA) IF A CP FALLS ON A FLAP,
C           OTHERWISE ZERO
      DIMENSION  FLP(8,1)
      NF     = 1
      TANDEL = 0.0
    7 IF(FLP(1,NF))2,1,2
    2 IF(FLP(3,NF)-ETA) 3,3,4
    3 IF(FLP(4,NF)-ETA) 4,5,5
    5 T       = FLP(5,NF) + (FLP(6,NF)-FLP(5,NF))*(ETA - FLP(3,NF)) /
     1          (FLP(4,NF) - FLP(3,NF))
      IF((XOC - T)* FLP(1,NF)) 6, 6, 4
    4 NF =  NF +1
      GOTO  7
    6 TANDEL = TAN( FLP(2,NF) * 0.0174 53292 51994 32957)
    1 RETURN
C     BLAINE D. GAITHER   OCT 72
      END
```

```
      SUBROUTINE FINDF(X,NX,E,NE,F,XC,NXC,EC,NEC,FC)
C
C  PROGRAM TO INTERPOLATE FOR A FUNCTION AT A GIVEN (X/C,ETA) POINT.
C  GRID UPON WHICH INTERPOLATION IS MADE IS GIVEN BY
C   X = GIVEN VALUES OF X/C.    NX=NUMBER OF X VALUES.
C   E = GIVEN VALUES OF ETA.    NE=NUMBER OF ETA VALUES.
C   THE INPUT GRID IS DESCRIBED BY ONE SET OF X/C VALUES
C     AND ONE SET OF ETA VALUES.
C   F = GIVEN VALUES OF A FUNCTION OF (X,E), GIVEN FIRST LONGITUDINALLY
C    AND THEN SPANWISE. (EX. IF NX=10,NE=5 , F(42) IS AT X(2) AND E(4).)
C     (NUMBER OF F VALUES TO BE AVAILABLE MUST EQUAL NX*NE.)
C
C  XC = XC ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
C  NXC = NUMBER OF XC GIVEN, TO MAXIMUM OF 20. (DIMENSION OF F1,F2 = 20)
C  EC = ETA ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
C  NEC = NUMBER OF EC GIVEN.
C  FC = INTERPOLATED FUNCTION VALUES, GIVEN CHORDWISE THEN SPANWISE.
C   NUMBER OF OUTPUT VALUES OF FC = NXC*NEC
C
C  INTERPOLATION IN THE X DIRECTION IS MADE BY SUBROUTINE CODIM.
C  INTERPOLATION IN THE ETA DIRECTION IS LINEAR.
C
      DIMENSION X(1),E(1),XC(1),EC(1),F(1),FC(1)
      DIMENSION F1(20),F2(20)
      NE1=NE-1
      L=0
      DO 50 J=1,NEC
      E1=EC(J)
      DO 5 IE=1,NE1
      IEL=IE
      IF(E1.GT.E(IE+1)) GO TO 5
      ETA1=E(IEL)
      ETA2=E(IEL+1)
      GO TO 6
    5 CONTINUE
    6 CONTINUE
      DELET=ETA2-ETA1
      D = E1-ETA1
      I1=(IEL-1)*NX+1
```

```
      I2=I1+NX
      CALL CODIM(X,F(I2),NX,XC,F2,NXC)
      CALL CODIM(X,F(I1),NX,XC,F1,NXC)
      DO 50 I=1,NXC
      L=L+1
      FC(L)=F1(I)+D*(F2(I)-F1(I))/DELET
50    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XLINE(X,Y,N,X1,Y1,N1)
C     THIS ROUTINE DOES LINEAR INTERPOLATION AT N1 (X1,Y1) POINTS
C      ON SEGMENTS BETWEEN N (X,Y) POINTS (GIVEN).
C     ALL X1 MUST BE WITHIN RANGE OF VALUES OF X.
C     VALUES OF X MUST BE ALGEBRAICALLY ASCENDING.
C
      DIMENSION X(1),Y(1),X1(1),Y1(1)
      L2=2
      DO 10 I=1,N1
 2    IF(X1(I).GT.X(L2)) GO TO 5
 3    L1=L2-1
      DX=X(L2)-X(L1)
      D1=X1(I)-X(L1)
      DY=Y(L2)-Y(L1)
      Y1(I)=D1*DY/DX+Y(L1)
      GO TO 10
 5    IF(L2.GE.N) GO TO 3
      L2=L2+1
      GO TO 2
 10   CONTINUE
      RETURN
      END
```

```
      PROGRAM INFLM
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2  ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON     /COMPRS/   BETAM
      COMMON/BODY/BB(31000)
      DIMENSION BODYRD(21000)
      COMMON/COMPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1         ,XN(1320),YN(1320),ZN(1320)
      EQUIVALENCE(B(1),BODYRD)
      EQUIVALENCE(DA(2),PANS)
      EQUIVALENCE (SYM,DA(3426))
      EQUIVALENCE (DA(7),XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
     1 ,(DA(11),BETA),(DA(12),PSTAR),(DA(13),QSTAR),(DA(14),RSTAR)
      COMMON/SCRAT/ XSOL(5000),BOUND(5000),AXB(5000)
     1              , AYB(5000),  AZB(5000)
      DIMENSION A(5000)
      EQUIVALENCE(A,AXB),(B,BB)
      DIMENSION B(5000)
      EQUIVALENCE(SYMB,DA(19))
      COMMON/PANINF/ PANSYM(10)
      LOGICAL ROOT,OUTER
      LOGICAL ROOTP,OUTERP
C     REWIND UNIT 11 IN THIS PROGRAM.  UNIT 11 WILL HAVE SXMT,SYMT,SZMT.
      REWIND 11
      REWIND 18
      REWIND 23
      REWIND 21
      REWIND 10
      REWIND 12
      NCT=MP1
```

```
      NPANS=PANS
C
C   TEMPORARY VALUES FOR BREF,CBAR
      BREF=1.0
      CBAR=10.0
      IF(NBODS.EQ.0) GO TO 1111
C   READ 2 RECORDS ON 18 TO SPACE OVER ALNGTH,ELNGTH ARRAYS
      READ(18) B
      READ(18) B
 1111 CONTINUE
C
      NBP=NBODS+NPANS
      M=0
      DO 10 KK=1,NBP
      LTPTS=LTC(KK)
      LNPTS=LNC(KK)
      DO 10 JJ=1,LTPTS
      DO 10 II=1,LNPTS
      M=M+1
      XX=XQ(M)-XCG
      YY = YQ(M) -YCG *BETAM
      ZZ = ZQ(M) -ZCG *BETAM
      VY    = -BETA*BETAM-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
      VZ    = ALPHA*BETAM+2.0*(PSTAR*YY/BREF+QSTAR*XX/CBAR)
      VX=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/BREF)
   10 BOUND(M)=-XN(M)*VX-YN(M)*VY-ZN(M)*VZ
      NB=0
      NCTV=0
      KCON=0
      IF(NBODS.EQ.0) GO TO 99
      DO 50 I=1,NBODS
      NB=NB+1
      SYML=SYMM(I)
      NBVV=NVL(I)
      NTVV=NVT(I)
      N3=NBVV+1
      N2=NLT(I)
      NSEG=LSEG(I)+TSEG(I)
      KCON=KCON+NSEG
```

```
      LNDIVB(I)=1                                                      3  0790
      LNDIV=1                                                          3  0800
      LTDIV=LTDIVB(I)                                                  3  0810
      READ(18) BB                                                      3  0820
      CALL INFL(NSEG,NBP)                                              3  0830
      REWIND 19                                                        3  0840
      REWIND 20                                                        3  0850
   50 CONTINUE                                                         3  0860
C    'A' MATRIX FOR INFLUENCE OF BODY VORTICES IS ON UNIT 21.          3  0870
C    AX,AY,AZ MATRICES FOR BODY INFLUENCES IS ON UNIT NTAPE (19 OR 20). 3  0880
C    UNIT MTAPE (20 OR 19) IS AVAILABLE FOR ANOTHER USE.               3  0890
      CALL MATA(KCON,NCTV,NTAPE,0)                                     3  0900
      IF(KCON.NE.0) REWIND 23                                          3  0910
   99 CONTINUE                                                         3  0920
      IF(NPANS.EQ.0) GO TO 101                                         3  0930
C                                                                      3  0940
      NCOLP=0                                                          3  0950
      DO 100 I=1,NPANS                                                 3  0960
      SYM=PANSYM(I)                                                    3  0970
      NB = NB+1                                                        3  0980
      NBVVP=NVL(NB)                                                    3  0990
      NTVVP=NVT(NB)                                                    3  1000
      ROOT=ROOTP(I)                                                    3  1010
      OUTER=OUTERP(I)                                                  3  1020
      WRITE(6,40) I,ROOT,OUTER                                         3  1030
      WRITE(6,40) I,(ROOTP(K),OUTERP(K),K=1,NPANS)                     3  1040
   40 FORMAT(*0I=*,I4/10L5)                                            3  1050
      NSPACE=NSPP(I)                                                   3  1060
      READ(18) BODYRC                                                  3  1070
      IF(.NOT.ROOT) GO TO 55                                           3  1080
      NLROOT=NSPACE+1                                                  3  1090
      N3=NBVVP+1                                                       3  1100
   55 IF(.NOT.OUTER) GO TO 60                                          3  1110
      NVTOUT=NTVVP-NSPACE                                              3  1120
   60 CONTINUE                                                         3  1130
      IP=I                                                             3  1140
      CALL PANMAT(XG,YG,ZG,NCT,NCOLP,IP)                               3  1150
  100 CONTINUE                                                         3  1160
C    MP2 = MUNIT FOR STORAGE OF AX,AY,AZ FOR PANEL VORTICES.           3  1170
```

```
      NZERO = 0
      CALL MATA(NZERO,NCOLP,MP2,1)
      REWIND 21
  101 CONTINUE
      REWIND 11
      END
```

```
      SUBROUTINE PANMAT(XQ,YQ,ZQ,NCP,NCOLP,IP)
      COMMON DA(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1  ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSURV(100),CHORD(100)
     2  ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4  ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5   ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6   ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
     7   ,XVS(100),YVS(100),ZVS(100)
      COMMON/CONPTS/  DUMQ(3960)
     1                 ,XN(1320),YN(1320),ZN(1320)
      COMMON/SCRAT/XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
      EQUIVALENCE(AX,AXB),(AY,AYB),(AZ,AZB),(X3,XSOL(1)),(Y3,XSOL(101))
     1            ,(Z3,XSOL(201)),(XT,XSOL(301)),(YT,XSOL(401))
     2            ,(ZT,XSOL(501)),(SUM,XSOL(601))
      DIMENSION AX(5000),AY(5000),AZ(5000),X3(100),Y3(100),Z3(100)
     1           ,XT(100),YT(100),ZT(100),SUM(100)
      DIMENSION B(31000)
      EQUIVALENCE (B,XVR(1,1))
C     FREE TRAILING VORTEX VARIABLES.......
C     XN,YN,ZN, USE, SAME SPACE AS UTV,VTV,WTV TO BE COMPUTED IN TRAIL.
C
      DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI( 500),YTVI(500)
     1           ,ZTVI(500),UTV(1000),VTV(1000),WTV(1000),V1(100),V2(100)
     2           ,V3(100),NIP(100),NTVE(100)
      EQUIVALENCE(B(15001),UTV),     (B(16001),VTV),    (B(17001),WTV)
     1            ,(B(18001),XTV),     (B(19001),YTV),    (B(20001),ZTV)
     2            ,(B(21001),XTVI),    (B(21501),YTVI),   (B(22001),ZTVI)
     3            ,(B(22501),V1),      (B(22601),V2),     (B(22701),V3)
     4            ,(B(22801),NIP),     (B(22901),NTVE)
     5           ,(B(23001),NV)
C     MP9=NTR=NO. OF FREE TRAILING VORTICES.
C     MP10 =NTRV=TOTAL NO. OF INITIAL PTS.
      EQUIVALENCE (MP9,NTR),(MP10,NTRV)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTPATT,NPRCLN,NPRCLT,NNCTXC,NNCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
```

```
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1    ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2    ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3    ,LNDIVP(7),LTDIVP(7),NSPP(7),ROOTP(7),OUTERP(7),SYMP(7)
      COMMON/PANINF/ PANSYM(10)
       EQUIVALENCE(DA(3432),XP0),   (DA(3433),YP0),   (DA(3434),ZP0)
      DIMENSION XQ(1),YQ(1),ZQ(1)
      DIMENSION SAVEC(6000), ASSP(1000),AS(1000)
      EQUIVALENCE(B(15001),SAVEC),(B(21001),ASSP),(B(22001),AS)
     1           ,(DA(3439),FNU),(DA(3440),FNW)
      DIMENSION YPD(200),ZPD(200)
      EQUIVALENCE(YPD(1),B(30001)),(ZPD(1),B(30201))
      REAL LLEGX,LLEGY,LLEGZ
      LOGICAL FLAG
      LOGICAL ROOT,OUTER
      DATA LSIG/0/
      DATA LDD/0/
      IF(IP.GT.1) GO TO 999
      NTR=0
      NTRV=0
      NU=FNU
      NW=FNW
  999 CONTINUE
      NVL1=NBVVP
      PI=3.141592654
      IC=0
      NBP1=NBVVP+1
      IF(.NOT.ROOT) GO TO 2
C
      XVR1=XVR(1,NBP1)
      IF(XVR1.GE.XVT(NNATT)) GO TO 2
      DO 1 I=1,NNATT
      IF(XVT(I).GT.XVR1) IC=IC+1
    1 CONTINUE
      IF(IC.EQ.1) IC=2
C  LET IC BE NUMBER OF LONGITUDINAL PANELS IN TRAILING VORTEX SECTION.
      N1=NSPACE+1
      DO 3 I=1,N1
      X3(I)=XVR(I,NBP1)
```

```
      Y3(I)=YVR(I,NBP1)
    3 Z3(I)=ZVR(I,NBP1)
      WRITE(6,101)(X3(I),Y3(I),Z3(I),I=1,N1)
  101 FORMAT(1H0/(1H ,1P3E20.6))
      CALL TVGM1(XVT,YVT,ZVT,NNATT,IC,X3,Y3,Z3,N1,XT,YT,ZT,NT)
      WRITE(6,100) NNATT,IC,N1,NT
  100 FORMAT( 7H PANMAT/10I5)
      WRITE(6,101)(XT(I),YT(I),ZT(I),I=1,NT)
      IF(IX.EQ.1) WRITE(6,502) NCP
  502 FORMAT(*0NCP=*,I4)
      NTX=NT/N1
      DO 503 I=1,N1
      NTR=NTR+1
  503 NIP(NTR)=NTX
      K=0
      DO 504 KK=1,N1
      DO 504 KK1=1,NTX
      K=K+1
      NTRV=NTRV+1
      XTVI(NTRV)=XT(K)
      YTVI(NTRV)=YT(K)
      ZTVI(NTRV)=ZT(K)
      IF(NTX.NE.KK1) GO TO 504
      LDD=LDD+1
      YPD(LDD)=YTVI(NTRV)
      ZPD(LDD)=ZTVI(NTRV)
  504 CONTINUE
    2 CONTINUE
      NIT=NTVVP+1-NSPACE
      DO 505 K=1,NIT
      IF(NSPACE.NE.0.AND.K.EQ.1) GO TO 505
      NTR=NTR+1
      NIP(NTR)=1
      NTRV=NTRV+1
      XTVI(NTRV)=XVS(K)
      YTVI(NTRV)=YVS(K)
      ZTVI(NTRV)=ZVS(K)
      LDD=LDD+1
      YPD(LDD)=YTVI(NTRV)
```

```
      ZPD(LDD)=ZTVI(NTRV)
      WRITE(6,7002) (I,YPD(I),ZPD(I),I=1,LDD)
 7002 FORMAT(*PANEL DRAG COORDINATES*/(I5,2F15.5))
  505 CONTINUE
      WRITE(6,800)(XTVI(I),YTVI(I),ZTVI(I),I=1,NTRV)
  800 FORMAT( 26HCXTVI,YTVI,ZTVI, IN PANMAT/(1P3E20.6))
C   SET NV=NO. OF FREE TRAILING VORTICES.
      NV=NTR
      WRITE(6,100) NSPACE,NVL1
      DO 1000 IX=1,NCP
      XC=XQ(IX)
      YC=YQ(IX)
      ZC=ZQ(IX)
      MA=0
      MS=0
      IF(.NOT.ROOT) GO TO 105
      DO 104 I=1,NSPACE
      DO 104 J=1,NVL1
      DO 5 K1=1,3
    5 SUM(K1)=0.0
      DO 50 K=1,4
      IF(J.EQ.NVL1.AND.K.EQ.4) GO TO 41
      IF(K.GT.1) GO TO 20
      X1=XVR(I,J+1)
      Y1=YVR(I,J+1)
      Z1=ZVR(I,J+1)
      X2=XVR(I,J)
      Y2=YVR(I,J)
      Z2=ZVR(I,J)
      GO TO 40
   20 X1=X2
      Y1=Y2
      Z1=Z2
      IF(K-3) 25,30,35
   25 X2=XVR(I+1,J)
      Y2=YVR(I+1,J)
      Z2=ZVR(I+1,J)
      GO TO 40
   30 X2=XVR(I+1,J+1)
```

```
      Y2=YVR(I+1,J+1)
      Z2=ZVR(I+1,J+1)
      GO TO 40
   35 X2=XVR(I,J+1)
      Y2=YVR(I,J+1)
      Z2=ZVR(I,J+1)
   40 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)
      GO TO 50
   41 CONTINUE
      IF(IC.EQ.0) GO TO 480
      SUM1=SUM(1)
      SUM2=SUM(2)
      SUM3=SUM(3)
      IF(I.EQ.1) GO TO 42
      RLEGX1=RLEGX
      RLEGY1=RLEGY
      RLEGZ1=RLEGZ
   42 DO 43 K1=1,3
   43 SUM(K1)=0.0
      DO 46 K1=1,IC
      IF(K1.EQ.1) GO TO 44
      X1=X2
      Y1=Y2
      Z1=Z2
      GO TO 45
   44 I1=I*(IC+1)+1
      X1=XT(I1)
      Y1=YT(I1)
      Z1=ZT(I1)
   45 I2=I*(IC+1)+1+K1
      X2=XT(I2)
      Y2=YT(I2)
      Z2=ZT(I2)
   46 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)
      RLEGX=SUM(1)
      RLEGY=SUM(2)
      RLEGZ=SUM(3)
      Y2H=Y2
      SYMPAS=-1.0
```

```
  409 CONTINUE
C  NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR RIGHT LEG.
      T1=SQRT((Y2-YC)**2+(Z2-ZC)**2)
      IF(T1-0.00001) 412,411,411
  411 CONTINUE
      T2=(X2-XC)/SQRT((X2-XC)**2+(Y2-YC)**2+(Z2-ZC)**2)
      QT=0.25*(1.0-T2)/(PI*T1)
      RLEGY=RLEGY-SYMPAS*QT*(Z2-ZC)/T1
      RLEGZ=RLEGZ+SYMPAS*QT*(Y2-YC)/T1
  412 CONTINUE
      IF(SYMPAS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 471
      Y2=Y2H
      GO TO 472
  471 SYMPAS=1.0
      Y2=-Y2
      GO TO 409
  472 CONTINUE
      IF(I.EQ.1) GO TO 421
      LLEGX=-RLEGX1
      LLEGY=-RLEGY1
      LLEGZ=-RLEGZ1
      GO TO 4721
  421 DO 431 K1=1,3
  431 SUM(K1)=0.0
      DO 461 K1=1,IC
      IF(K1.EQ.1) GO TO 441
      X2=X1
      Y2=Y1
      Z2=Z1
      GO TO 451
  441 X2=XT(1)
      Y2=YT(1)
      Z2=ZT(1)
  451 I1=K1+1
      X1=XT(I1)
      Y1=YT(I1)
      Z1=ZT(I1)
  461 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)
      LLEGX=SUM(1)
```

```
      LLEGY=SUM(2)
      LLEGZ=SUM(3)
      Y1H=Y1
      SYMPAS=-1.0
 4091 CONTINUE
C  NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR LEFT LEG.
      T1=SQRT((Y1-YC)**2+(Z1-ZC)**2)
      IF(T1-0.00001) 470,415,415
 415  CONTINUE
      T2=(X1-XC)/SQRT((X1-XC)**2+(Y1-YC)**2+(Z1*ZC)**2)
      QT=0.25*(1.0-T2)/(PI*T1)
      LLEGY=LLEGY+SYMPAS*QT*(Z1-ZC)/T1
      LLEGZ=LLEGZ-SYMPAS*QT*(Y1-YC)/T1
 470  CONTINUE
      IF(SYMPAS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 4711
      Y1=Y1H
      GO TO 4721
 4711 SYMPAS=1.0
      Y1=-Y1
      GO TO 4091
 4721 CONTINUE
      SUM(1)=SUM1 + RLEGX+LLEGX
      SUM(2)=SUM2 + RLEGY+LLEGY
      SUM(3)=SUM3 + RLEGZ+LLEGZ
 480  CONTINUE
C  PROVIDE HERE FOR ST. LINE TR. VORTICES WHERE PANEL TR. EDGE IS
C   EITHER AT END OF BODY OR BEHIND IT.  (IC=0 CASE)
 50   CONTINUE
      MA=MA+1
      AX(MA)=SUM(1)
      AY(MA)=SUM(2)
      AZ(MA)=SUM(3)
      DO 90 IS=1,2
      MS=MS+1
      IV=0
      JV=0
      T=TS(MS)
      DZ=DZS(MS)
      DY=DYS(MS)
```

```
      KSOL=1
      CALL PVSK(T,DY,DZ,XC,YC,ZC,MS,MA,IV,JV,KSOL)
   90 CONTINUE
  104 CONTINUE
  105 CONTINUE
C  DO OUTER PANEL INFLUENCE EQUATIONS AND SOURCE INFLUENCE EQUATIONS.
      NVTOUT=NTVVP-NSPACE
      DO 200 I=1,NVTOUT
      DO 200 J=1,NBVVP
      MS=MS+1
      MA=MA+1
      KSOL=2
  110 T=TS(MS)
      DZ=DZS(MS)
      DY=DYS(MS)
      CALL PVSK(T,DY,DZ,XC,YC,ZC,MS,MA,I,J,KSOL)
      IF(KSOL.EQ.1) GO TO 115
      KSOL=1
      MS=MS+1
      GO TO 110
  115 CONTINUE
  200 CONTINUE
      IF(DA(5000).LT.0) GO TO 351
      WRITE(6,300) IX,(I,AX(I),AY(I),AZ(I),I=1,MA)
  300 FORMAT(///'0INFLUENCE COEFFICIENTS FOR CONTROL PT.'I3//
     1   (I3,1P3E18.6))
      WRITE(6,350) IX,(I,SXMT(I),SYMT(I),SZMT(I),I=1,MS)
  350 FORMAT(///'0SOURCE COEFFICIENTS FOR CONTROL PT,'I3//
     1   (I3,1P3E18.6))
  351  CONTINUE
C    IN SUBROUTINE PANMAT, WRITE THE SXMT,SYMT,SZMT MATRICES ON UNIT 11.
C    THERE WILL BE NCT ROWS OF SXMT FOR EACH PANEL.
C    NCT = TOTAL NUMBER OF CONTROL POINTS.
      WRITE(11)(SXMT(I),SYMT(I),SZMT(I),I=1,MS)
      XN1=XN(IX)
      YN1=YN(IX)
      ZN1=ZN(IX)
      BN=0.0
      BNX=0.0
```

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```
      BNY=0.0
      J=LSIG
      DO 355 I=1,NS
      J=J+1
      BNX=SIGMA(J)*SXMT(I)+BNX
      BNY=SIGMA(J)*SYMT(I)+BNY
  355 BN=SIGMA(J)*(XN1*SXMT(I) + YN1*SYMT(I) + ZN1*SZMT(I)) + BN
      IF(NBODS.EQ.0) GO TO 358
      IF(IX.LE.NCPTS(1)) GO TO 359
  358 BOUND(IX)=BOUND(IX)-BN
  359 CONTINUE
      IF(NU+NW.EQ.0) GO TO 3060
CALL  CONFUN FOR CONSTRAINT MATRIX MULTIPLY.
      LOC1=(IP-1)*1000+1
      LOC2=LOC1+400
      LOC3=LOC2+400
      FLAG=.FALSE.
      IF(NW.EQ.0)FLAG=.TRUE.
      CALL CONFUN(AX,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
     1            ,ASSP,NTVVP,NBVVP,AS,NW,FLAG,NU)
      IF(FLAG) GO TO 1050
      MA=NU*NW
      DO 1005 I=1,MA
 1005 AX(I)=ASSP(I)
      GO TO 1060
 1050 MA=NU*NTVVP
      DO 1055 I=1,MA
 1055 AX(I)=AS(I)
 1060 CONTINUE
      CALL CONFUN(AY,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
     1            ,ASSP,NTVVP,NBVVP,AS,MA,FLAG,NU)
      IF(FLAG) GO TO 2050
      DO 2005 I=1,MA
 2005 AY(I)=ASSP(I)
      GO TO 2060
 2050 DO 2055 I=1,MA
 2055 AY(I)=AS(I)
 2060 CONTINUE
      CALL CONFUN(AZ,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
```

```
     1          ,ASSP,NTVVP,NTVVP,AS,N..,FLAG,..)
      IF(FLAG) GO TO 3050
      DO 3005 I=1,MA
 3005 AZ(I)=ASSP(I)
      GO TO 3060
 3050 DO 3055 I=1,MA
 3055 AZ(I)=AS(I)
 3060 CONTINUE
      LSIGH=J
      IF(IP.NE.1) GO TO 400
      IF(IX.NE.1) GO TO 360
      NCOLP=NCOLP+MA
      MUNIT=MTAPE
  360 CONTINUE
C
C    MUNIT IS THE FILE TO BE WRITTEN.
C    MUNPR IS THE FILE TO BE READ.
C    FOR IP=1, MUNIT=MTAPE=19(OR 20).
C    FOR IP=2, MUNIT=24
C              MUNPR=19(OR 20).
C    THEREAFTER, FOR IP EVEN, MUNIT AND MUNPR SAME AS FOR IP=2.
C                FOR IP ODD,  MUNIT AND MUNPR VALUES ARE REVERSED.
C
      IF(IX.EQ.1) WRITE(6,500) IP,MTAPE,NCOLP
  500 FORMAT(*0IP,MTAPE,NCOLP*,3I5)
      WRITE(MTAPE)(AX (I),AY(I),AZ(I),I=1,NCOLP)
      GO TO 900
  400 IF(2*(IP/2).NE.IP) GO TO 401
      MUNIT=24
      GO TO 402
  401 MUNIT=MTAPE
  402 MC1=NCPRE+1
      MC2=NCPRE+MA
      J=0
      DO 404 I=MC1,MC2
      J=J+1
      AX(I)=AX(J)
      AY(I)=AY(J)
  404 AZ(I)=AZ(J)
```

```
      IF(IX.EQ.1) WRITE(6,501) IP,MUNPR,MUNIT,NCPRE,MA
501   FORMAT(*0IP,MUNPR,MUNIT,NCPRE,MA=*,5I5)
      READ(MUNPR)(AX(I),AY(I),AZ(I),I=1,NCPRE)
      IF(IX.EQ.1)  NCOLP=NCPRE + MA
      WRITE(MUNIT)(AX(I),AY(I),AZ(I),I=1,NCOLP)
900   CONTINUE
1000  CONTINUE
      LSIG=LSIGH
      NCPRE=NCOLP
      MUNPR=MUNIT
      MP2=MUNIT
      MP3=NCOLP
      REWIND MUNPR
      REWIND MTAPE
      REWIND 24
      RETURN
      END
```

```
      SUBROUTINE INFL(KCN,NSP)
C     COMPUTE AX,AY,AZ MATRICES AND BOUNDARY CONDITIONS.
      COMMON DA(5000)
     1   ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2    ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1    ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2    ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3    ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/ XV1(151,31),YV(151,31),ZV(151,31)
     1              ,TNX(1320),TNY(1320),TNZ(1320)
     2              ,TTX(1320),TTY(1320),TTZ(1320)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1                 ,XN(1320),YN(1320),ZN(1320)
      COMMON/SCRAT/ XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
      COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2
      DIMENSION SUM(3)
      EQUIVALENCE (DA(7),XCG), (DA(8),YCG), (DA(9),ZCG),(DA(10),ALPHA)
     1   ,(DA(11),BETA), (DA(12),PSTAR), (DA(13),QSTAR), (DA(14),RSTAR)
      EQUIVALENCE(DA(19),SYMB)
      DIMENSION XQQ(1320),YQQ(1320),ZQQ(1320)
      DIMENSION BB(31000)
      DIMENSION B (5000)
      EQUIVALENCE(BB,XV1(1,1))
      EQUIVALENCE(B ,XV1(1,1))
      EQUIVALENCE (XQQ,XQ),(YQQ,YQ),(ZQQ,ZQ)
      EQUIVALENCE(N,NCTV),(M,NCT)
      EQUIVALENCE(ALNGTH,XSOL)
      DIMENSION ALNGTH(5000)
C     IF KCN DOES NOT = 0, ALNGTH IS NEEDED FOR AXYZRL.
      IF(KCN.EQ.0) GO TO 5
C    REWIND 18 AND THEN READ ALNGTH ARRAY.
      REWIND 18
      READ(18) ALNGTH
C    NOW SKIP 2 RECORDS SO THAT 18 IS POSITIONED TO READ PANEL DATA.
      READ(18) B
      READ(18) BB
    5 CONTINUE
      PI4=12.5663704
```

```
      NSTART=NCTV
      NCT=0
      DO 65 KK=1,NBP
      LTPTS=LTC(KK)
      LNPTS=LNC(KK)
      DO 65 JJ=1,LTPTS
      DO 65 II=1,LNPTS
      NCT=NCT+1
      L=M
      X=XQ(M)
      Y=YQ(M)
      Z=ZQ(M)
      N=NSTART
      DO 60 J=1,NTVV
      DO 60 I=1,NBVV
      N=N+1
      TOT1=0.0
      TOT2=0.0
      TOT3=0.0
      DO 55 K=1,4
      DO 8 KS=1,3
8     SUM(KS)=0.0
      GO TO (10,20,30,40),K
10    CONTINUE
12    II1=LNDIV*(I-1)+1
      IF1=II1
      II2=(J-1)*LTDIV
      DO 14 L=1,LTDIV
      II2=II2+1
      IF2=II2+1
14    CALL VORTEX(SUM)
      GO TO 50
20    II2=1+J*LTDIV
      IF2=II2
      II1=(I-1)*LNDIV
      DO 24 L=1,LNDIV
      II1=II1+1
      IF1=II1+1
24    CALL VORTEX(SUM)
```

```
      GO TO 50
33    II1=IF1
      II2=II2+1
      DO 34 L=1,LTDIV
      II2=II2-1
      IF2=II2-1
      IF(I.NE.NBVV) GO TO 33
C
C    ADD TRAILING VORTEX CONTRIBUTION.
C    RIGHT SIDE
      SYMLOO=1.0
301   CONTINUE
      DX=XV1(II1,II2)-X
      DY=SYMLOO*YV(II1,II2)-Y
      DZ=ZV(II1,II2)-Z
      TYZ=DY**2+DZ**2
      T1=SQRT(TYZ)
      T2=DX/SQRT(DX**2+TYZ)
      QT=(1.0-T2)/T1
      QTT=SYMLOO*QT
      SUM(2)=SUM(2)+QTT*DZ/T1
      SUM(3)=SUM(3)-QTT*DY/T1
C    LEFT SIDE
      DX=XV1(IF1,IF2)-X
      DY=SYMLOO*YV(IF1,IF2)-Y
      DZ=ZV(IF1,IF2)-Z
      TYZ=DY**2+DZ**2
      T1=SQRT(TYZ)
      T2=DX/SQRT(DX**2+TYZ)
      QT=(1.0-T2)/T1
      QTT=SYMLOO*QT
      SUM(2)=SUM(2)-QTT*DZ/T1
      SUM(3)=SUM(3)+QTT*DY/T1
C
      IF(SYMB.NE.0.0) GO TO 34
      IF(SYMLOO.EQ.-1.0) GO TO 34
      SYMLOO=-1.0
      GO TO 301
34    CALL VORTEX(SUM)
```

```
34    CONTINUE
      GO TO 50
40    CONTINUE
42    II2=IF2
      III=III+1
      DO 44 L=1,LNDIV
      III=III-1
      IF1=III-1
44    CALL VORTEX(SUM)
50    TOT1=TOT1+SUM(1)
      TOT2=TOT2+SUM(2)
      TOT3=TOT3+SUM(3)
55    CONTINUE
      AXB(N)=TOT1/PI4
      AYB(N)=TOT2/PI4
      AZB(N)=TOT3/PI4
      IF(I.NE.1) GO TO 60
      AXB(N)=0.0
      AYB(N)=0.0
      AZB(N)=0.0
60    CONTINUE
C    IF BODY HAS CONSTRAINT FUNCTIONS, CONVERT AX TO AXRL. SAME FOR AY,AZ
      IF(KCN.EQ.0) GO TO 601
      NS1=NSTART+1
      CALL AXYZRL(NS1,ALNGTH,AXB,AYB,AZB)
601   CONTINUE
      N1=1
      IF(NB.EQ.1) GO TO 62
      NB1=NB-1
      N2=0
      DO 61  J=1,NB1
61    N2=N2+NVL(J)*NVT(J)
      READ(NTAPE)(AXB(I),AYB(I),AZB(I),I=N1,N2)
62    WRITE(MTAPE)(AXB(I),AYB(I),AZB(I),I=N1,NCTV)
65    CONTINUE
      MTAPE=39-MTAPE
      NTAPE=39-NTAPE
      RETURN
      END
```

```
      SUBROUTINE CONFUN(A, SCRTCH, SCRCH, TETA, NROOT, ASSP, NETA, NXOC,
     1 AS, NW, FLAG, NCOL)
C
C     A      -  A MATRIX (ONE ROW FROM FRED)
C     SCRTCH -  T(XOC)
C     SCRCH  -T(XOC) FOR USE IN ROOT SECTION
C      TETA   -  T(ETA)
C     NROOT NUMBER OF ETAS IN ROOT SECTION OF PANEL
C     ASSP   -  OUTPUT   A//
C     NETA   -  NUMBER OF ETAS
C     NXOC   -  NUMBER OF XOCS
C     AS     -  CHORDWIZE TRANSFORMED              MATRIX  A/
C     NW     -  NUMBER OF LATERAL CONSTRAINT FUNCTIONS
C     FLAG   -  IF FALSE SOLVE FOR COMPLETELY CONSTRAINED
C     NCOL   -  NU + NF + NK (NUMBER OF COL.S OF AS)
      DIMENSION A(NXOC, NETA), SCRTCH(NXOC, NCOL), SCRCH(NXOC, NCOL),
     1 TETA(NETA,NW), AS(NCOL, NETA), ASSP(NCOL,NW)
      LOGICAL FLAG
      IF(NROOT) 41, 41, 1
    1 DO 40 IETA=1, NROOT
      DO 40 IC = 1, NCOL
      TEMP =0.0
      DO 30 K=1,NXOC
   30 TEMP = TEMP + SCRCH(K,IC)*A(K,IETA)
   40 AS(IC,IETA)= TEMP
   41 IS = NROOT + 1
      IF(IS.GT.NETA) RETURN
      DO 60 IETA=IS, NETA
      DO 60 IC=1, NCOL
      TEMP =0.0
      DO 50 K=1, NXOC
   50 TEMP = TEMP + SCRTCH(K,IC)*A(K,IETA)
   60 AS(IC,IETA)= TEMP
      IF( FLAG) RETURN
C     SUBROUTINE TO SOLVE COMPLETLY CONSTRAINED MATRIX
      DO 70 INW=1,NW
      DO 70 IUKF=1, NCOL
      TEMP = 0.0
      DO 20 IETA=1, NETA
```

```
   20 TEMP = TEMP + TETA(IETA,INW)*AS(IUKF,IETA)
      ASSP(IUKF,INW) = TEMP
   70 CONTINUE
      RETURN
C     BLAINE D. GAITHER 9/72
      END
```

```
      SUBROUTINE TVGM1(X,Y,Z,N,NS,X1,Y1,Z1,N1,XT,YT,ZT,NT)
CALCULATES FIRST ITERATION TRAILING VORTEX GEOMETRY.
C  X,Y,Z = JUNCTURE TRAILING VORTEX.  N=NO.OF POINTS.
C  X1,Y1,Z1 = PANEL TRAILING EDGE.  N1 =NO.OF POINTS.
C  XT,YT,ZT = COMPUTED TRAILING VORTEX POINTS.  NT=NO.OF POINTS.
C
      DIMENSION X(1),Y(1),Z(1),X1(1),Y1(1),Z1(1),XT(1),YT(1),ZT(1)
      DIMENSION XS(21),YS(21),ZS(21),SA(21),SJ(21)
      DIMENSION DCX(20),DCY(20),DCZ(20),XTT(21),YTT(21),ZTT(21)
        WRITE(6,100)(I,X(I),Y(I),Z(I),I=1,N)
      WRITE(6,100)(I,X1(I),Y1(I),Z1(I),I=1,N1)
  100 FORMAT(1H0/(I3,1P3E18.5))
      NT=(NS+1)*N1
      DX=(X(N)-X1(1))/NS
      XS(1)=X1(1)
      DO 1 I=1,NS
    1 XS(I+1)=XS(I)+DX
      NS1=NS+1
      CALL CODIM(X,Y,N,XS,YS,NS1)
      CALL CODIM(X,Z,N,XS,ZS,NS1)
      SAI=0.0
      SA(1)=0.0
      DO 5 I=1,NS
      DY= YS(I+1)-YS(I)
      DZ= ZS(I+1)-ZS(I)
      SAI =   SAI+ SQRT(DX**2 + DY**2 + DZ**2)
    5 SA(I+1)=SAI
      SD=SAI/NS
      SJ(1)=0.0
      DO 10 I=1,NS
   10 SJ(I+1)=SJ(I)+SD
      CALL CODIM(SA,XS,NS1,SJ,XT,NS1)
      CALL CODIM(SA,YS,NS1,SJ,YT,NS1)
      CALL CODIM(SA,ZS,NS1,SJ,ZT,NS1)
C
C  DIVIDE LAST BOUND VORTEX INTO EQUAL SPACES.
      N11=N1-1
      XF=X(N)
      YF=Y1(N1)
```

```
      ZF=Z1(N1)
      DX=0.0
      DY=(YF-Y(N))/N11
      DZ=(ZF-Z(N))/N11
      DO 15 I=2,N1
      IT1=(I-1)*NS1
C     (IT1+1) PT. IS ON FIRST BOUND VORTEX.
      N3=IT1+1
      XT(N3)=X1(I)
      YT(N3)=Y1(I)
      ZT(N3)=Z1(I)
      IT2= I*NS1
      XT(IT2)=XT(IT1)
      YT(IT2)=YT(IT1)+DY
   15 ZT(IT2)=ZT(IT1)+DZ
      CALL NORM(DX,DY,DZ)
      N2=N1-1
      NN=NS1*N11+1
C
COMPUTE DX ALONG LAST TRAILING VORTEX.
      DXL=(XF-X1(N1))/NS
      XT(NN)=X1(N1)
      DO 16 I=1,NS
      NN=NN+1
      XT(NN)=XT(NN-1)+DXL
      YT(NN)=Y1(N1)
   16 ZT(NN)=Z1(N1)
      NP=N11*NS1
      N4=N2-1
      DO 20 J=2,NS
      CSX=XT(NP+J)-XT(J)
      CSY=YT(NP+J)-YT(J)
      CSZ=ZT(NP+J)-ZT(J)
      CS2=CSX**2+CSY**2+CSZ**2
      PER=(J-1.0)/NS
      DENOM=0.0
      DO 18 I=1,N2
      DXLE=X1(I+1)-X1(I)
      DYLE=Y1(I+1)-Y1(I)
```

```
      DZLE=Z1(I+1)-Z1(I)
      CALL NORM(DXLE,DYLE,DZLE)
      DIRX=DXLE+(DX-DXLE)*PER
      DIRY=DYLE+(DY-DYLE)*PER
      DIRZ=DZLE+(DZ-DZLE)*PER
      CALL NORM(DIRX,DIRY,DIRZ)
      DCX(I)=DIRX
      DCY(I)=DIRY
      DCZ(I)=DIRZ
   18 DENOM=DENOM + DOT(DIRX,DIRY,DIRZ,CSX,CSY,CSZ)
      DS=CS2/DENOM
      NN=NS1+J
      DO 20 I=1,N4
      N3=NN-NS1
      XT(NN)=XT(N3)+DS*DCX(I)
      YT(NN)=YT(N3)+DS*DCY(I)
      ZT(NN)=ZT(N3)+DS*DCZ(I)
   20 NN=NN+NS1
      DO 30 I=2,N11
      NN=(I-1)*NS1
      SAI=0.0
      SA(1)=0.0
      DO 22 J=1,NS
      N2=NN+J
      N4=N2+1
      DX= XT(N2)-XT(N4)
      DY= YT(N2)-YT(N4)
      DZ= ZT(N2)-ZT(N4)
      SAI= SAI+ SQRT(DX**2 +DY**2 +DZ**2)
   22 SA(J+1)=SAI
      SD=SAI/NS
      DO 23 J=1,NS1
      N2=NN+J
      XTT(J)=XT(N2)
      YTT(J)=YT(N2)
   23 ZTT(J)=ZT(N2)
      SJ(1)=0.0
      DO 24 J=1,NS
   24 SJ(J+1)=SJ(J)+SD
```

```
      CALL CODIM(SA,XTT,NS1,SJ,DCX,NS)                                  3   8480
      CALL CODIM(SA,YTT,NS1,SJ,DCY,NS)                                  3   8490
      CALL CODIM(SA,ZTT,NS1,SJ,DCZ,NS)                                  3   8500
      DO 30 J=2,NS                                                      3   8510
      N2=NN+J                                                           3   8520
      XT(N2)=DCX(J)                                                     3   8530
      YT(N2)=DCY(J)                                                     3   8540
   30 ZT(N2)=DCZ(J)                                                     3   8550
      RETURN                                                            3   8560
      END                                                               3   8570
```

```
      SUBROUTINE MATCON
C
C   PERFORM PHI,THETA MATRIX MULTIPLY ON AXB,AYB,AZB MATRICES.
C   NO REPLACEMENT IS MADE FOR BODIES HAVING  NO CONSTRAINT FUNCTIONS.
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2   ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/ ARRAY(31000)
      DIMENSION BX(5000),BY(5000),BZ(5000),PHTH(5000)
      EQUIVALENCE(BX,ARRAY(1)),(BY,ARRAY(5001)),(BZ,ARRAY(10001))
     1          ,(PHTH,ARRAY(15001))
      COMMON/SCRAT/XSCL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
      DO 1000 I=1,NCT
      READ(NTAPE)(AXB(K),AYB(K),AZB(K),K=1,NCTV)
      M=0
      IV=0
      DO 100 NB=1,NBODS
      NVP=NVL(NB)*NVT(NB)
      IF(LSEG(NB).EQ.0.0.AND.TSEG(NB).EQ.0.0) GO TO 50
      KODE=1
      NC=ITAPE
      GO TO 70
   50 NC=NVP
      KODE=0
      DO 60 I1=1,NC
      M=M+1
      IV=IV+1
      BX(M)=AXB(IV)
      BY(M)=AYB(IV)
   60 BZ(M)=AZB(IV)
      GO TO 100
   70 CONTINUE
      IS=IV
      DO 80 I2=1,NC
      SX=0.0
```

```
      SY=0.0
      SZ=0.0
      READ(23)(PHTH(KC),KC=1,NVP)
      M=M+1
      DO 75 I1=1,NVP
      ISI1=IS+I1
      SX=SX+PHTH(I1)*AXB(ISI1)
      SY=SY+PHTH(I1)*AYB(ISI1)
   75 SZ=SZ+PHTH(I1)*AZB(ISI1)
      IV=IV+1
      BX(M)=SX
      BY(M)=SY
   80 BZ(M)=SZ
  100 CONTINUE
      IF(KODE.EQ.1) REWIND 23
 1000 WRITE(MTAPE)(BX(K),BY(K),BZ(K),K=1,M)
C    CHANGE MEANING OF NCTV FROM TOT. NO. OF VORTICES ON BODIES
C      TO NUMBER OF COLUMNS OF 'A' MATRIX. (FOR DISCRETE CASE,NO CHANGE.)
      NCTV=M
      NTAPE=39-NTAPE
      MTAPE=39-MTAPE
      REWIND 19
      REWIND 20
      RETURN
      END
```

```
      SUBROUTINE MATA(KCON,NCOLS,IUNIT,IBP)
C  KCON = 0 FOR PANEL SIDE.  KCON = 0 OR NSEG FOR BODY SIDE (SEE INFLM.)
C  NCOLS = NO. OF AXB,AYB,AZB. (NO. OF COLS OF BODY OR PANEL SIDE.)
      COMMON DA(5000)
     1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2   ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/CONPTS/XQ(1320),YQ(1320),ZQ(1320)
     1              ,XN(1320),YN(1320),ZN(1320)
      COMMON/SCRAT/ XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
      DIMENSION A(5000)
      EQUIVALENCE(A,AXB)
      EQUIVALENCE(DA(2),PANS)
      NPANS=PANS
      IWR=21
      IF(IBP.EQ.1) IWR=10
      NBP=NBODS + NPANS
      IF(KCON.EQ.0) GO TO 100
      CALL MATCON
  100 L=0
      DO 300 I=1,NBP
      LTPTS=LTC(I)
      LNPTS=LNC(I)
      DO 300 JJ=1,LTPTS
      DO 300 II=1,LNPTS
      L=L+1
      XNN=XN(L)
      YNN=YN(L)
      ZNN=ZN(L)
      READ(IUNIT)(AXB(K),AYB(K),AZB(K),K=1,NCOLS)
      DO 200 K=1,NCOLS
  200 A(K)=XNN*AXB(K)+YNN*AYB(K)+ZNN*AZB(K)
  300 WRITE(IWR)(A(K),K=1,NCOLS)
      REWIND IWR
      REWIND NTAPE
      JTAPE=21
```

```
      RETURN                                                  3   9880
      END                                                     3   9890
```

```
      SUBROUTINE AXYZRL(KSTART,ALNGTH,AXB,AYB,AZB)
      DIMENSION ALNGTH(1)
      DIMENSION AXB(1),AYB(1),AZB(1)
      COMMON DA(5000)
     1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
     2  ,LNDIV,LTDIV,LNPTS,LTPTS
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3  ,LNDIVB(7),LTDIVB(7),NSPP(7),RCOTP(7),OUTERP(7),SYMM(7)
      DO 100 J1=1,NTVV
      I2=J1*NBVV
      I3=KSTART-1+I2
      ADDX=0.0
      ADDY=0.0
      ADDZ=0.0
      DO 100 I1=1,NBVV
      ADDX=AXB(I3)+ADDX
      AXB(I3)=ADDX*ALNGTH(I3)
      ADDY=AYB(I3)+ADDY
      AYB(I3)=ADDY*ALNGTH(I3)
      ADDZ=AZB(I3)+ADDZ
      AZB(I3)=ADDZ*ALNGTH(I3)
      I3=I3-1
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PVSK(T,DYY,DZZ,XC,YC,ZC,MS,MA,IV,JV,KSOL)
      COMMON DA(5000)
      EQUIVALENCE(DA(3426),SYM)
      COMMON/PANINF/ PANSYM(10)
      COMMON/SCRAT/XSOL(5000),BOUND(5000),AX(5000),AY(5000),AZ(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5  ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6  ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1 .,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
C
C     KSOL=1  FOR SOURCE PTS. ONLY
C     KSOL=2  FOR BOTH SOURCE PTS. AND VORTEX POINTS.
C     MS=SUBSCRIPT OF SOURCE PT.
C     XC,YC,ZC  --- CONTROL PT.
C     IV = LATERAL VORTEX SUBSCRIPT.
C     JV = LONGITUDINAL VORTEX SUBSCRIPT.
C    INSERT SPECIFICATION STATEMENTS HERE.
      REAL I1,I2,I3,I4
      YVV=0.5*SQRT(DYY**2+DZZ**2)
      YV2=2.0*YVV
      PI=3.141592654
      YV=YVV
      GO TO (10,20),KSOL
   10 YK=YSS(MS)
      ZK=ZSS(MS)
      XK=XSS(MS)
      GO TO 25
   20 IF(MS-2*(MS/2).EQ.0) GO TO 10
```

```
      YK=YVO(IV,JV)
      ZK=ZVO(IV,JV)
      XK=XVO(IV,JV)
   25 CONTINUE
      SUMX=0.0
      SUMY=0.0
      SUMZ=0.0
      UTSUM=0.0
      VTSUM=0.0
      WTSUM=0.0
      SIGN=1.0
      DZ=ZC-ZK
      X=XC-XK
   50 CONTINUE
      DY=YC-SIGN*YK
      RY=DYY/YV2
      RZ=DZZ/YV2
      Y=RY*DY+RZ*DZ
      Z=-RZ*DY+RY*DZ
      R12=(Y+YV)**2+Z**2
      R22=(X-T*Y)**2+Z**2*(1.0+T*T)
      R32=(Y-YV)**2+Z**2
      R4= SQRT((X-T*YV)**2+(Y-YV)**2+Z**2)
      R5= SQRT((X+T*YV)**2+(Y+YV)**2+Z**2)
      I1=(X+T*YV)/R5
      I2=(Y+T*X+YV*(1.0+T**2))/R5
      I3=-(Y+T*X-YV*(1.0+T**2))/R4
      I4=(X-T*YV)/R4
      IF(ABS(Z).GT.YV2) GO TO 42
      Z=0.0
      R6D=(X+T*YV)**2+(Y+YV)**2
      R7D=(X-T*YV)**2+(Y-YV)**2
      RXTY=(X-T*Y)**2
      R6=RXTY/R6D
      R7=RXTY/R7D
      IF(R6.GE.0.0075968656) GO TO 41
      IF(R7.GE.0.0075968656) GO TO 41
      IF(ABS(Y).LE.YV) GO TO 41
      TERM1=ABS(1.0/R7D-1.0/R6D)*0.5/P
```

```
      GO TO 43
   41 IF(ABS(Y).GT.YV) GO TO 42
      IF(ABS(X-T*Y).GE.0.25*ABS(PLL(MA))) GO TO 42
      TERM1=0.0
      GO TO 43
   42 TERM1=(I2+I3)/R22
   43 CONTINUE
      TERM2=(I1+1.0)/R12
      TERM3=(I4+1.0)/R32
      TERM4=1.0/R4-1.0/R5
      P=SQRT(1.0+T*T)
      EUS=(T*TERM4+(X-T*Y)*TERM1)/P
      EVS=(TERM4-T*(X-T*Y)*TERM1)/P
      EWS=P*Z*TERM1
      US=0.25*EUS/PI
      VS=0.25*EVS/PI
      WS=0.25*EWS/PI
      UT=US
      VT=VS*RY-WS*RZ
      WT=VS*RZ+WS*RY
      UTSUM=UT+UTSUM
      VTSUM=VT+VTSUM
      WTSUM=WT+WTSUM
      IF(KSOL.EQ.1) GO TO 45
      EU=Z*TERM1
      EV=Z*(-T*TERM1+TERM2-TERM3)
      EW=-(X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
      UV=0.25*EU/PI
      VV=0.25*EV/PI
      WV=0.25*EW/PI
      UI=UV
      VI=RY*VV-RZ*WV
      WI=RZ*VV+RY*WV
      SUMX=UI+SUMX
      SUMY=VI+SUMY
      SUMZ=WI+SUMZ
   45 CONTINUE
C     FOR SYMMETRY, GET IMAGE CONTRIBUTION.
      IF(SYM.NE.0.0) GO TO 60
```

```
      IF(SIGN.LT.0.0) GO TO 60
      SIGN=-1.0
      DZZ=-DZZ
      T=-T
      GO TO 50
   60 CONTINUE
      SXMT(MS)=UTSUM
      SYMT(MS)=VTSUM
      SZMT(MS)=WTSUM
      IF(KSOL.EQ.1) RETURN
      AX(MA)=SUMX
      AY(MA)=SUMY
      AZ(MA)=SUMZ
      RETURN
      END
```

```
      SUBROUTINE VORPAN(SUM,XI,YI,ZI,XF,YF,ZF,X,Y,Z)
      COMMON DA(5000)
      EQUIVALENCE(DA(3426),SYM)
      DIMENSION SUM(1)
      R4PI=0.07957747
      YIH=YI
      YFH=YF
      XFQ=XF-X
      ZFQ=ZF-Z
      XIQ=XI-X
      ZIQ=ZI-Z
      SYMLOO=1.0
   10 YFQ=YF-Y
      YIQ=YI-Y
      DELX=XF-XI
      DELY=YF-YI
      DELZ=ZF-ZI
      RXS1=YFQ*DELZ-ZFQ*DELY
      RXS2=ZFQ*DELX-XFQ*DELZ
      RXS3=XFQ*DELY-YFQ*DELX
      RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)
      TERM1=SQRT(DELX**2+DELY**2+DELZ**2)
      TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)
      TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)
      TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ
      RATIO = TERM4/TERM1**2
      COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)
      COSB = TERM4/(TERM1*TERM2)
      CC=COSB-COSA
      HX=XFQ-RATIO*DELX
      HY=YFQ-RATIO*DELY
      HZ=ZFQ-RATIO*DELZ
      H=SQRT(HX*HX+HY*HY+HZ*HZ)
      IF(H-0.00001) 11,12,12
   11 COEF=0.0
      GO TO 13
   12 CONTINUE
      HRXS=H*RXS
      COEF=R4PI*SYMLOO*CC/HRXS
```

```
13    CONTINUE
      SUM(1)=COEF*RXS1 + SUM(1)
      SUM(2)=COEF*RXS2 + SUM(2)
      SUM(3)=COEF*RXS3 + SUM(3)
      YI=YIH
      YF=YFH
      IF(SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN
      SYMLOO=-1.0
      YI=-YI
      YF=-YF
      GO TO 10
      END
```

```
      SUBROUTINE VORTEX(SUM)                                           3 11730
      COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)                    3 11740
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)                        3 11750
      COMMON DA(5000)                                                  3 11760
      EQUIVALENCE (DA(19),SYM)                                         3 11770
      COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2                              3 11780
      DIMENSION SUM(1)                                                 3 11790
      XI=XV(II1,II2)                                                   3 11800
      YI=YV(II1,II2)                                                   3 11810
      ZI=ZV(II1,II2)                                                   3 11820
      XF=XV(IF1,IF2)                                                   3 11830
      YF=YV(IF1,IF2)                                                   3 11840
      ZF=ZV(IF1,IF2)                                                   3 11850
      XFQ=XF-X                                                         3 11860
      ZFQ=ZF-Z                                                         3 11870
      XIQ=XI-X                                                         3 11880
      ZIQ=ZI-Z                                                         3 11890
      SYMLOO=1.0                                                       3 11900
   10 YFQ=YF-Y                                                         3 11910
      YIQ=YI-Y                                                         3 11920
      DELX=XF-XI                                                       3 11930
      DELY=YF-YI                                                       3 11940
      DELZ=ZF-ZI                                                       3 11950
      RXS1=YFQ*DELZ-ZFQ*DELY                                           3 11960
      RXS2=ZFQ*DELX-XFQ*DELZ                                           3 11970
      RXS3=XFQ*DELY-YFQ*DELX                                           3 11980
      RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)                               3 11990
      TERM1=SQRT(DELX**2+DELY**2+DELZ**2)                              3 12000
      TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)                                 3 12010
      TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)                                 3 12020
      TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ                                3 12030
      RATIO = TERM4/TERM1**2                                           3 12040
      COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)                  3 12050
      COSB = TERM4/(TERM1*TERM2)                                       3 12060
      CC=COSB-COSA                                                     3 12070
      HX=XFQ-RATIO*DELX                                                3 12080
      HY=YFQ-RATIO*DELY                                                3 12090
      HZ=ZFQ-RATIO*DELZ                                                3 12100
      H=SQRT(HX*HX+HY*HY+HZ*HZ)                                        3 12110
```

```
      IF(H-0.00001) 11,12,12
   11 COEF=0.0
      GO TO 13
   12 CONTINUE
      HRXS=H*RXS
      COEF=SYMLOO*CC/HRXS
   13 CONTINUE
      SUM(1)=COEF*RXS1 + SUM(1)
      SUM(2)=COEF*RXS2 + SUM(2)
      SUM(3)=COEF*RXS3 + SUM(3)
      IF(SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN
      SYMLOO=-1.0
      YI=-YI
      YF=-YF
      GO TO 10
      END
```

```
      PROGRAM SOL
      COMMON/SCRAT/XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/BODY/ AA(31000)
      DIMENSION PHTH(5000),ALNGTH(5000),GA(5000)
      EQUIVALENCE(PHTH,AA(1)),(ALNGTH,AA(5001)),(GA,AA(10001))
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      DIMENSION  B1(1000), P1(5000), SAVEC(6000), DUM(2000)
      DIMENSION  SAVEC1(6000),ATET(1000)
      EQUIVALENCE  (B1,AXB(1)), (P1,AXB(1001)), (SAVEC,AYB(1001))
      EQUIVALENCE  (DUM,AZB(2001)), (ATET,AZB(4001)), (DA(2),PANS)
      EQUIVALENCE  (SAVEC1,AA(15001))
      COMMON DA(5100)
      DIMENSION LT(10),LN(10),NP1(10)
      COMMON/PANINF/ PANSYM(10),ASOL(600)
      NPANS = PANS
      IF(NPANS .EQ. 0) GO TO 10
       DO 8 I=1,6000
    8 SAVEC(I) = SAVEC1(I)
   10 CONTINUE
      REWIND 23
C    REWIND 18 IN ORDER TO READ ALNGTH ARRAY.
      REWIND 18
      WRITE(6,1) NCTV,MP3,NCT
    1 FORMAT(*0SURROUTINE SOL*/3I5)
      WRITE(6,3) (BOUND(I),I=1,NCT)
    3 FORMAT(20H0BOUNDARY CONDITIONS/(1P10E13.5))
    2 FORMAT(*0SOLUTION*/(1P10E13.5))
      NCOLS=NCTV+MP3
      DO 20 I=1,NCOLS
      IF(BOUND(I).NE.0.0) GO TO 22
   20 CONTINUE
      DO 21 I=1,NCOLS
```

```
21    XSOL(I)=0.0
       DO 211 I=1,600
 211  ASOL(I) = 0.0
      GO TO 200
22    CONTINUE
      CALL MSOLX(NCTV,MP3,NCT,BOUND,XSOL,DUM)
      DO 2305 I=1,600
2305  ASOL(I)=XSOL(I)
23    CONTINUE
      NR=NCOLS
      KCN=LSEG(1)+TSEG(1)
      IF(KCN.EQ.0) GO TO 111
      READ(18) ALNGTH
      REWIND 18
      NTVV=NVT(1)
      NBVV=NVL(1)
      NR=NTVV*NBVV
      NC=ITAPE
      WRITE(6,231)(XSOL(I),I=1,NCOLS)
 231  FORMAT(50H0SOLUTION FOR COEFFICIENTS OF CONSTRAINT EQUATIONS/
     1           (1P10E13.5))
C
CONVERT FIRST NC XSOL TERMS INTO NR K'S.
C    NOW COMPUTE THE PRODUCT OF PHTH*A = GA, A COLUMN MATRIX
      DO 101 I=1,NR
 101  GA(I)=0.0
      DO 102 J=1,NC
      READ(23)(PHTH(I),I=1,NR)
      DO 102 I=1,NR
 102  GA(I)=GA(I)+PHTH(I)*XSOL(J)
C
C    NOW DO  R*L MATRIX PRE-MULTIPLY OF GA VECTOR... RESULT IS K VECTOR.
      K=0
      DO 110 J=1,NTVV
      TL=0.0
      DO 110 I=1,NBVV
      K=K+1
      AL1=ALNGTH(K-1)
      AL2=ALNGTH(K)
```

```
      AL3=ALNGTH(K+1)
      IF(I.EQ.1) AL1=0.0
      IF(I.EQ.NBVV) AL3=0.0
      AL=0.5*(0.75*AL1+AL2+0.25*AL3)
      XSOL(K)=AL*GA(K)+TL
      TL=XSOL(K)
  110 CONTINUE
  111 CONTINUE
      NB1=NVT(1)*NVL(1)
      IF(KCN .NE. 0) NB1 = ITAPE
      IF(NBODS.EQ.0) NB1 = 0
      IF(NBODS .EQ. 0) GO TO 301
      DO 300 I=1,NB1
  300 B1(I) = XSOL(I)
  301 IF(NPANS .EQ. 0) GO TO 700
       DO 305 I=1,NPANS
      NBP1 = NBODS+I
      LT(I) = NVT(NBP1)
      LN(I) = NVL(NBP1)
      IF(LFUNC(I+1) .NE. 0) LN(I) = LFUNC(I+1)
      IF(TFUNC(I+1) .NE. 0) LT(I) = TFUNC(I+1)
 6000 FORMAT(6I5,3X,2F15.5)
  305 NP1(I) = LT(I) * LN(I)
      IP1 = NB1
       DO 310 I=1,NPANS
      IT = (I-1) *1000
      M = NP1(I)
      WRITE(6,6000) I,NP1(I),IT,M,IP1
       DO 308 K=1,M
      P1(IT+K) = XSOL(IP1+K)
  308 CONTINUE
  310 IP1 = IP1 +M
C        NOW DO ANY CONVERSION NECESSARY FOR PANEL A'S TO K'S
       DO 500 IP=1,NPANS
      I=IP
      M = NP1(I)
      IT = (I-1) *1000
      LOC1 = (IP-1) *1000
      LOC2 = LOC1 +400
```

```
      LOC3 = LOC2 +400
      NBP1 = NBODS + IP
      IF(LN(I) .EQ. NVL(NBP1) .AND. LT(I) .EQ. NVT(NBP1)) GO TO 499
       DO 315 K=1,M
  315 DUM(K) = P1(IT+K)
      NTVVP = NVT(NBP1)
      NU = LN(IP)
      NW=LT(IP)
      NBVVP = NVL(NBP1)
      IF(LT(I) .NE. NVT(NBP1)) GO TO 330
      L = 0
       DO 320 J=1,NTVVP
       DO 320 I=1,NBVVP
      L = L + 1
      IF(J .GT. NSPP(IP)) GO TO 317
      SUM = 0.0
       DO 316 K=1,NU
  316 SUM = SUM + SAVEC(LOC2+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
      GO TO 319
  317 SUM = 0.0
      DO 318 K=1,NU
 6001 FORMAT(I5,1PE20.6)
  318 SUM = SUM + SAVEC(LOC1+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
  319 XSOL(L) = SUM
  320 CONTINUE
      GO TO 361
  330 CONTINUE
      L = 0
       DO 355 J=1,NTVVP
       DO 355 I=1,NU
      L = L + 1
      SUM = 0.0
      DO 346 K=1,NW
  346 SUM = SUM +DUM((K-1)*NU+I) *SAVEC((K-1)*NTVVP+J+LOC3)
      ATET(L) = SUM
      WRITE(6,6001) L,ATET(L)
  355 CONTINUE
      L = 0
      DO 360 J=1,NTVVP
```

```
      DO 360 I=1,NBVVP
      L = L+ 1
      IF(J .GT. NSPP(IP)) GO TO 357
      SUM = 0.0
       DO 356 K=1,NU
  356 SUM = SUM +SAVEC(LOC2+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
      GO TO 359
  357 SUM = 0.0
       DO 358 K=1,NU
  358 SUM = SUM + SAVEC(LOC1+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
  359 XSOL(L) = SUM
      WRITE(6,6001) L,XSOL(L)
  360 CONTINUE
 361  CONTINUE
      IT=(IP-1)*1000
      WRITE(6,6001) IT
       DO 370 I=1,L
  370 P1(IT+I) = XSOL(I)
 499  CONTINUE
  500 CONTINUE
      L = 0
      IF(NBODS .EQ. 0) GO TO 600
      NB1=NVT(1)*NVL(1)
       DO 550 I=1,NB1
      L = L + 1
 550  XSOL(L) = B1(I)
  600 IF(NPANS .EQ. 0) GO TO 700
       DO 650 I=1,NPANS
      IT = (I-1)*1000
      M=NVT(NBODS+I )*NVL(NBODS+I )
       DO 625 K=1,M
      L = L + 1
  625 XSOL(L) = P1(IT+K)
  650 CONTINUE
      WRITE(6,50)
 50   FORMAT(47H0CONVERT SOLUTION FOR BODY CONSTRAINT FUNCTIONS)
      NRM=NR+NMOVE
      WRITE(6,150) (XSOL(I),I=1,L)
 150  FORMAT(16H0SOLUTION MATRIX/(1P10E13.5))
```

```
200   CONTINUE                                                        4   1960
700   CONTINUE                                                        4   1970
      WRITE(6,2)(XSOL(I),I=1,NCOLS)                                   4   1980
      END                                                             4   1990
```

```
      SUBROUTINE MSOLX(NB,NP,NQT,B,XSOL,IL)
C     NKTB = NO. OF COLUMNS IN BODY INFLUENCE MATRIX.
C     NKTP = NO. OF COLUMNS IN PANEL INFLUENCE MATRIX.
      DIMENSION B(1),XSOL(1),IL(1)
      COMMON/BODY/ A(28000),AR(500),IXC(500)
      COMMON DA(5000)
      EQUIVALENCE (FUNCL,DA(34)), (FNB, DA(30)), (FNT,DA(31))
      NBVV = FNB
      NTVV = FNT
      WRITE(6,1000)
 1000 FORMAT(*1A MATRIX*)
      NKT=NB+NP
      IF(NB.EQ.0.OR.FUNCL.NE.0.0) GO TO 5
      NKT=NKT-NTVV
 5    CONTINUE
      NKTP=NKT+1
      N=(NKT*(NKT+3))/2
      DO 10 I=1,N
   10 A(I)=0.0
      NKT2=NKT+2
      DO 60 K=1,NQT
      IF(NB.EQ.0) GO TO 1
      READ(21)(AR(L),L=1,NB)
      IF(FUNCL.NE.0.) GO TO 23
      L2 = 1
      L1 =1
      NBM1 = NBVV -1
      DO 22 J=1, NTVV
      DO 21 I=1, NBM1
      AR(L1) = AR(L1+L2)
   21 L1 = L1 + 1
   22 L2 = L2 + 1
   23 CONTINUE
      IF(K.GT.20) GO TO 20
 1001 FORMAT('0ROW'I3'     BODY ON BODY'/(1P10E13.4))
 20   CONTINUE
 1    IF(NP.EQ.0) GO TO 2
      NN=NB+1
      IF( NB .EQ. 0.OR.FUNCL.NE.0.)GOTO 24
```

```
      NN = NN - NTVV
   24 CONTINUE
      READ(10)(AR(L),L=NN,NKT)
      IF(K.GT.20) GO TO 30
 1003 FORMAT('0ROW'I3'    WING ON BODY'/(1P10E13.4))
 30   CONTINUE
 2    CONTINUE
      AR(NKT+1)=B(K)
      IXI=1
      DO 50 I=1,NKT
      R=SQRT(A(IXI)**2+AR(I)**2)
      IF(R.EQ.0.) GO TO 50
      C=A(IXI)/R
      S=AR(I)/R
      IXJ=IXI
      DO 40 J=I,NKTP
      T2=C*A(IXJ)+S*AR(J)
      AR(J)=-S*A(IXJ)+C*AR(J)
      A(IXJ)=T2
   40 IXJ=IXJ+1
   50 IXI=IXI+NKT2-I
   60 CONTINUE
      REWIND 21
      REWIND 10
      II=1
      IXI=1
      DO 80 I=1,NKT
      IXC(I)=IXI
      XSOL(I)=0.
      IL(I)=0
      IF(A(IXI).LE.0.0000001) GO TO 80
      IL(I)=II
      II=II+1
   80 IXI=IXI+NKT2-I
      II=NKT
      DO 210 J=1,NKT
      IF(IL(II).LE.0) GO TO 210
      JI=IL(II)
      JS=IXC(JI)-JI
```

```
      JXI=JS+II
      JXN=JS+NKTP
      IF(II-NKT) 170,200,220
  170 IK=II+1
      JXK=JXI
      DO 180 K=IK,NKT
      JXK=JXK+1
  180 XSOL(II)=XSOL(II)-A(JXK)*XSOL(K)
  200 XSOL(II)=(XSOL(II)+A(JXN))/A(JXI)
  210 II=II-1
  220 CONTINUE
      IF(NB.EQ.0.OR.FUNCL.NE.0.0) GOTO 310
      L2 = 1
      L1 = 1
      DO 301 J=1, NTVV
      DO 300 I=1, NBVV
      IF(I.NE.1) GOTO 299
      AR(L1) = 0.0
      GOTO 300
  299 AR(L1) = XSOL(L1-L2)
  300 L1 = L1 + 1
  301 L2 = L2 + 1
      IF(NP.EQ.0) GOTO 304
      DO 303 I=1, NP
      AR(L1) = XSOL(L1-NTVV)
  303 L1 = L1 +1
  304 L1 = L1 -1
      DO 305 I=1, L1
  305 XSOL(I) = AR(I)
  310 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SFSOL (M,N)
C              COEFFICIENT MATRIX ON UNIT 23 BY ROWS
C               M ROWS
C               N COLUMNS
C              B VECTOR (RIGHT SIDE) IN BOUND
      COMMON /BODY/S(150,150),X(1000),  BWORK(150),XWORK(150),COL(150),
     1TOPCOL(150),WORK(150),IWORK(150),ATA(1000), ATB(1000), A(1000),
     2PAD(600)
      COMMON /SCRAT/XSOL(5000),BOUND(5000)
      INTEGER UNIT
      REWIND 10
C              GENERATE A TRANSPOSE A ON UNIT
      CALL ATRAN (A,M,N,BOUND,ATB,ATA,UNIT)
C              NOW SOLVE N X N SYSTEM
C              DETERMINE NROW, MAXIMUM PARTITION IS 150 ROWS
C              THERE MUST BE AT LEAST TWO PARTITION ROWS
      IF (N - 150) 5,5,7
    5 NROW = 2
      GO TO  9
    7 NROW = N /150
      IF (N - (NROW*150)) 8, 9,8
    8 NROW = NROW + 1
C              DETERMINE SIZES OF SUBMATRICES
C              INTERIOR SUBMATRICES
    9 NPO = N /NROW
C              UPPER LEFT SUBMATRIX
      NP1 = N  - NPO * NROW + NPO
C              STORE SUBMATRICES IN UNIT 10
      DO 15 KROW = 1,NROW
C              SUBMATRIX SIZE FOR KROW .NE. 1
      NP = NPO
C              SUBMATRIX SIZE FOR KROW = 1
      IF (KROW .EQ. 1) NP = NP1
C              ROW NUMBER OF TOP ELEMENT IN SUBMATRIX
      KTOP = N  - NP - (NROW-KROW)*NPO +1
C              ROW NUMBER OF BOTTOM ELEMENT IN SUBMATRIX
      KBOT = KTOP + NP - 1
C              STORE SUBMATRIX COLUMNS
C              READ ONE COLUMN AT A TIME FROM UNIT, N ELEMENTS
```

```
C                 LOAD JCOL OF COEFFICIENT MATRIX FROM KTOP TO KBOT
      DO 10 JCOL = 1,N
      READ (UNIT) (A(I), I=1,N)
   10 WRITE (10) (A(I), I = KTOP,KBOT)
C                 LOAD CONSTANT VECTOR FROM KTOP TO KBOT
      WRITE (10) (ATB(I), I = KTOP,KBOT)
   15 REWIND UNIT
      REWIND 10
C                 PUT SOLUTION IN X
   18 CALL XSOLVE (1000,150,N, NPO, NP1, NROW, S, XSOL, IWORK, WORK,
     1BWORK, XWORK, COL, TOPCOL)
      RETURN
      END
```

```
      SUBROUTINE XSOLVE(MX,NPX,N,NPO,NP1,NROW,Z,X,IWORK,WORK,
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C ***** SUBROUTINE FOR SOLUTION BY PARTITIONING USING FILE STORAGE *****
C
C ***** WRITTEN BY D.G. ELLIOTT USING SCHEME SUGGESTED BY C.L. LAWSON *
C ***** JET PROPULSION LABORATORY, PASADENA, NOVEMBER 1, 1972 *****
     *             BWORK,XWORK,COL,TOPCOL)
      INTEGER IWORK(NPX),UNIT1,UNIT2,UNIT3
      REAL WORK(NPX)
      REAL     Z(NPX,NPX),X(MX),BWORK(NPX),XWORK(NPX),
     *         COL(NPX),TOPCOL(NPX)
C
C ***** FORTRAN UNITS 10, 11, AND 12 USED FOR STORAGE *****
C
C ***** TRIANGULARIZATION *****
C
    6 NCXS = 0
      NCX1 = N+1-NP1
C                               TRIANGULARIZE THE NROW ROWS OF SUBMATRICES
       DO 90 KROW=1,NROW
C                     SET UP UNITS
      IF (KROW - 1) 2,2,3
    2 UNIT1 = 10
      UNIT2 = 21
      UNIT3 = 22
      GO TO 4
    3 JU = UNIT3
      UNIT3 = UNIT2
      UNIT2 = UNIT1
      UNIT1 = JU
C                               CONTAINS OLD LOWER-RIGHT MATRIX
C                               TO RECEIVE TRANSFORMED TOP ROW
C                               TO RECEIVE NEW LOWER-RIGHT MATRIX
C
C                               SIZE OF INTERIOR SUBMATRICES
    4 NP = NPO
C                               SIZE OF UPPER-LEFT SUBMATRIX
      IF(KROW .EQ. 1) NP = NP1
```

```
C                                 NUMBER OF COLUMNS BEYOND FIRST MATRIX IN KRO
      NCX = N+1-NP1-(KROW-1) *NPO
         DO 10 J=1,NP
C                                 READ MATRIX IN KROW
 10      READ (UNIT1) (Z(K,J),K=1,NP)
C                                 TRIANGULARIZE FIRST MATRIX
      CALL DECOMP(NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
C                                 TRANSFORM REST OF ROW, INCLUDING RT-SIDE VCT
         DO 20 J=1,NCX
C                                 NEXT COLUMN IN ROW
         READ (UNIT1) (BWORK(K),K=1,NP)
C                                 PUT TRANSFORMED COLUMN IN XWORK
              CALL SOLVE (NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
C                                 TRIANGULARIZATION COMPLETE
      IF(KROW .EQ. NROW) GO TO 100
C                                 STORE TRANSFORMED COLUMN OF KROW
   20    WRITE (UNIT2) (XWORK(K),K=1,NP)
C                                 NO PREVIOUS ROW TO STACK IN 2
      IF(KROW .EQ. 1) GO TO 40
C
      NNP = NPO
C                                 STACK TRANSFORMED ROWS IN REVERSE ORDER
         DO 30 J=1,NCXS
         IF((NCXS-J) .LT. NCX1) NNP = NP1
C                                 READ PRBVIOUSLY TRANSFORMED ROWS
         READ (UNIT3) (COL(K),K=1,NNP)
C                                 AND WRITE AFTER NEW ONE
   30    WRITE (UNIT2) (COL(K),K=1,NNP)
C
C                                 FOR WRITING NEW LOWER-RT MATRIX IN NEXT OPER
      REWIND UNIT3
C
   40 KROWP1 = KROW +1
C                                 SUBTRACT MULTIPLES OF TOP ROW FROM EACH ROW
         DO 80 KKROW=KROWP1,NROW
            DO 50 J=1,NP
C                                 READ FIRST MATRIX IN KKROW
   50       READ (UNIT1) (Z(K,J),K=1,NPO)
C                                 REPOSITION AT START OF TOP ROW
```

```
      REWIND UNIT2
C                             SUBTRACT MULTIPLES OF TOP COLUMNS FROM EA CO
      DO 70 J=1,NCX
C                             NEXT COLUMN IN KKROW OF OLD MAT
      READ (UNIT1) (COL(K),K=1,NPO)
C                             NEXT COLUMN IN TOP ROW
      READ (UNIT2) (TOPCOL(K),K=1,NP)
C
C                             SUBTRACT FROM EACH ELEMENT IN COLUMN
      DO 60 K=1,NPO
C                             SUBTRACT FIRST MAT TIMES TOP ROW
      DO 60 JJ=1,NP
   60 COL(K) = COL(K) -Z(K,JJ) *TOPCOL(JJ)
C
C                             NEXT COL IN NBW MATRIX
   70 WRITE (UNIT3) (COL(K),K=1,NPO)
   80 CONTINUE
      REWIND UNIT1
      REWIND UNIT2
      REWIND UNIT3
C
C                             TOTA NUMBER OF COLUMNS STORED IN UNIT2
      NCXS = NCXS +NCX
   90 CONTINUE
C
C  ***** BACK SUBSTITUTION *****
C
  100 DO 105 K=1,N
  105 X(K) = 0
C                             ZERO X-VECTOR
      DO 110 K=1,NPO
C                             BOTTOM SUBVECTOR IN X
  110 X(N-NPO+K) = XWORK(K)
      NROWM1 = NROW -1
C                             BACK SUBSTITUTE FROM BOTTOM
      DO 140 KRX=1,NROWM1
      KROW = NROWM1 -KRX +1
      NP = NPO
      NMKROW = NROW -KROW
```

```
      IF(KROW .EQ. 1) NP = NP1
      KSTART = N-NP-NMKROW *NP0
C                         SOLVE FOR KROW SUBVECTOR
          DO 130 KCOL=1,NMKROW
          KCOLM1 = KCOL -1
              DO 120 J=1,NP0
C                         NEXT MATRIX IN KROW
  120         READ (UNIT3) (Z(K,J),K=1,NP)
C                         FORM PRODUCT OF MATRICES AND SUBVECTORS
              DO 130 K=1,NP
                  DO 130 JJ=1,NP0
  130         X(KSTART+K) = X(KSTART+K) -Z(K,JJ) *X(KSTART+NP+KCOLM1
     * *NP0+JJ)
C                         READ RIGHT SIDE VECTOR
      READ (UNIT3) (BWORK(K),K=1,NP)
C                         ADD RIGHT-SIDE VECTOR TO COMPLETE X FOR KROW
          DO 140 K=1,NP
  140     X(KSTART+K) = X(KSTART+K) +BWORK(K)
      RETURN
      END
```

```
      SUBROUTINE ATRAN (A,M,N,B,ATB,ATA, UNIT)
C          M    A TRANSPOSE TIMES A (ATRAN)
C               A STORED BY ROWS ON UNIT 23
C                M ROWS, N COLUMNS, M IS GREATER THAN OR EQUAL N
C               A TRANSPOSE A STORED BY COLUMNS (ROWS DUE TO SYMMETRY)
C               UNIT IS AUXILIARY UNIT CONTAINING A TRANSPOSE A
      DIMENSION A(N), ATA(N), B(M), ATB(N)
      INTEGER UNIT, UNITN, UNITN1
C               PREPARE AUXILIARY STORAGE
    1 UNITN = 21
      UNITN1 = 22
      IFLAG = -1
      REWIND 21
      REWIND 22
      REWIND 23
      DO 5 I = 1,N
      DO 4 J = 1,N
    4 ATA(J) = 0.0
    5 WRITE (21) ATA
      REWIND 21
C               INITIALIZE STORAGE FOR A TRANSPOSE B VECTOR
      DO 6 I = 1,N
    6 ATB(I) = 0.0
C               READ A ROW OF A
      DO 40 I = 1,M
      READ (23)  A
C               READ PARTIAL VALUE OF ELEMENTS IN A COLUMN OF ATA
        DO 30 J = 1,N
        READ (UNITN) ATA
C               GENERATE NEXT TERM FOR ELEMENTS IN COLUMN O ATA
          DO 20 L = 1,N
   20     ATA(L) = ATA(L) + A(J) *A(L)
          WRITE (UNITN1) ATA
   30   CONTINUE
C               GENERATE NEXT TERM FOR ELEMENTS OF ATB
      DO 25 K = 1,N
   25 ATB(K) = ATB(K) + A(K) * B(I)
      IFLAG = IFLAG * (-1)
      UNITN = UNITN + IFLAG
```

```
      UNITN1=UNITN1 - IFLAG                                             4   5370
      REWIND UNITN                                                      4   5380
      REWIND UNITN1                                                     4   5390
   40 CONTINUE                                                          4   5400
      UNIT = UNITN                                                      4   5410
      RETURN                                                            4   5420
      END                                                               4   5430
```

```
      SUBROUTINE DECOMP(MX,N,UL,B,X,SCALES,IPS)
C
C  ***** SUBROUTINE FOR SIMULTANEOUS EQUATION SOLVING *****
C  ***** REF' G. FORSYTHE AND C. MOLER, 'COMPUTER SOLUTION OF LINEAR
C              ALGEBRAIC SYSTEMS', PRENTICE-HALL, 1967 *****
C
C  ***** NOMENCLATURE' UL=COEFF MATRIX, B=RIGHT-SIDE VCTR, X=UNKNOWN VCT
C  ***** CALL TO DECOMP TRIANGULARIZES THE COEFFICIENT MATRIX *****
C
      REAL    UL(MX,MX),B(MX),X(MX),PIVOT,EM,SUM
      DIMENSION SCALES(MX),IPS(MX)
C
C INITIALIZE IPS AND SCALES
          DO 5 I = 1,N
          IPS(I) = I
          ROWNRM = 0.0
              DO 2 J = 1,N
2             ROWNRM =AMAX1( ROWNRM,  ABS(UL(I,J)) )
          IF (ROWNRM) 3,4,3
3         SCALES(I) = 1.0/ROWNRM
          GO TO 5
4         PRINT 20
          SCALES(I) = 0.0
5         CONTINUE
C
C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
      NM1=N-1
          DO 17 K=1,NM1
          BIG = 0.0
              DO 11 I = K,N
              IP = IPS(I)
              SIZE =  ABS(UL(IP,K))*SCALES(IP)
              IF (SIZE-BIG) 11,11,10
10            BIG = SIZE
              IDXPIV = I
11            CONTINUE
          IF (BIG) 13,12,13
12        PRINT 25
          GO TO 17
```

```
13          IF (IDXPIV-K) 14,15,14
14          J = IPS(K)
            IPS(K) = IPS(IDXPIV)
            IPS(IDXPIV) = J
15          KP = IPS(K)
            PIVOT = UL(KP,K)
            KP1 = K+1
                 DO 16 I = KP1,N
                 IP = IPS(I)
                 EM = -UL(IP,K)/PIVOT
                 UL(IP,K) = -EM
                 DO 161 J1= KP1,N
                 UL(IP,J1)= UL(IP,J1) + EM*UL(KP,J1)
  161            CONTINUE
16               CONTINUE
17          CONTINUE
       KP = IPS(N)
       IF (UL(KP,N)) 19,18,19
18     PRINT 20
 19     GO TO 500
20     FORMAT (* MATRIX WITH ZERO ROW IN DECOMP*)
25     FORMAT (* SINGULAR MATRIX IN DECOMP*)
C
C  ***** THIS ENTRY BACK-SUBSTITUTES THE RIGHT SIDE *****
C
       ENTRY SOLVE
       NP1 = N + 1
C
       IP = IPS(1)
       X(1) = B(IP)
            DO 30 I = 2,N
            IP = IPS(I)
            SUM = 0.0
            IM1=I-1
                 DO 27 J=1,IM1
27               SUM = SUM + UL(IP,J)*X(J)
30          X(I) = B(IP) - SUM
C
       IP = IPS(N)
```

```
      X(N) = X(N)/UL(IP,N)
         DO 40 IBACK = 2,N
         I = NP1-IBACK
C I GOES (N-1),...,1
         IP = IPS(I)
         IP1 = I+1
         SUM = 0.0
            DO 35 J = IP1,N
35          SUM = SUM + UL(IP,J)*X(J)
40       X(I) = (X(I)-SUM)/UL(IP,I)
 500  RETURN
      END
```

```
      PROGRAM VEL
      REWIND 12
      CALL VELOC
      END
      SUBROUTINE VELOC
      COMMON/SLOPE/SIGMAP(500),  DZDXT(500),DZDXC(500),TANP1(500)
      EQUIVALENCE  (B(11871),SIGMA)
      DIMENSION SIGMA(1000)
      COMMON/SCRAT/  AK(5000),XG(11,151),GAMT(11,151),GAMB(1500)
     1   ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
     2   ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
     3   ,CP(1320)
      EQUIVALENCE(XCCO,B(2421))
      DIMENSION XCCO(20),XOC(20)
      COMMON/BODY/ B(31000)
      EQUIVALENCE(B(14044),TMX),   (B(15364),TMY),   (B(16684),TMZ)
     1           ,(B(18004),TTX),   (B(19324),TTY),   (B(20644),TTZ)
      EQUIVALENCE(DA(3),FMACH)
      DIMENSION TMX(1320),TMY(1320),TMZ(1320)
     1          ,TTX(1320),TTY(1320),TTZ(1320)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1               ,XN(1320),YN(1320),ZN(1320)
      COMMON DA(5000)
      COMMON      /COMPRS/   BETAM
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
C
C  WHEN SOURCE MATRIX IS READ, GAMMA ARRAYS WILL BE DESTROYED.
      EQUIVALENCE (SMX,XG),(SMY,GAMT),(SMZ,GAMB)
      DIMENSION SMX(800),SMY(800),SMZ(800)
C
      DIMENSION VX(1320),VY(1320),VZ(1320)
      EQUIVALENCE(VX,B(22001)),(VY,B(23321)),(VZ,B(24641))
C  VX,VY,VZ  EQUIVALENCING SHOULD NOT DESTROY SIGMA ARRAY.
C
      EQUIVALENCE (DA(7), XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
     1 ,(DA(11),BETA), (DA(12),PSTAR),(DA(13),QSTAR),(DA(14),RSTAR)
```

```
      COMMON/PANEL/ NPAN,IPSYM,INC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1  ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2  ,NTHET,NTIP,CHTIP,ROOT,OUTER,NMATT
     3  ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      DIMENSION AX(4681),AY(4681),AZ(4681)
      EQUIVALENCE(B(1),AX),(B(4682),AY),(B(9363),AZ)
      EQUIVALENCE  (B,BB,BP)
      EQUIVALENCE(AKTB,B(24001)),(AKTP,B(24501)),(DA(2),PANS)
      DIMENSION AKTB(500),AKTP(500)
      DIMENSION BB(22000),BP(13000)
C
C    BB IS READ INTO CORE TO GET TMX - TTZ ARRAYS INTO CORE.
C    BB IS NOT LONG ENOUGH TO DESTROY SIGMA ARRAY.
C
C    MP2 IS UNIT FOR PANEL INFLUENCE MATRIX
C    MP3 IS NUMBER OF AX,AY,AZ TERMS IN EACH ROW OF THE MATRIX ON MP2.
C    NCB = NO. OF CONTROL POINTS ON BODY.
C
      COMMON/PANINF/ PANSYM(10),ASOL(600)
      DO 1 I=1,20
    1 XOC(I)=XCCO(I)
C
C    TEMPORARY BREF,CBAR
      BREF=1.0
      CBAR=10.0
C
      GAM14=1.4
      IF(NBODS.NE.0) REWIND NTAPE
      IF(PANS.NE.0.0) REWIND MP2
CALL GAMMA FOR CAPITAL GAMMA ARRAYS.
      CALL GAMMA
C
CALL DELV FOR DELVM AND DELVI ARRAYS.
      CALL DELV
      WRITE(6,5000) (DELVT(I),I=1,200)
 5000 FORMAT( 6H0DELVT//(1H ,10F12.6))
      WRITE(6,5001) (DELVM(I),I=1,200)
 5001 FORMAT( 6H0DELVM//(1H ,10F12.6))
      IF(NBODS.EQ.0) GO TO 4
```

```
C
      READ(18)
      READ(18)
      READ(18) BB
    4 CONTINUE
C
C
      NCB=NCPTS(1)
      IF(NBODS.EQ.0) NCB=0
      NPANS=PANS
      DO 5 I=1,NCT
      XX=XQ(I)-XCG
      YY=YQ(I)-YCG
      ZZ=ZQ(I)-ZCG
      VX(I)=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/BREF)
      VY(I)=-BETA-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
      VZ(I)=ALPHA+2.0*(PSTAR*YY/BREF+QSTAR*XX/CBAR)
    5 CONTINUE
      NPWT=(NCT-NCB)*2+NCB
      DO 6 I=1,NPWT
      VM(I)=0.0
    6 VT(I)=0.0
      IF(NPANS.EQ.0) GO TO 2005
      ISIG=0
C   READ PANEL DATA TO  GET SIGMA
      DO 7 I=1,NPANS
    7 READ(18) BP
      REWIND 18
      WRITE(6,8000) (ASOL (I),I=1,220)
 8000 FORMAT(1H ,1P10E13.4)
      WRITE(6,8000) (SIGMA(I),I=1,280)
      WRITE(6,8000) (SIGMAP(I),I=1,280)
      WRITE(6,8000) (TANP1(I),I=1,140)
      WRITE(6,8000) (DZDXT(I),I=1,140)
      WRITE(6,8000) (DZDXC(I),I=1,140)
      DO 2000 I=1,NPANS
      NSIG=NVPTS(NBODS+I)*2
C
COMPUTE TERMS INVOLVING SOURCE MATRICES.
```

```
      DO 1000 IC=1,NCT
      T1=0.0
      T2=0.0
      T3=0.0
      T4=0.0
      T5=0.0
      T6=0.0
      READ(11)(SMX(J),SMY(J),SMZ(J),J=1,NSIG)
      DO 50 J=1,NSIG
      IS=ISIG+J
      T1=T1+(SMX(J)*TMX(IC)+SMY(J)*TMY(IC)+SMZ(J)*TMZ(IC))*SIGMA(IS)
      T2=T2+(SMX(J)*TTX(IC)+SMY(J)*TTY(IC)+SMZ(J)*TTZ(IC))*SIGMA(IS)
      T3=T3+SMX(J)*SIGMA(IS)
      SIGDIF=SIGMA(IS)-SIGMAP(IS)
      T4=T4+SMX(J)*SIGDIF
      TERM=ZN(IC)*SMY(J)-YN(IC)*SMZ(J)
      T5=T5+TERM*SIGMA(IS)
      T6=T6+TERM*SIGDIF
   50 CONTINUE
C   NOW A4D MORE TERMS TO THOSE ABOVE.
      IF(I.NE.NPANS) GO TO 1000
      IF(9C.GT.NCB) GO TO 500
      VM(IC)=T1
      VT(IC)=T2
      WRITE(6,8003) IC,VM(IC),VT(IC)
 8003 FORMAT(I4,1P6E20.5)
      GO TO 600
C
  500 IX=2*IC-NCB-1
COMPUTE SUBSCRIPT FOR TOP CONTROL POINT ON PANEL.
C   TOP WILL HAVE VELOCITY(IX)
C   BOTTOM WILL HAVE VELOCITY(IX+1).
      ICP=IC-NCB
      VM(IX)=T3+DELVM(IC)*(1.0+SQRT(TANP1(ICP))*T4)
      VM(IX+1)=T3-DELVM(IC)*(1.0+SQRT(TANP1(ICP))*T4)
      TERM = T5+ZN(IC)*VY(IC)*BETAM -YN(IC)*VZ(IC)*BETAM
      TERM2=DELVT(IC)*(1.0+SQRT(TANP1(ICP))*T6)
      VT(IX)=TERM+TERM2
      VT(IX+1)=TERM-TERM2
```

```
      WRITE(6,8003)IC,YN(IC),ZN(IC),VY(IC),VZ(IC)
      WRITE(6,8003) IX,VM(IX),VT(IX),T3,T4,T5,T6
      IXXX=IX+1
      WRITE(6,8003) IXXX,VM(IXXX),VT(IXXX),DELVM(IC),DELVT(IC)
  600 CONTINUE
 1000 CONTINUE
      ISIG=IS
 2000 CONTINUE
 2005 NB1=NBODS+1
      DO 3000 IC=1,NCT
      FLAG=1.0
      IF(NPANS.EQ.0) GO TO 2014
      IF(IC-NCB-NCPTS(NB1).GT.0) NB1=NB1+1
      LPTS=LNC(NB1)
      ICR=IC-((IC-1)/LPTS)*LPTS
 2014 CONTINUE
      IX=2*IC-NCB-1
      IF(NBODS.EQ.0) GO TO 2050
      READ(NTAPE)(AX(I),AY(I),AZ(I),I=1,NCTV)
      IF(IC.GT.NCB) GO TO 2020
      TERM=0.0
      DO 2015 I=1,NCTV
 2015 TERM=TERM+(TMX(IC)*AX(I)+TMY(IC)*AY(I)+TMZ(IC)*AZ(I))*ASOL(I)
      VM(IC)=VM(IC)+TERM
      TERM=0.0
      DO 2016 I=1,NCTV
 2016 TERM=TERM+(TTX(IC)*AX(I)+TTY(IC)*AY(I)+TTZ(IC)*AZ(I))*ASOL(I)
      VT(IC)=VT(IC)+TERM
      VM(IC) = VM(IC)+BETAM**2*TMX(IC)*VX(IC)+BETAM*TMY(IC)*VY(IC)+
     1BETAM*TMZ(IC)*VZ(IC) +DELVM(IC)
      VT(IC) = VT(IC)+BETAM**2*TTX(IC)*VX(IC)+BETAM*TTY(IC)*VY(IC)+
     1BETAM*TTZ(IC)*VZ(IC) +DELVT(IC)
      DBETA=SQRT(BETAM**2*(TMX(IC)**2+TMY(IC)**2+TMZ(IC)**2))
      VM(IC) = VM(IC) /BETAM/DBETA
      DBETA=SQRT(BETAM**2*(TTX(IC)**2+TTY(IC)**2+TTZ(IC)**2))
      VT(IC) = VT(IC) /BETAM/DBETA
      GO TO 2050
 2020 TERM=0.0
      DO 2025 I=1,NCTV
```

```
2025 TERM=TERM+AX(I)*ASOL(I)
     WRITE(6,8001) IC,TERM
8001 FORMAT(1H0,I5,1PE15.5)
     VM(IX)=VM(IX)+TERM
     VM(IX+1)=VM(IX+1)+TERM
     TERM=0.0
     DO 2026 I=1,NCTV
2026 TERM=TERM+(ZN(IC)*AY(I)-YN(IC)*AZ(I))*ASOL(I)
     WRITE(6,8001) IC,TERM
     VT(IX)=VT(IX)+TERM
     VT(IX+1)=VT(IX+1)+TERM
2050 CONTINUE
     IF(NPANS.EQ.0) GO TO 2900
     REA4(MP2)(AX(I),AY(I),AZ(I),I=1,MP3)
     IF(IC.GT.NCB) GO TO 2070
     TERM=0.0
     DO 2065 I=1,MP3
     IK=NCTV+I
2065 TERM=TERM+(TMX(IC)*AX(I)+TMY(IC)*AY(I)+TMZ(IC)*AZ(I))*ASOL(IK)
     VM(IC)=VM(IC)+TERM
     TERM=0.0
      DO 2066 I=1,MP3
     IK=NCTV+I
2066 TERM=TERM+(TTX(IC)*AX(I)+TTY(IC)*AY(I)+TTZ(IC)*AZ(I))*ASOL(IK)
     VT(IC)=VT(IC)+TERM
     GO TO 2900
2070 TERM=0.0
     DO 2075 I=1,MP3
     IK=I+NCTV
2075 TERM=TERM+AX(I)*ASOL(IK)
     WRITE(6,8001) IC,TERM
     VM(IX)=VM(IX)+TERM
     VM(9X+1)=VM(IX+1)+TERM
     TERM=0.0
     DO 2076 I=1,MP3
     IK=I+NCTV
2076 TERM=TERM+(ZN(IC)*AY(I)-YN(IC)*AZ(I))*ASOL(IK)
     WRITE(6,8001) IC,TERM
     VT(IX)=VT(IX)+TERM
```

```
      VT(IX+1)=VT(IX+1)+TERM
      IF(IC.LE.NCB) GO TO 2900
      ICP=IC-NCB
      TOP=1.0/SQRT(1.0+TANP1(ICP)*(DZDXT(ICP)+DZDXC(ICP))**2)
      BOTTOM=1.0/SQRT(1.0+TANP1(ICP)*(DZDXT(ICP)-DZDXC(ICP))**2)
      VM(9X) = TOP*(VM(IX)/BETAM**2 +VX(IC))
      VM(IX+1) = BOTTOM*(VM(IX+1)/BETAM**2 + VX(IC))
      VT(IX) = TOP* VT(IX)/BETAM
      VT(9X+1) = BOTTOM*VT(IX+1)/BETAM
      WRITE(6,8002) IX,VM(IX),VM(IX+1),VT(IX),VT(IX+1)
 8002 FORMAT(I4,1P4E20.5//)
 2900 IF(IC-NCB) 2901,2901,2902
 2901 ICX=IC
      GO TO 2903
 2902 ICX=IX
 2903 A=1.0-VT(ICX)**2-VM(ICX)**2
      AM2=A*FMACH**2
      IF(6MACH-0.1) 2905,2905,2904
 2904 CP(9CX)=(2.0/(GAM14*FMACH**2))*((1.0+(GAM14-1.0)*0.5*AM2)
     1   **(GAM14/(GAM14-1.0))-1.0)
      GO TO 2906
 2905 CP(ICX)=A*(1.0+0.25*AM2*(1.0-0.1*AM2))
 2906 CONTINUE
      IF(FLAG.EQ.-1.0) GO TO 2908
      IF(IC-NCB) 2908,2908,2907
 2907 FLAG=-1.0
      ICX=ICX+1
      GO TO 2903
 2908 CONTINUE
 3000 CONTINUE
      IXX=IX+1
      IF(NPANS.EQ.0) IXX=IC-1
      REWIND 11
      IF(NBODS.NE.0) REWIND NTAPE
      IF(PANS.NE.0.0) REWIND MP2
C
C     SET TRAILING EDGE K VALUES.
      L=0
      IV=0
```

```
      IF(NBODS.EQ.0) GO TO 310
      NT=NVT(1)
      NL=NVL(1)
      IV=NT*NL
      DO 305 I=1,NT
      J=I*NL
      LTD=LTDIV(1)
      DO 305 I1=1,LTD
      L=L+1
305   AKTB(L)=AK(J)
      WRITE(6,7000) (I,AKTB(I),I=1,L)
7000  FORMAT(*0AKTB*/(I5,1PE20.6))
310   IF(PANS.EQ.0.0) GO TO 400
      L=0
      DO 350 K=1,NPANS
      NT=NVT(NBODS+K)
      NL=NVL(NBODS+K)
      DO 340 I=1,NT
      J=I*NL+IV
      L=L+1
340   AKTP(L)=AK(J)
      WRITE(6,7001) (I,AKTP(I),I=1,L)
7001  FORMAT(*0AKTP*/(I5,1PE20.6))
350   IV=IV+NT*NL
400   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DELV
      COMMON DA(5000)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/SCRAT/XSOL(5000),XG(11,151),GAMT(11,151),GAMB(1500)
     1   ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
     2   ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
     3   ,CP(1320),XGW(50,20),GAMTW(50,20)
      EQUIVALENCE(DA(2),PANS)
      EQUIVALENCE (DA(2800),FLNC),  (DA(3100),FLTC)
      DIMENSION FLNC(150),FLTC(40)
      COMMON/BODY/ B(31000)
      EQUIVALENCE (ALNGTH,B(20000)),(BLNGTH,B(25000))
      DIMENSION ALNGTH(5000),BLNGTH(5000)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1             ,XN(1320),YN(1320),ZN(1320)
      COMMON/CONTRV/ LNN(3,40),LTT(3,40)
      COMMON/AF/ ATTFB(10),ATTFW(50)
      IF(NBODS.EQ.0) GO TO 5
C
      REW9ND 18
      NNV=NVL(1)*NVT(1)
      READ(18) (ALNGTH(I),I=1,NNV)
      READ(18) (BLNGTH(I),I=1,NNV)
      REWIND 18
    5 CONTINUE
C
      NPANS=PANS
      KC=0
      WRITE(6,4998)(GAMB(I),I=1,100)
 4998 FORMAT( 5HOGAMB//(1P10E12.4))
      IF(NBODS.EQ.0) GO TO 100
C
COMPUTE DELVM,DELVT AT BODY CONTROL POINTS.
      LTC3=LTC(1)
      LNCC=LNC(1)
      DO 50 J=1,LTCC
```

```
C
C   IT1,IT2 ARE TRAILING VORTICES ON WHICH INTERPOLATED GAMT ARE FOUND.
      IT1=FLTC(J)+0.01
      IT2=IT1+1
      DO 50 I=1,LNCC
      KC=KC+1
C    IB1,IB2 ARE BOUND VORTICES.
      IB1=FLNC(I)+0.01
      IB2=IB1+1
      L2=(IT1-1)*NVL(1)+IB1-1
      IF(IB1.NE.1) GO TO 10
      ALM2=0.0
      GO TO 12
   10 ALM2=ALNGTH(L2)
   12 L5=L2+1
      ALM5=ALNGTH(L5)
      IF(IB1.NE.NVL(1)) GO TO 14
      ALM8=0.0
   13 ALM11=0.0
      GO TO 20
   14 L8=L5+1
      ALM8=ALNGTH(L8)
      IF(IB2.GE.NVL(1)) GO TO 13
      ALM11=ALNGTH(L8+1)
   20 CONTINUE
      NGAM1=NVL(1)
      NGAM2=NGAM1
      IF(ATTFB(IT1).EQ.99.0) NGAM1=20
   25 CONTINUE
      IF(ATTFB(IT2).EQ.99.0) NGAM2=20
   26 CONTINUE
      DO 261 M=1,NGAM1
      XD(M)=XG(IT1,M)
  261 GD(M)=GAMT(IT1,M)
      IF(XQ(KC).GT.XD(NGAM1))   XD(NGAM1) = XG(KC)
      CALL CODIM(XD,GD,NGAM1,XQ(KC),GAM1,1)
      DO 262 M=1,NGAM2
      XD(M)=XG(IT2,M)
  262 GD(M)=GAMT(IT2,M)
```

```
      IF(XQ(KC).GT.XD(NGAM2))  XD(NGAM2) = XQ(KC)
      CALL CODIM(XD,GD,NGAM2,XQ(KC),GAM2,1)
      L2=(IT1-1)*NVL(1)+IB1
      L1=L2-NVL(1)
      L3=L2+NVL(1)
      BL2=BLNGTH(L2)
      IF(IT1.NE.1) GO TO 35
      BL1=BL2
      BL3=BLNGTH(L3)
      GO TO 40
35    IF(IT1.NE.NVT(1)) GO TO 36
      BL1=BLNGTH(L1)
      BL3=BL2
      GO TO 40
36    BL1=BLNGTH(L1)
      BL3=BLNGTH(L3)
40    CONTINUE
      D =   0.75*ALM5+0.25*ALM8
      IF(IB1-NVL(1)) 42,41,42
41    GAMB2=0.0
      GO TO 43
42    GAMB2=GAMB(L2+1)
43    CONTINUE
6002  FORMAT(8I5)
6001  FORMAT(1H0,1P4E20.4)
      DELVM(KC)=GAMB(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
     1         +GAMB2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D
      DELVT(KC)=-0.5*(GAM1/(BL1+BL2) +GAM2/(BL2+BL3))
50    CONTINUE
100   CONTINUE
      IF(NPANS.EQ.0) GO TO 300
      KTRV=0
      IF(NBODS) 101,101,102
101   NBVORT=0
      GO TO 103
102   NBVORT=NVL(1)*NVT(1)
103   CONTINUE
      DO 250 II=1,NPANS
      NBP=NBODS+II
```

```
COMPUTE D5LVM,DELVT ON PANELS.
C
      LTCC=LTC(NBP)
      LNCC=LNC (NBP)
      DO 200 J=1,LTCC
      IT1=LTT(II,J)+0.01+KTRV
      IT2=IT1+1
      DO 200 I=1,LNCC
      KC=KC+1
      IB1=LNN(II,I)+0.01
      IB2=IB1+1
      IT1L=IT1-KTRV
      IT2L=IT1L+1
      L2=(IT1L-1)*NVL(NBP)+IB1-1+NBVORT
      L5=L2+1
      L8=L5+1
      L11=L8+1
      ALM2=(ALNGTH(L2))
      ALM5=(ALNGTH(L5))
      ALM8=(ALNGTH(L8))
      ALM11=(ALNGTH(L11))
      IF(IB1.EQ.1) ALM2=ALM5
      IF(IB1.EQ.NVL(NBP)) ALM8=ALM5
      IF(IB2.EQ.NVL(NBP)) ALM11=ALM8
      NGAM1=NVL(NBP)
      NGAM2=NGAM1
      IF(ATTFW(IT1).EQ.99.0) NGAM1=20
125   CONTINUE
      IF(ATTFW(IT2).EQ.99.0) NGAM2=20
126   CONTINUE
      DO 361 M=1,NGAM1
      XD(M)=XGW(IT1,M)
361   GD(M)=GAMTW(IT1,M)
      CALL CODIM(XD,GD,NGAM1,XQ(KC),GAM1,1)
      DO 362 M=1,NGAM2
      XD(M)=XGW(IT2,M)
362   GD(M)=GAMTW(IT2,M)
      CALL CODIM(XD,GD,NGAM2,XQ(KC),GAM2,1)
      L2=(IT1L-1)*NVL(NBP)+IB1 +NBVORT
```

```
      L1=L2-NVL(NBP)
      L3=L2+NVL(NBP)
      BL2=BLNGTH(L2)
      IF(IT1L.NE.1) GO TO 135
      BL1=BL2
      BL3=BLNGTH(L3)
      GO TO 140
  135 IF(IT1L.NE.NVT(NBP)) GO TO 136
      BL1=BLNGTH(L1)
      BL3=BL2
      GO TO 140
  136 BL1=BLNGTH(L1)
      BL3=BLNGTH(L3)
  140 CONTINUE
      D=0.75*ALM5+0.25*ALM8
      IF(IB1-NVL(NBP)) 142,141,142
  141 GAMB2=0.0
      GO TO 143
  142 GAMB2=GAMB(L2+1)
  143 CONTINUE
      DELVM(KC)=GAMB(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
     1         +GAMB2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D
      TERMP=-DELVM(KC)*SLOPEC(L2)
C
C  ADD TERMP ONLY FOR PANEL.
      DELVT(KC)=-0.5*(GAM1/(BL1+BL2)+GAM2/(BL2+BL3)) + TERMP
  200 CONTINUE
      KTRV=KTRV+NVT(NBP)+1
      NBVORT=NVPTS(NBP)+NBVORT
  250 CONTINUE
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GAMMA
C
C   THIS PROGRAM COMPUTES GAMMA'S FOR BOUND AND TRAILING VORTICES.
C
C,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
C   INSERT NATT
      COMMON/PANATT/ NATT(30),XATT(200)
C,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/SCRAT/  AK(5000),XG(11,151),GAMT(11,151),GAMB(1500)
     1   ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
     2   ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
     3   ,CP(1320),XGW(50,20),GAMTW(50,20)
      COMMON DA(5000)
      COMMON/AF/ ATTFB(10),ATTFW(50)
      EQUIVALENCE (DA(2),PANS)
      DATA L/0/,KBODS/0/,NTBODS/0/
      REWIND 18
      IF(NBODS.EQ.0) GO TO 1111
C   SKIP ALNGTH AND BLNGTH  ARRAYS
      READ(18)
      READ(18)
 1111 CONTINUE
      NPANS=PANS
      IF(NBODS.EQ.0) GO TO 100
      DO 10 I=1,NBODS
      IF(I-1) 1,1,2
 1    M=1
      GO TO 3
 2    M=NCPTS(I-1)+1
 3    CONTINUE
      CALL BODGAM(I,AK(M),NVL(I),NVT(I),L)
      KBODS=KBODS+NCPTS(I)
 10   CONTINUE
 100  IF(NPANS.EQ.0) GO TO 205
      KPI=KBODS+1
```

```
      DO 200 I=1,NPANS
      KP=KPI
      INB=I+NBODS
      CALL PANGAM(I,AK,NVL(INB),NVT(INB),KP,L,NTBODS)
      WRITE(6,7007) (AK(IX),IX=1,100)
 7007 FORMAT(*0AK AFTER PANGAM*/(1P10E13.4))
      KPI=KPI+NCPTS(INB)
  200 CONTINUE
  205 CONTINUE
      REW9ND 18
      IF(NPANS.EQ.0) RETURN
      NTRVP=0
      N1PRE=0
      DO 300 I=1,NPANS
      NA=NATT((I-1)*3+2)
C   NA=COMPONENT (BODY OR PANEL) TO WHICH PANEL I IS ATTCHED.
      IF(NA.EQ.0) GO TO 260
C
C   DETERMINE WHICH TRAILING VORTICES NEED TO HAVE MODIFIED GAMT'S.
C   THESE WILL BE THE VORTICES NUMBERED N1 AND N2.
      N1=0
      IF(NA.EQ.1) GO TO 190
      N11=NA-1
      DO 180 K=1,N11
  180 N1=NVT(K)+N1+1
  190 N1=N1+NATT(3*I)
      N2=NTRVP+1
C   N1=TRAILING VORTEX LINE TO WHICH PANEL I IS ATTACHED.
C   N2=IN2OARD TRAILING VORTEX LINE OF ATTACHED PANEL I.
C   DETERMINE XMIN AND XMAX OF TR. VORTICES N1,N2.
      IF(XG(N1,1)-XGW(N2,1)) 202,202,203
  202 XMIN=XG(N1,1)
      GO TO 204
  203 XMIN=XGW(N2,1)
  204 NVOR1=NVL(NA)
      NVOR2=NVL(NBODS+I)
      IF(N1.EQ.N1PRE) NVOR1=20
      IF(XG(N1,NVOR1)-XGW(N2,NVOR2)) 206,206,207
  206 XMAX=XGW(N2,NVOR2)
```

```
      GO TO 208
  207 XMAX=XG(N1,NVOR1)
  208 CONTINUE
      N20=19
      XINT=XMAX-XMIN
      DX=XINT/N20
      N21=N20+1
      DO 210 M=1,N21
  210 XAA(M)=XMIN+(M-1)*DX
      DO 211 M=1,NVOR1
      XD(M)=XG(N1,M)
  211 GD(M)=GAMT(N1,M)
      CALL CODIM(XD,GD,NVOR1,XAA,GAMMA1,N21)
      DO 230 M=1,N21
      IF(XAA(M) -XG(N1,1)) 222,224,224
  222 GAMMA1(M)=0.0
      GO TO 230
  224 IF(XAA(M)-XG(N1,NVOR1)) 228,228,226
  226 GAMMA1(M)=GLAST
      GO TO 230
  228 GLAST=GAMMA1(M)
  230 CONTINUE
      DO 213 M=1,NVOR2
      XD(M)=XGW(N2,M)
  213 GD(M)=GAMTW(N2,M)
      CALL CODIM(XD,GD,NVOR2,XAA,GAMMA2,N21)
      DO 240 M=1,N21
      IF(XAA(M)-XGW(N2,1)) 232,234,234
  232 GAMMA2(M)=0.0
      GO TO 240
  234 IF(XAA(M)-XGW(N2,NVOR2)) 238,238,236
  236 GAMMA2(M)=GLAST
      GO TO 240
  238 GLAST=GAMMA2(M)
  240 CONTINUE
      DO 250 M=1,N21
      XG(N1,M)=XAA(M)
      XGW(N2,M)=XAA(M)
      WRITE(6,261) N1,N2,N1PRE
```

```
261   FORMAT(*0N1,N2,N1PRE* 3I5)
      WRITE(6,262) M,GAMMA1(M),GAMMA2(M)
262   FORMAT(*0M,GAMMA1,GAMMA2 = *I3,1P2E20.5)
      IF(N1.NE.N1PRE) GO TO 249
      GAMT(N1,M)=          GAMMA2(M)+GAMT(N1,M)
      GO TO 250
249   GAMT(N1,M)=GAMMA1(M)+GAMMA2(M)
250   GAMTW(N2,M)=GAMT(N1,M)
C
      ATTFB(N1)=99.0
      ATTFW(N2)=99.0
      WRITE(6,5998)(XG(N1,M),M=1,20)
5998  FORMAT(*0X-COORDINATES FOR ATTACHMENT LINE*/(1P10E13.4))
      WRITE(6,5999) N1,N2
5999  FORMAT(*0BODY ATTACHMENT LINE IS*,I3/
     1       *0PANEL ATTACHMENT LINE IS*,I3)
      WRITE(6,6000)(GAMT(N1,M),M=1,20)
6000  FORMAT(*0BODY GAMT AT ATTACHMENT LINE*/(1P10E13.4))
      WRITE(6,6001)(GAMTW(N2,M),M=1,20)
6001  FORMAT(*0PANEL GAMT AT ATTACHMENT LINE*/(1P10E13.4))
260   CONTINUE
      NTRVP=NTRVP+NVT(NBODS+I)+1
      N1PRE=N1
300   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PANGAM(IP,AK,NL,NT,KP,L,NTBP)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1  ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
     2  ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4  ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5   ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6   ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
      EQUIVALENCE (B,BP,XVR)
      DIMENSION BP(13000)
      DIMENSION B(30000),ALNGTH(5000),BLNGTH(5000),AK(1)
      EQUIVALENCE(B(20000),ALNGTH),(B(25000),BLNGTH)
      COMMON/SCRAT/DUMM(5000),XG(11,151),GAMT(11,151),GAMB(1500)
     1   ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
     2   ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
     3   ,CP(1320),XGW(50,20),GAMTW(50,20)
      COMMON/PANINF/PANSYM(10)
      DATA LENGTH/0/
      L1=0
      NSPACE = NSPP(IP)
      SYM=PANSYM(IP)
      LS=1
      IF(NBODS) 14,13,14
   13 NBVORT=0
      GO TO 15
   14 NBVORT=NVL(1)*NVT(1)
   15 CONTINUE
      READ(18) BP
      WRITE(6,6000) IP,NL,NT
      WRITE(6,6000) KP,L,NTBP
 6000 FORMAT(3I5,1P2E16.4)
      DO 305 J=1,NT
      IF(J.LE.NSPACE) GO TO 305
      SUM=0.0
      DO 300 I=1,NL
```

```
      I1=(J-1)*NL+I+KP-1
      SUM= SUM+AK(I1)
      DUMM(I1)=SUM
      WRITE(6,6000) J,I,I1,SUM,AK(I1)
300   AK(I1)=SUM
305   CONTINUE
      WRITE(6,7007) (DUMM(I),I=1,100)
7007  FORMAT(*0DUMM*/(1P10E13.4))
      NS1=NSPACE+1
      N1=NT+1
      DO 100 J=1,N1
      JMNS=J-NSPACE
      JT=J+NTBP
      DO 100 I=1,NL
      IF(J.EQ.N1) GO TO 45
      LENGTH=LENGTH+1
      KK=NBVORT+LENGTH
      L1=L1+1
      ALNGTH(KK)=PLL(L1)
      IF(JMNS) 41,41,42
41    BLNGTH(KK)=PLT(L1)
      GO TO 43
42    BLNGTH(KK)=2.0*YSUBV(JMNS)
43    CONTINUE
      SLOPEC(LENGTH)=TS(LS)
45    CONTINUE
      IF(JMNS) 11,11,21
11    XGW(JT,I)=0.5*(XVR(J,I)+XVR(J,I+1))
      GO TO 30
21    CONTINUE
      IF(J-N1) 22,24,24
22    FACT=-0.5
      JTR=JMNS
      GO TO 251
24    LS=NL*(NT-1)*2+1
      FACT=+0.5
      JTR=JMNS-1
251   IF(I-1) 26,26,28
26    X1=XVO(JTR ,I)+FACT*TS(LS)*SQRT(DYS(LS)**2+DZS(LS)**2)
```

```
      GO TO 29
28    X1=X2
29    CONTINUE
      IF(I.EQ.NL) GO TO 291
      LS2=LS+2
      X2=XVO(JTR ,I+1)+FACT*TS(LS2)*SQRT(DYS(LS2)**2+DZS(LS2)**2)
      GO TO 292
291   X2=X1+DELX
292   XGW(JT,I)=0.5*(X2+X1)
      DELX=X2-X1
30    CONTINUE
      K1=(J-1)*NL+I+KP-1
      LS=LS+2
      IF(J-1) 1,1,5
1     IF(SYM) 2,9,2
2     GAMTW(JT,I)=-AK(K1)
      GO TO 10
5     IF(J-N1) 6,7,6
6     GAMTW(JT,I)= AK(K1-NL)-AK(K1)
      GO TO 10
7     GAMTW(JT,I)=AK(K1-NL)
      GO TO 10
9     IF(NBODS.NE.0) GO TO 2
      GAMTW(JT,I)=0.0
10    CONTINUE
      IF(J.EQ.N1) GO TO 40
      L=L+1
      IF(I-1) 25,20,25
20    GAMB(L)=AK(K1)
      GO TO 40
25    GAMB(L)=AK(K1)-AK(K1-1)
40    CONTINUE
100   CONTINUE
      NTBP=NTBP+NVT(IP+NBODS)+1
      RETURN
      END
```

```
      SUBROUTINE BODGAM(IB,AK,NL,NT,L)
      COMMON DA(5000)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/BODY/B(30000)
      DIMENSION XV(151,31)
      EQUIVALENCE(XV,B)
      COMMON/SCRAT/XSOL(5000),XG(11,151),GAMT(11,151),GAMB(1500)
     1   ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
     2   ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
     3   ,CP(1320)
      DIMENSION AK(1)
      EQUIVALENCE (SYM,DA(19))
      READ(18) B
      N1=NT+1
      LTDIV=LTDIVB(IB)
      DO 100 J=1,N1
      J1=(J-1)*LTDIV+1
      DO 100 I=1,NL
      K1=(J-1)*NL+I
      XG(J,I)=0.5*(XV(I,J1)+XV(I+1,J1))
      IF(J-1) 1,1,5
    1 IF(SYM) 2,4,2
    2 GAMT(J,I)=-AK(K1)
      GO TO 10
    4 GAMT(J,I)=0.0
      GO TO 10
    5 IF(J-N1) 6,7,6
    6 GAMT(J,I)=AK(K1-NL)-AK(K1)
      GO TO 10
    7 IF(SYM) 8,9,8
    8 GAMT(J,I)=AK(K1-NL)
      GO TO 10
    9 GAMT(J,I)=0.0
   10 CONTINUE
      IF(J.EQ.N1) GO TO 40
      L=L+1
```

```
      IF(I-1) 25,20,25
 20   GAMB(L)=AK(K1)
      GO TO 40
 25   GAMB(L)=AK(K1)-AK(K1-1)
 40   CONTINUE
100   CONTINUE
      IF(SYM.EQ.0) GO TO 15
      DO 14 I=1,NL
      GAMT(1,I)=GAMT(1,I)+GAMT(N1,I)
 14   GAMT(N1,I)=GAMT(1,I)
 15   CONTINUE
      RETURN
      END
```

```
      PRO7RAM FORCES
C
C     MAIN PROGRAM FOR FORCES.
      COMMON/BODY/ XAREA(5000),YAREA(5000),ZAREA(5000)
     1            ,  XCP(5000),  YCP(5000),  ZCP(5000),  XTL(100)
     2            ,FLNC(150),FLTC(40),XCON(100)
      COMMON     /COMPRS/   BETAM
      DIMENSION YPD(200),ZPD(200),YV(100),ZV(100),PIND(10)
C     SEE EQUIVALENCE IN PANMAT FOR YPD,ZPD
      EQUIVALENCE(YPD(1),XTL(1)),(ZPD(1),FLNC(101))
      EQUIVALENCE (YV(1),XAREA),(ZV(1),YAREA)
      COMMON /PANINF/ PSYM(10), DUM11(600), PANREF(10), PCHORD(10)
      DIMENSION TBD(10),TPD(10)
      EQUIVALENCE (DA(19),BSYM)
      DIMENSION AKTB(500),AKTP(500)
      EQUIVALENCE (AKTB(1),YCP(4001)),(AKTP(1),YCP(4501))
      DIMENSION CP(1320)
      COMMON/SCRAT/ D(25000)
      DIMENSION VM(1500),VT(1500)
      EQUIVALENCE (VM(1),D(14323)),(VT(1),D(12823))
      DIMENSION CPBODY(1500)
      EQUIVALENCE(D(18076),CP),(D(20000),CPBODY)
      DIMENSION E   (50),XCCO(20),DYI(400),DZI(400)
     1         ,TWIST(20),CPU(400),CPL(400),DSLE(20),XLE(20),YLE(20)
     2         ,ZLE(20),SPCF(40),CHC(20)
      EQUIVALENCE (E,D(5001)),(XCCO,D(5101)),(DYI,D(5201)),(DZI,D(5701))
     1           ,(TWIST,D(6201)),(CPU,D(6301)),(CPL,D(6801))
     2           ,(DSLE,D(7301)), (XLE,D(7401)),(YLE,D(7501))
     3            ,(ZLE,D(7601)),(SPCF,D(7701)),(CHC,D(7801))
     4       ,(XCG,DA(7)),(YCG,DA(8)),(ZCG,DA(9)),(PANS,DA(2))
     5   ,(VMU,D(8001)),(VML,D(8201)),(VTU,D(8401)),(VTL,D(8601))
      DIMENSION VMU(200),VML(200),VTL(200),VTU(200)
      COMMON DA(5000)
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1   ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2   ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3   ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
      COMMON/SLOPE/ SIGMAP(500),DZDXT(500),DZDXC(500),TANPI(500)
      DIMENSION ZERO(50)
```

```
      DIMENSION SB(5,8), PS(10,6)
      EQUIVALENCE (BR, DA(16)), (PR, DA(3421)), (PSY, DA(3426)),
     1 (PCH, DA(3422)), (BCH, DA(17))
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
     1              ,XN(1320),YN(1320),ZN(1320)
      DIMENSION BP(13000)
      DIMENSION XCC(20),XCV(20),XSS(1000),YSS(1000),ZSS(1000),ET(20)
     1         ,TS(1000),SIGMA(1000)
      EQUIVALENCE(BP,XAREA),(XCC,BP(2421)),(XCV,BP(2401))
     1   ,(XSS,BP(8871)),(YSS,BP(9871)),(ZSS,BP(10871)),(TS,BP(7871))
     2    ,(SIGMA,BP(11871))
      DIMENSION CPNET(1000),CDS(100),CDA(100),CDT(100),CDTH(100)
      EQUIVALENCE  (SPAN,DA(3423)),(PLVI,DA(3431))
      EQUIVALENCE(D(23001),CPNET),(D(24101),CDS),(D(24201),CDA)
     1         ,(D(24301),CDT),  (D(24401),CDTH)
      COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
      DATA PIND/10*1.0/
      NC=0
      IF(NBODS.EQ.0) GO TO 50
      N2=LTDIVB(1)*NVT(1)+1
      READ(12)(YV(I),ZV(I),I=1,N2)
50    CONTINUE
      NPANS=PANS
      IF(PANS.EQ.0.0) GO TO 55
      WRITE(6,7002) (I,YPD(I),ZPD(I),I=1,20)
7002  FORMAT(*0PANEL DRAG COORDINATES*/(I5,2F15.5))
      WRITE(6,7001) (I,AKTP(I),I=1,20)
7001  FORMAT(*0 AKTP*/(I5,1PE20.6))
      WRITE(6,6001) NVT(NBODS+1),NVT(NBODS+2)
      WRITE(6,6002) PSYM
      WRITE(6,6002) PIND
55    CONTINUE
      NSP=0
      DO 101 IS=1,LTCPP
      CDS(IS)=0.
      CDA(IS)=0.
      CDT(IS)=0.
101   CDTH(IS)=0.
      CDST=0.
```

```
      CDAT=0.
      CDTT=0.
      CDTHT=0.
      IF(DA(3424).NE.1.) GO TO 58
      IF(NBODS.NE.0) GO TO 58
      IF(NPANS.NE.1) GO TO 58
      BETAM=1.0
      NCPT=NCPTS(1)
      DO 56 I=1,NCPT
      IF(ZN(I).NE.1.0) GO TO 58
   56 CONTINUE
      IF(PLVI.NE.0.0) GO TO 58
      REWIND 18
      READ(18) BP
      REWIND 18
      HSPAN=NTVVP*(YSS(NBVVP*2+1)-YSS(1))
      DO 41 I=1,LTCPP
   41 ET(I)=YQ((I-1)*LNCPP+1)/HSPAN
       DO 59 I=1,NCPT
       IU=(I-1)*2+1
      IL=IU+1
   59 CPNET(I)=CP(IL)-CP(IU)
      NVPT=NBVVP*NTVVP*2
      DO 61 I=1,NVPT
      YSS(I)=YSS(I)/BETAM
      ZSS(I)=ZSS(I)/BETAM
      TS(I) =BETAM*TS(I)
   61 SIGMA(I)=SIGMA(I)/BETAM
      CALL "FDRAG(NBVVP,NTVVP,LNCPP,LTCPP,XCC,XCV,ET,    XSS,YSS,ZSS
     1  ,TS,SIGMA,CPNET,NSP,CDS,CDA,CDT,CDTH,CDST,CDAT,CDTT,CDTHT)
   58 CONTINUE
      DO 551 I=1,100
      YV(I)=YV(I)/BETAM
  551 ZV(I)=ZV(I)/BETAM
      DO 552 I=1,200
      YPD(I)=YPD(I)/BETAM
  552 ZPD(I)=ZPD(I)/BETAM
      CALL FFDRAG(NBODS,NPANS,YV,ZV,AKTR,N2-1,BSYM,YPD,ZPD,AKTP
     1 ,NVT(NBODS+1),PSYM,PIND,TPD,TPD,NCP)
```

```
      IF(NBODS.EQ.0.) GO TO 100
      READ(12) XAREA,YAREA,ZAREA
      READ(12) XCP,YCP,ZCP,XTL,XCON,FLNC,FLTC,NBV,NTV,LNPTS,LTPTS
     1       ,LNDIV,LTDIV,NBVV,NTVV,CHORD,XCG,YCG,ZCG,ALPHA,PREF
 6000 FORMAT( 1P9E12.4)
 6001 FORMAT(10I5)
      NC=NCPTS(1)
      DO 60 I=1,NC
 60   CPBODY(I)=CP(I)
 6002 FORMAT(*0*/(1P10E13.4))
      CALL BPRINT(LNPTS,LTPTS,XQ,YQ,ZQ,VM,VT,CP,1)
      CALL REFLCT(DA(19), NBV, NTV, XAREA, YAREA, ZAREA, XCP, YCP, ZCP,
     1  LNPTS,LTPTS,FLTC,CPBODY,LTDIV)
      NPAA = NBV*NTV
       DO 30 I=1,NPAA
      XAREA(I) = XAREA(I)/BETAM**2
      YAREA(I) = YAREA(I)/BETAM
      ZAREA(I) = ZAREA(I)/BETAM
      YCP(I) = YCP(I)/BETAM
   30 ZCP(I) = ZCP(I)/BETAM
      CALL BINTEG(XAREA,YAREA,ZAREA,XCP,YCP,ZCP,FLNC,LNPTS,FLTC,LTPTS
     1       ,CPBODY,LNDIV,LTDIV,XTL,NBVV,D(1001),XCON,D(1)
     2   ,D(101),D(201),CHORD,D(301),D(401),D(501),D(601),D(701),D(801)
     3   , NBV, NTV, D(951), XCG, YCG, ZCG, ALPHA, BREF,
     4  SB(1,1), SB(1,2), SB(1,3), SB(1,4), SB(1,5), SB(1,6),TPP(1),1
     5)
      SB(1,7) = BR
      SB(1,8) = BCH
C
C     THE FIRST SUBSCRIPTS OF SB(I,J) SHOULD BE THE BODY(PANEL) NUMBER
 100  IF(PANS.EQ.0.) GO TO 200
      NPANS=PANS
      NCP1=NC
      NPBZ=1
      DO 554 I=1,500
      DZDXC(I)=DZDXC(I)/BETAM
 554  DZDXT(I)=DZDXT(I)/BETAM
      N11=NC+1
      DO 555 I=N11,1320
```

```
      YQ(I)=YQ(I)/BETAM
  555 ZQ(I)=ZQ(I)/BETAM
      DO 150 K=1,NPANS
      READ(12) B,REFA,LTCPP,E,  LNCPP,XCCO,DYI,DZI,TWIST,CHC
     1        ,CHRM,(XLE(I),YLE(I),ZLE(I),I=1,LTCPP),FLTC,SPCF,BS,DSLE
      DO 105 I=1,LTCPP
      K1=FLTC(I)
      DSLE(I)=DSLE(K1)/BETAM
      YLE(I)=YLE(I)/BETAM
  105 ZLE(I)=ZLE(I)/BETAM
      NPTS=NCPTS(NBODS+K)
      DO 110 I=1,NPTS
      IU=(I-1)*2+1+NCP1
      IL=IU+1
      VMU(I)=VM(IU)
      VTU(I)=VT(IU)
      VML(I)=VM(IL)
      VTL(I)=VT(IL)
      CPL(I)=CP(IL)
  110 CPU(I)=CP(IU)
      N1=NC+1
      DO 111 I=1,50
  111 ZERO(I)=0.0
      CALL PPRINT(E,XCCO,LTCPP,LNCPP,VMU,VML,VTU,VTL,CPU,CPL,NBODS+K)
      B=B/BETAM
      BS=B/BETAM
      DO 553 I=1,400
      DYI(I)=DYI(I)/BETAM
  553 DZI(I)=DZI(I)/BETAM
      CALL PANINT(B,REFA,LTCPP,E,  LNCPP,XCCO,DYI,DZI,TWIST,BS,SPCF
     1   ,CPU,CPL,DSLE,XCG,YCG,ZCG,CHC,CHRM,DZDXC(NPDZ),DZDXT(NPDZ)
     2          ,XQ(N1),YQ(N1),ZQ(N1),XLE,YLE,ZLE,D(8000)
     3  ,TPD(K),CDA,CDS,CDTH,CDT,PS(K,1),PS(K,2),PS(K,3),
     4   PS(K,4), PS(K,5), PS(K,6), K+NBODS, CDAT, CDST, CDTHT, CDTT)
      NC=NC+NPTS
      NCP1=NCP1+2*NPTS
      NPDZ=NPDZ+NPTS
  150 CONTINUE
  200 CONTINUE
```

```
C                                                                      6  1960
      CALL CCINTG( SB(1,1), SB(1,2), SB(1,3), SR(1,4), SR(1,5), SR(1,6),6  1970
     1   SB(1,7), PS(1,1), PS(1,2), PS(1,3), PS(1,4), PS(1,5), PS(1,6), 6  1980
     2   PANREF, PSYM, PCHORD, SB(1,8)                                  6  1990
     3)                                                                 6  2000
      STOP 6543                                                         6  2010
      END                                                               6  2020
```

```
      0SUBROUTINE CCINTG (CXB, CYB, CZB, CMXB, CMYB, CMZB, ARB , CXP, CYP
     1, CZP, CMXP, CMYP, CMZP, AP, FS, CP, CB
C
C     THIS ROUTINE INTEGRATES THE TOTAL CONFIGURATION
C
C     CXB CYB CZB         -ARRAYS OF C S OF BODIES
C     CMXB CMYB CMZB      -ARRAYS OF CM S OF BODIES
C     ARB                 -ARRAY  OF REFERENCE AREAS OF BODIES
C     CXP CYP CZP         -ARRAYS OF C S OF PANELS
C     CMXP CMYP CMZP      -ARRAYS OF CMS OF PANELS
C     AP                  -ARRAY  OF  PANEL AREAS
C     CB                  -ARRAY  OF  BODY CHORD LENGTHS
C     CP                  -ARRAY  OF  PANEL CHORD LENGTHS
C     NB                  -NUMBER OF  BODIES
C     NP                  -NUMBER OF  PANELS
C     C                   -TOTAL CONFIGURATION CHORD LENGTH
C     XCRCG YCRCG ZCRCG-X Y AND Z ( /CR)CG
C     AR          -  TOTAL CONFIGURATION REFERENCE AREA
C     FS          -  SYMETRY INDICATOR 0.0 IS SYMETRICAL
C
      0DIMENSION         CXB(1),       CYB(1),       CZB(1),       CMXB(1),
     1         CMYB(1),  CMZB(1),      ARB(1),
     2         CXP(1),   CYP(1),       CZP(1),       CMXP(1),      CMYP(1),
     3         CMZP(1),  AP(1),        FS(1),        CP(1),        CB(1)
       EQUIVALENCE         (ARBAR,CBJC),               (CBJC,APJAR),(CBJC,CPJC)
       COMMON DA(5100)
       EQUIVALENCE (DA(5),C), (DA(1), BN), (DA(2), PN)
     1  , (AR, DA(4)), (XCRCG, DA(7)), (YCRCG,DA(8)), (ZCRCG, DA(9))
       NB = BN
       NP = PN
       PRINT 6
     6 FORMAT(1H1,35X,*TOTAL CONFIGURATION LOADS*)
       CX    =0.0
       CY    =0.0
       CZ    =0.0
       CMX   =0.0
       CMY   =0.0
       CMZ   =0.0
C
```

```
C      SUM OVER BODIES
       IF(NB .LE. 0) GOTO 300
       DO 100 J=1, NB
       ARBAR = ARB(J)/AR
       CX    = CX + ARBAR * CXB(J)
       CY    = CY + ARBAR * CYB(J)
       CZ    = CZ + ARBAR * CZB(J)
       CBJC  = CB(J)/ C *  ARBAR
       CMX   = CMX + CBJC*CMXB(J)
       CMY   = CMY + CBJC*CMYB(J)
       CMZ   = CMZ + CBJC*CMZB(J)
   100 CONTINUE
C
C      SUM OVER PANELS
   300 IF(NP .LE. 0) GOTO 400
       DO 200 J=1, NP
       FS0   = 1.
       FS2   = 1.
       IF(6S(J)) 102, 101, 102
   101 FS0   = 0.0
       FS2   = 2.0
   102 APJ1R = AP(J)7AR
       CX    = CX + APJAR * FS2 * CXP(J)
       CY    = CY + APJAR * FS0 * CYP(J)
       CZ    = CZ + APJAR * FS2 * CZP(J)
       CPJC  = APJAR * CP(J)/C
       CMX   = CMX + CPJC*FS0*CMXP(J)
       CMY   = CMY + CPJC*FS2*CMYP(J)
       CMZ   = CMZ + CPJC*FS0*CMZP(J)
   200 CONTINUE
   400 CONTINUE
       PRINT 206, CX, CY, CZ, CMX, CMY, CMZ
   206 FORMAT(1H0,8X,*CX*,13X,*CY*,13X,*CZ*,13X,*CMX*,12X,*CMY*,12X,*CMZ*
      1 /1X,6F15.7)
       CPJC  =  CX*CX  + CY*CY + CZ*CZ
       XCR   =  XCRCG + (CY*CMZ - CZ*CMY)/CPJC
       YCR   =  YCRCG + (CZ*CMX - CX*CMZ)/CPJC
       ZCR   =  ZCRCG + (CX*CMY - CY*CMX)/CPJC
       PRINT 216, XCR, YCR, ZCR
```

```
216 FORMAT(1H0,3X,*(X/CR)  CP*,6X,*(Y/CR) CP*,6X,*(Z/CR) CP*/1X,3F15.76
   1)                                                                  6
     RETURN                                                            6
     END                                                               6
```

```
      SUBROUTINE PANINT(B,REFA,NETA,ETA,NXOC,XOC,DY,DZ,EPSILO,BS,DA       6   2850
     1          ,CPU,CPL,DSLE,XCG,YCG,ZCG,C,CHORD,DZC,DZT,X,Y,Z           6   2860
     2 , XLE, YLE, ZLE, DS, CDI, CDTO, CDLIC, CTAVRG, CDTICA, CX, CY,     6   2870
     3  CZ, CMX, CMY, CMZ, NC,CDT,CDL,CT,CTDI)                             6   2880
C                                                                          6   2890
C     CODED BY B. D. GAITHER                                              6   2900
C     THE GREAT NUMBER OF ARGUMENTS WERE USED TO  ALLOW GREATER EASE  IN6   2910
C     ADDING THIS ROUTINE TO THE OTHERS. AT A LATER DATE THESE SHOULD BE6  2920
C     REPLACED BY  COMMON REFERENCES.                                     6   2930
C                                                                          6   2940
C     THIS ROUTINE INTEGRATES PANELS WITHCONTROL SURFACES                 6   2950
C                                                                          6   2960
C     B         - SPAN                                                     6   2970
C     REF1      -  REFERENCE AREA                                          6   2980
C     NET1      - NUMBER OF ETAS  WHERE CPS ARE GIVEN.                     6   2990
C     ETA       - ETAS FOR GIVEN CPS.                                      6   3000
C     NXO3      - NUMBER OF X/C STATION FOR GIVEN CPS.                     6   3010
C     XOC       - X/C ARRAY                                                6   3020
C     DY        - DELTA Y FOR VORTEX WHERE CP IS GIVEN.                    6   3030
C     DZ        - DELTA Z FOR VORTEX WHERE CP IS GIVEN.                    6   3040
C     EPSILO    - TWIST ARRAY                                              6   3050
C     BS        - SURFACE SPAN LENGTH (FOR 2-D SURFACE, BS=B).             6   3060
C     SPCF      - SPECIAL LONGITUDINAL CONTROL FUNCTIONS                   6   3070
C     CPU       - CP ARRAY FOR UPPER SURFACE.                              6   3080
C     CPL       - CP ARRAY FOR LOWER SURFACE.                              6   3090
C     DSL5      - DELTA S ON LEADING EDGE FOR SPANWISE VORTEX SECTIONS     6   3100
C                      WHERE CPS ARE GIVEN.                                6   3110
C     XCG       -  X CENTER OF GRAVITY                                     6   3120
C     YCG       -  Y CENTER OF GRAVITY                                     6   3130
C     ZCG       -  Z CENTER OF GRAVITY                                     6   3140
C     C         - CHORD ARRAY FOR SPANWISE STATIONS WHERE CPS ARE GIVEN.   6   3150
C     CHORD     - MEAN AERODYNAMIC CHORD (GIVEN IN INPUT).                 6   3160
C     DZC       - DZ/DX FOR CAMBER.                                        6   3170
C     DZT       - DZ/DX FOR THICKNESS.                                     6   3180
C     X Y Z     -  CO ORDINATES AT CPS                                     6   3190
C     XLE       - X ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.           6   3200
C     YLE       - Y ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.           6   3210
C     ZLE       - Z ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.           6   3220
C     DS        - A SCRATCH ARRAY                                          6   3230
C     CDTO          -          (CDT=0 C)/CAVG
```

```
C       CDL9C       -                (CDLIC)/CAVG                      6   3240
C       CTAVRG      -                (CTC)/CAVG                        6   3250
C       CDTICA      -                (CDTIC)/CAVG                      6   3260
C       CXCAVG      -      CXC/CAVG                                    6   3270
C       CYCAVG      -      CYC/CAVG                                    6   3280
C       CZCZVG      -      CZC/CZVG                                    6   3290
C       CMX3AV      -      CMXC/CAVG                                   6   3300
C       CMYCAV      -      CMYC/CAVG                                   6   3310
C       CMZCAV      -      CMZC/CAVG                                   6   3320
C       CNCAVG      -      CNC/CAVG                                    6   3330
C       CMLEXC      -      CMLEXC/CAVG                                 6   3340
C       CMLEYC      -      CMLEYC/CAVG                                 6   3350
C       CMLEZC      -      CMLEZC/CAVG                                 6   3360
C       XCPLE       -      XCP/C LE                                    6   3370
C       YCPLE       -      YCP/C LE                                    6   3380
C       ZCPLE       -      ZCP/C LE                                    6   3390
C       T1 2 3 .. 8 -      SCRTCH ARRAYS NETA LONG WHERE DEPENDENT VARIABLE6   3400
C                          S TO INTEGRAT ARE STORE                     6   3410
C       T1 1 2 .. 8 -      SCRTCH VARIABLES WHERE VALUES OF INTEGRALS ARE  6   3420
C                   -      STORED                                      6   3430
C       CMX CMY CMZ -       CMX CMY CMZ                                6   3440
C       CX CY CZ    -       CX CY CZ                                   6   3450
C       XCPC        -      XCP/C                                       6   3460
C       YCP3        -      YCP/C                                       6   3470
C       ZCPC        -      ZCP/C                                       6   3480
C       DS          -      DELTA S                                     6   3490
C       DSLE        -      DELTA S L.E.                                6   3500
        DIMENSION DSLE(30), DS(NXOC,NETA),              DY(NXOC,NETA), 6   3510
       1DZ(NXOC,NETA), CPU(NXOC, NETA), CPL(NXOC,NETA), EPSILO(     NETA),6   3520
       2ETA(NETA), XOC(NXOC), X(NXOC,NETA), Y( NXOC,NETA), Z(NXOC,NETA),  6   3530
       3T1(30),T2(30), T3(30), T4(30), T5(30), T6(30), T7(30), T8(30),    6   3540
       4 CXCAVG(30), CYCAVG(30), CZCAVG(30), CMXCAV(30), CMYCAV(30), CMZCA6   3550
       5V(30), CNCAVG(30), CMLEXC(30), CMLEYC(30), CMLEZC(30), XCPLE(30), 6   3560
       6YCPLE(30),ZCPLE(30),CDTO(NETA), CDLIC(NETA), CTAVRG(NETA), CDTICA(6   3570
       7   NETA), DA(8,1), XLE(1), YLE(1), ZLE(1)                         6   3580
        DIMENSION DZC(NXOC,NETA),DZT(NXOC,NETA)                        6   3590
        DIMENSION C(1), XSLCS(20), YSLCS(20), ZSLCS(20), NSLCS(20)      6   3600
       1      , XSMAHL(20), YSMAHL(20), ZSMAHL(20), XSCPCS(20), YSCPCS(20)6   3610
       2      , ZSCPCS(20), CMHC(20)                                    6   3620
```

```
      EQU9VALENCE (CXCAVG,XSLCS),    (YSLCS,CYCAVG),
     1       (CZCAVG,ZSLCS),          (CNCAVG,NSLCS),
     2       (CMXCAV,XSMAHL),         (CMYCAV,YSMAHL),
     3       (CMZCAV,ZSMAHL),         (XCPLE,XSCPCS),
     4       (YCPLE,YSCPCS),          (ZCPLE,ZSCPCS), (CMLEXC,CMHC)
      RLINT(XI, XO, ETAI, ETAO, ATA)
     1      = XI + (XO - XI) * (ATA - ETAI) / (ETAO - ETAI)
      PRINT 26, NC
   26 FORMAT(1H1, 36X, *PANEL SECTIONAL LOADS FOR COMPONENT*, I5/
     11H0,5X, 3HETA, 7X,
     2*CXC/CAVG*,5X,*CYC/CAVG*, 5X, *CZC/CAVG*, 5X, *CNC/CAVG*
     3,4X,*CMLEXC/CAVG*, 2X, *CMLEYC/CAVG*, 2X, *CMLEZC/CAVG* )
C      SET UP DELTA S AND DELTA S L.E.
      BT = B*B
      AR = BT/REFA
      DO 20 K=1, NETA
      DO 10 I=1, NXOC
      DS(I,K)=SQRT(DY(I,K)**2+DZ(I,K)**2)
   10 CONTINUE
   20 CONTINUE
      ARBSB2 = AR*BS/(BT)
C      STEP SPAN WIZE
      DO 500 K=1, NETA
      DYOPIT=DY(1,K)/DS(1,K)
      DZOPIT=DZ(1,K)/DS(1,K)
C      STEP CHORD WIZE
      DO 100 I=1, NXOC
C      FILL DEPENDENT VARIABLE ARRAYS
      DSDSLE=DS(I,K)/DSLE(K)
      TCON = CPL(I,K)*
     1TAN1LF(        DZT(I,K),DZC(I,K),EPSILO(  K),DA,ETA(K),X(I,K),1.0)
     2 -CPU(I,K) *
     3TAN1LF(        DZT(I,K),DZC(I,K),EPSILO(  K),DA,ETA(K),X(I,K),-1.)
      CPLCPU=CPL(I,K)-CPU(I,K)
      T1(I)=DSDSLE*TCON
      T2(I)=DSDSLE*CPLCPU
      YIKYCG=Y(I,K)-YCG
      ZIKZCG=Z(I,K)-ZCG
      XIKXCG=X(I,K)-XCG
```

```
      T3(9)=DSDSLE*(YIKYCG*DYOPIT+ZIKZCG*DZOPIT)*CPLCPU
      T4(I)=DSDSLE*(ZIKZCG*TCON-XIKXCG*DYOPIT*CPLCPU)
      T5(I)=DSDSLE*(-XIKXCG*DZOPIT*CPLCPU-YIKYCG*TCON)
      XIKXLE=     X(I,K) - XLE(K)
      YIKYLE=     Y(I,K) - YLE(K)
      ZIKZLE=     Z(I,K) - ZLE(K)
      T6 (I)=DSDSLE*(YIKYLE*DYOPIT+ZIKZLE*DZOPIT)*CPLCPU
      T7(I)=DSDSLE*(ZIKZLE*TCON-XIKXLE*DYOPIT*CPLCPU)
      T8(I)=DSDSLE*(-XIKXLE*DZOPIT*CPLCPU-YIKYLE*TCON)
  100 CONTINUE
C     INTEGRATE CHORD WIZE
      TI1=POLINT(XOC,T1,NXOC,0.0,1.0)
      TI2=POLINT(XOC,T2,NXOC,0.0,1.0)
      TI3=POLINT(XOC,T3,NXOC,0.0,1.0)
      TI4=POLINT(XOC,T4,NXOC,0.0,1.0)
      TI5=POLINT(XOC,T5,NXOC,0.0,1.0)
      TI6=POLINT(XOC,T6,NXOC,0.0,1.0)
      TI7=POLINT(XOC,T7,NXOC,0.0,1.0)
      TI8=POLINT(XOC,T8,NXOC,0.0,1.0)
      CARBB=C(K)*ARBSB2
      CTAVRG(K)=CTAVRG(K)*ARBSB2
      CXCAVG(K) = CARBB*(TI1)-CTAVRG(K)
      CYCAVG(K)=-CARBB*DZOPIT*TI2
      CZC1VG(K)=CARBB*DYOPIT*TI2
      CARBBC=CARBB/CHORD
      CMXCAV(K)=CARBBC*TI3
      CMYCAV(K) = CARBBC*TI4-(Z(1,K)-ZCG)*CTAVRG(K)/CHORD
      CMZCAV(K) = CARBBC*TI5+(Y(1,K)-YCG)*CTAVRG(K)/CHORD
      CNCAVG(K)=-DZOPIT*CYCAVG(K)+DYOPIT*CZCAVG(K)
      TPIT=CXCAVG(K)**2+CYCAVG(K)**2+CZCAVG(K)**2
      CMLEXC(K)=ARBSB2*TI6
      CMLEYC(K)=ARBSB2*TI7
      CMLEZC(K)=ARBSB2*TI8
      CDT0(K)=CDT0(K)*ARBSB2
      CDLIC(K)=CDLIC(K)*ARBSB2
      CDTICA(K)=CDTICA(K)*ARBSB2
      IF(ABS(TPIT).GT.1.0E-7) GOTO 131
      XCPLE(K)=0.0
      YCPLE(K)=0.0
```

```
      ZCPLE(K)=0.0
      GOTO  132
  131 CONTINUE
      XCPLE(K)=(CYCAVG(K)*CMLEZC(K)-CZCAVG(K)*CMLEYC(K))/TPIT
      YCPLE(K)=(CZCAVG(K)*CMLEXC(K)-CXCAVG(K)*CMLEZC(K))/TPIT
      ZCPLE(K)=(CXCAVG(K)*CMLEYC(K)-CYCAVG(K)*CMLEXC(K))/TPIT
  132 CONTINUE
      PRINT 156 , ETA(K), CXCAVG(K), CYCAVG(K), CZCAVG(K), CNCAVG(K),
     1CMLEXC(K), CMLEYC(K), CMLEZC(K)
  156 FORMAT(1H ,   F10.6, 7F13.6)
  500 CONTINUE
      PRINT 506
  506 FORMAT(1H0,5X,3HETA, 7X,
     1*XLE/C CP*, 5X, *YLE/C CP*, 5X, *ZLE/C CP*, 4X, *CDT=0C/CAVG*
     2, 3X, *CDLIC/CAVG*, 4X, *CTC/CAVG*, 4X, *CDTIC/CAVG*)
      PRINT 156, ( ETA(K), XCPLE(K), YCPLE(K), ZCPLE(K), CDTO(K), CDLIC
     1(K),  CTAVRG(K),  CDTICA(K), K=1, NETA)
C     INTEGRATE SPAN WIZE
      CX=POLINT(ETA,CXCAVG,NETA,0.0,1.)
      CY=POLINT(ETA,CYCAVG,NETA,0.0,1.)
      CZ=POLINT(ETA,CZCAVG,NETA,0.0,1.)
      CMX=POLINT(ETA,CMXCAV,NETA,0.0,1.)
      CMY=POLINT(ETA,CMYCAV,NETA,0.0,1.)
      CMZ=POLINT(ETA,CMZCAV,NETA,0.0,1.)
      PRINT 606, CX, CY, CZ, CMX, CMY, CMZ
      CDI = CDI / REFA
      CDT=CDT/REFA
      CDL=CDL/REFA
      CT=CT/REFA
      CTDI=CTDI/REFA
  606 FORMAT(///1H0, 44X, *TOTAL PANEL LOADS*/
     1        1H0,7X,*CX*,13X,*CY*,13X,*CZ*,13X,*CMX*,12X,*CMY*,12X,*CMZ*
     2/1H ,6F15.6)
      XYZ  = CX*CX + CY*CY + CZ*CZ
      IF(ABS(XYZ).GT. 1.0E-7) GOTO 611
      XCPC  = 0.0
      YCPC  = 0.0
      ZCPC  = 0.0
      GOTO  612
```

```
  511 CONTINUE
      XCPC=XCG/CHORD+(CY*CMZ-CZ*CMY)/XYZ
      YCPC=YCG/CHORD+(CZ*CMX-CX*CMZ)/XYZ
      ZCPC=ZCG/CHORD+(CX*CMY-CY*CNX)/XYZ
  612 CONTINUE
      PRINT 616, CDT, CDL, CDI,CT, CTDI, XCPC, YCPC, ZCPC
  616 FORMAT(1H0,5X,*CDT=0*,10X,*CDLI*,12X,*CDI*,13X,*CT*,12X,*CDTI*,11X
     T,
     1/1H , 5F15.6/1H0, 5X,
     2  *X/C CP*, 9X, *Y/C CP*, 9X, *Z/C CP*/1H , 3F15.6)
C
C     THIS SECTION STEPS THROUGH THE CONTROL SURFACES WHILE SOLVING THE
C     CONTROL SURFACE EQUATIONS
      NCS = 1
  709 IF(DA(1,NCS).EQ.0) RETURN
      PRINT 706, NCS
  706 FORMAT(1H1,35X,15HCONTROL SURFACE, I3,16H SECTIONAL LOADS)
C
C     FIN4 THE ETAS AT WHICH THIS CONTROL SURFACE STARTS AND ENDS
      DO 710 KS=1, NETA
      IF(ETA(KS)-DA(3,NCS)) 710,711,711
  710 CONTINUE
  711 DO 720 KE=KS, NETA
      IF(ETA(KE)-DA(4,NCS)) 720, 721, 722
  720 CONTINUE
  722 KE = KE - 1
      IF(KE .LE. 0) KE = KS
  721 NESPN = KE-KS + 1
      TFS   = (DA(1,NCS)+1.)/2.0
      ETAD = DA(4,NCS) - DA (3,NCS)
C     STEP  SPANWISE THROUGH THIS PANEL
      DO 900 K=KS, KE
      T7(K) = (ETA(K)-DA(3,NCS))/ETAD
      DYOPIT= DY(1,K)/DS  (K)
      DZOPIT= DZ(1,K)/DS  (K)
C     FIN4 THE XOCS AT WHICH THIS CONTROL SURFACE STARTS OR  ENDS
      XKF= RLINT(DA(5,NCS),DA(6,NCS), DA(3,NCS), DA(4,NCS),  ETA(K))
      XH = RLINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS),  ETA(K))
      XHP =RLINT(DA(7,NCS),DA(8,NCS),DA(3,NCS),DA(4,NCS),ETA(K)+1.E-7)
```

```
      XHM =RLINT(DA(7,NCS),DA(8,NCS),DA(3,NCS),DA(4,NCS),ETA(K)-1.E-7)
      XHD = XHP - XHM
      DO 740 ISE=1, NXOC
      IF(X(ISE,K)-XKF) 740,741,7422
  740 CONTINUE
 7422 ISE = ISE -1
  741 IS     = 1
      IE     = ISE
      CCS    = X(IE,K) - X(1,K)
      IF(  TFS    ) 742, 742, 743
  742 IS     = IE
      IF( X(IS,K) .NE. XKF) IS = IS + 1
      IE     = NXOC
      CCS    = X(1,K) + C(K) - X(IS,K)
C
C     FIT FOR Y AND Z OF HING LINE
  743 DO 760 I=2, NXOC
      IF(X(I,K)-XH) 760,761,761
  760 CONTINUE
  761 YH     = RLINT(Y(I,K), Y(I-1,K), X(I,K), X(I-1,K), XH)
      ZH     = RLINT(Z(I,K), Z(I-1,K), X(I,K), X(I-1,K), XH)
      YHP =RLINT(Y(I,K),Y(I-1,K),X(I,K),X(I-1,K),XHP)
      YHM =RLINT(Y(I,K),Y(I-1,K),X(I,K),X(I-1,K),XHM)
      YHD = YHP - YHM
      ZHP =RLINT(Z(I,K),Z(I-1,K),X(I,K),X(I-1,K),XHP)
      ZHM =RLINT(Z(I,K),Z(I-1,K),X(I,K),X(I-1,K),XHM)
      ZHD = ZHP - ZHM
      T   =  SQRT(YHD*YHD + XHD*XHD + ZHD*ZHD)
      HZ  = ZHD/T
      HY  = YHD/T
      HX  = XHD/T
      TT = C(K) / CCS
      DO 770 I=IS, IE
      DSDSLE= DS(I,K)/DSLE(K)
      XIKXH = X(I,K) - XH
      YIKYH = Y(I,K) - YH
      ZIKZH = Z(I,K) - ZH
      T = TAN(DA(2,NCS))
      TCON = CPL(I,K) * (T + DZT(I,K) - DZC(I,K) + EPSILO(K)) /
```

```
      1       (1.- (DZT(I,K) - DZC(I,K) + EPSILO(K)) * T)
      2       - CPU(I,K) * (T - DZT(I,K) - DZC(I,K) + EPSILO(K)) /
      3       (1. -T*(-DZT(I,K) - DZC(I,K) + EPSILO(K)))
       T1(I) = DSDSLE * TCON
       CPLCPU = CPL(I,K) - CPU(I,K)
       T2(I) = DSDSLE*CPLCPU
       T3(I) = DSDSLE*(YIKYH*DYOPIT + ZIKZH*DZOPIT)*CPLCPU
       T4(I) =DSDSLE*(ZIKZH*TCON-XIKXH*DYOPIT*CPLCPU)
       T5(I) =DSDSLE*(XIKXH*DZOPIT*CPLCPU - YIKYH*TCON)
       IF(TFS) 768, 768, 762
  768  T8(I) = TT*(XOC(I)+1./TT-1.)
       GOTO 763
  762  T8(I) = TT * XOC(I)
  763  CONTINUE
  770  CONTINUE
C      INTEGRATE CHORDWIZE
       NX     = IE -IS +1
       TI1      = POLINT( T8(IS),T1(IS),  NX, 0.0, 1.0)
       TI2      = POLINT( T8(IS),T2(IS),  NX, 0.0, 1.0)
       TI3      = POLINT( T8(IS),T3(IS) , NX, 0.0, 1.0)
       TI4      = POLINT( T8(IS),T4(IS) , NX, 0.0, 1.0)
       TI5      = POLINT( T8(IS),T5(IS) , NX, 0.0, 1.0)
       CARBB    = CCS *ARBSB2
       XSLCS(K) = CARBB*TI1 - CTAVRG(K)
       YSLCS(K) =-CARBB*DZOPIT*TI2
       ZSLCS(K) = CARBB*DYOPIT*TI2
       CARBB = CARBB/CHORD
       NSLCS(K) =-YSLCS(K)*DZOPIT + ZSLCS(K)*DYOPIT
       XSMAHL(K)= CARBB*TI3
       YSMAHL(K)= CARBB*TI4-TFS*(Z(1,K) - ZH)*CTAVRG(K)/CHORD
       ZSMAHL(K)= CARBB*TI5-TFS*(Y(1,K) - YH)*CTAVRG(K)/CHORD
       T = XSLCS(K)**2 + YSLCS(K)**2 + ZSLCS(K)**2
       T= T * C(K) / CHORD
       XSCPCS(K) = (YSLCS(K)*ZSMAHL(K) - ZSLCS(K)*YSMAHL(K))/T
       YSCPCS(K) = (ZSLCS(K)*XSMAHL(K) - XSLCS(K)*ZSMAHL(K))/T
       ZSCPCS(K) = (XSLCS(K)*YSMAHL(K) - YSLCS(K)*XSMAHL(K))/T
       CMHC(K) = XSMAHL(K)*HX + YSMAHL(K)*HY + ZSMAHL(K)*HZ
  900  CONTINUE
       CHX = POLINT(T7(KE),XSLCS(KE), NESPN,0.0,1.0)
```

```
      CHY = POLINT(T7(KE),YSLCS(KE),NESPN,0.0,1.0)
      CHZ = POLINT(T7(KE),ZSLCS(KE),NESPN,0.0,1.0)
      CMH = POLINT(T7(KE), CMHC(KE),NESPN,0.0,1.0)
      PRINT 806
  806 FORMAT(1H0, 7X, 3HETA, 6X, 9HCHXC/CAVG, 3X, 9HCHYC/CAVG, 3X,
     1       9HCHZC/CAVG, 2X,
     2       34HCMHXC/CAVG  CMHYC/CAVG  CMHZC/CAVG, 3X, 9HCMHC/CAVG)
  816 FORMAT(///1H0, 10X,27HCONTROL SURFACE TOTAL LOADS/
     1        1H0,48H          ETA        (X/C) CP     (Y/C) CP     (Z/C) CP)
  826 FORMAT(47H0         CHX           CHY           CHZ           CHM)
  836 FORMAT(1H ,8F12.6)
  846 FORMAT(F13.6, 7(12H    0.000000))
      KK    = KS-1
      IF(KS.EQ. 1) GOTO 841
      DO 840 K=1, KK
  840 PRINT 846, ETA(K)
  841 DO 850 K=KS, KE
  850 PRINT 836, ETA(5), XSLCS(K), YSLCS(K), ZSLCS(K), XSMAHL(K),
     1       YSMAHL(K),  ZSMAHL(K), CMHC(K)
      IF(KE.EQ.NETA) GO TO 861
      KP    =KE+1
      DO 860 K=KP, NETA
  860 PRINT 846, ETA(K)
  861 PRINT 816
      IF(KS .EQ.1) GOTO 871
      DO 870 K=1,KK
  870 PRINT 876, ETA(K)
  876 FORMAT(F13.6, 4(12H    0.000000))
  871 DO 880 K=KS, KE
  880 PRINT 836, ETA(K), XSCPCS(K), YSCPCS(K), ZSCPCS(K)
      IF(KE .EQ. NETA)  GOTO 891
      DO 890 K=KP, NETA
  890 PRINT 876, ETA(K)
  891 CONTINUE
      NCS = NCS + 1
      GOTO  709
      END
```

```
OSUBROUTINE  PPRINT(ETA, XOC, NETA, NXOC, VMU , VML , VTU , VTL     6
1  , CPU, CPL, NC)                                                    6
ODIMENSION ETA(NETA), XOC(NXOC), VMU(NXOC, NETA), VML(NXOC, NETA), 6
1       VTU(NXOC, NETA), VTL(NXOC, NETA), CPU(NXOC, NETA),          6
2       CPL(NXOC, NETA)                                               6
 PRINT 1, NC                                                          6
10FORMAT(1H1,20X,*PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPO6
1NENT*, I4  )                                                         6
 DO 10 I=1, NETA                                                      6
 PRINT 2, ETA(I)                                                      6
 PRINT 3                                                              6
 DO 10 J=1, NXOC                                                      6
 CP     = CPL(J,I) - CPU(J,I)                                         6
100PRINT 4,  XOC(J), VMU(J,I), VML(J,I), VTU(J,I), VTL(J,I), CPU(J,I)6
 1      , CPL(J,I), CP                                                6
2 FORMAT(5HOETA=, F9.6)                                               6
4 FORMAT(8F12.6)                                                      6
30FORMAT(5X,*X/C*, 7X, *VM/V UP*, 5X, *VM/V LO*, 5X, *VT/V UP*, 5X, 6
1       *VT/V LO*, 6X, *CP UP*, 7X, *CP LO*, 6X, *CP L-U*)           6
 RETURN                                                               6
C    BLAINE D. GAITHER 11/72                                          6
 END                                                                  6
```

```
      SUBROUTINE BINTEG(DAX, DAY, DAZ, XC, YC, ZC, LONPNA, LONLEN,LATPAN6   6560
     1 , LATLEN, CPD, NSSTAP, NSSECP, XTL, NSTAT, S, X,                   6   6570
     2   PANS, SLX, DXC, BCHRDL, CX, CY, CZ, CMX, CMY, CMZ                6   6580
     3  ,NSSTA, NSSEC, LONPAN, XCG, YCG, ZCG, ALPHA, ARBJ,                6   6590
     4  TSXL, TSYL, TSZL, TMX, TMY, TMZ, CDI, NC                          6   6600
     5)                                                                   6   6610
C                                                                         6   6620
C                                                                         6   6630
C                                                                         6   6640
C                                                                         6   6650
C     DAX DAY DAZ  -   AREAS OF SUBPANELS VIEWED FROM X, Y, Z             6   6660
C     XC  YC  ZC   -   X Y Z VALS OF CENTROIDS OF SUB PANELS              6   6670
C     NSSTA        -   NUMBER OF SUBSTATIONS                              6   6680
C     NSSEC        -   NUMBER OF SUBSECTIONS                              6   6690
C     LONPNA       -   CONTAINS NUMBER LONGITUAINALLY OF THE PANELS       6   6700
C                  WHERE CPS ARE SPECIFIED                                6   6710
C     LATPNA       -  IS LATERAL COUNTER PART TO LONPAN                   6   6720
C     LONLEN       -   NUMBER OF STATIONS WHERE CPS ARE GIVE              6   6730
C     LATLEN       -   NUMBER OF SECTIONS WHERE CPS ARE GIVE              6   6740
C     CPD          -   CONTAINS CP VALS AT PANELS SPECIFIED BY LONPAN     6   6750
C                     AND LATPAN                                          6   6760
C     NSSECP       -   NUMBER OF SUBSECTIONS PER PANEL                    6   6770
C     NSSTAP       -   NUMBER OF SUBSTATIONS PER PANEL                    6   6780
C     XTL          -   VECTOR OF PANEL BOUNDRIES                          6   6790
C     NSTAT        -   NUMBER OF STATIONS                                 6   6800
C     BCHRDL       -   BODY  CHORD LENGTH                                 6   6810
C     ARBJ      - REFERENCE AREA FOR THIS BODY                            6   6820
C     X         - ARRAY OF X VALS FOR CP VAL.S OF CPD                     6   6830
C     S SS XX   - SCRATCH ARRAYS                                          6   6840
C     PANS         -   SET TO INTEGRAL OF EACH STATION                    6   6850
C     SLX          -   XS  WHERE SECTION LOADS                            6   6860
C     DXC          -   DELTA X OVER C                                     6   6870
C     CX CY CZ     -   SECTION LOADS                                      6   6880
C     CMX CMY CMZ  -   SECTION MOMENTS                                    6   6890
C     LONPAN LATPAN SCRATCH                                               6   6900
C                                                                         6   6910
C                                                                         6   6920
C                                                                         6   6930
      DIMENSION  DAX(NSSTA, NSSEC),    DAY(NSSTA, NSSEC),                 6   6940
     1DAZ(NSSTA, NSSEC),    XC(NSSTA, NSSEC),    YC(NSSTA, NSSEC),        6   6940
```

```
      2ZC(NSSTA,NSSEC), LONPNA(LONLEN),
     3 CPD(LONLEN, LATLEN), X(LONLEN), XX(49), LONPAN(LONLEN), LATPAN
     4(LATLEN),  XTL(1), SLX(NSTAT),   DXC(NSTAT),CX(NSTAT), CY(NSTAT),
     5CZ(NSTAT), CMX(NSTAT),CMY(NSTAT),CMZ(NSTAT), PANS(NSTAT),
     6 S(NSSEC), SS(49), SLSS(50), SLXX(50)
      EQUIVALENCE (SLSS(2),SS), (SLXX(2), XX)
      REAL LATPAN, LONPNA
      NAMELIST /LA/ SX, SY, SZ  /LS/ NSTA,NBOT, NTOP /LC/ LT,LL
     1 /LD/ AXT, AYT, AZT, ATX, ATY, ATZ
C     CONVERT LATPAN * LONPAN SO THEY  POINT TO THE CORRECT SUB-PANEL
C     INSTEAD OF PANEL
      LT = NSSTAP/ 2 + 1
      DO 10 I=1, LONLEN
   10 LONPAN(I)= (LONPNA(I)-1)*NSSTAP + LT
      SX = 0.
      SY = 0.
      SZ = 0.
      DO 21 J=1,NSSEC
      DO 21 I=1,NSSTA
      SX = SX + ABS(DAX(I,J)        )
      SY = SY + ABS(DAY(I,J)     )
   21 SZ = SZ + ABS(DAZ(I,J)        )
      SX = SX/2.0
      SY = SY/2.0
      SZ = SZ/2.0
      DO 400 NSTA=1, NSTAT
      NBOT = (NSTA-1)*NSSTAP + 1
      NTOP = NBOT + NSSTAP - 1
      PANSUM = 0
      AXT = 0.
      AYT = 0.
      AZT = 0.
      ATZ = 0.
      ATY = 0.
      ATX = 0.
C     START ON THIS STATION
      DO 390 NST=NBOT, NTOP
C     FIND CPS FOR THIS RING(SUB-STATION)
      DO 315 I=1, LATLEN
```

```
C     FIND CP FOR A MERIDIAN
      LT= LATPAN(I)
  942 FORMAT(1H , I5, G11.4,5X, 3G11.4,5X,  3G11.4)
      IF(LONLEN .GT. 3) GOTO305
      T = XC(NST,LT)
      NB = 1
      NT =2
      IF(T.LE.X(2).OR.LONLEN.LT.3) GOTO325
      NB = 2
      NT = 3
  325 XX(I) = (T - X(NB)) * (CPD(NT,I) - CPD(NB,I)) / (X(NT)-X(NB)) +
     1 CPD(NB,I)
      GOTO 315
  305 XX(I) = CODIM1(XC(NST,LT),       X, CPD(1,I), LONLEN,-1.0)
  315 CONTINUE
  941 FORMAT(10G11.4)
   31 IF(LATLEN.GT.1) GOTO 32
      PANSUM = PANSUM + NSSECP*XX(1)
      DO 500  I=1, NSSEC
      AXT = AXT + XX(1)*DAX(NST,I)
      AYT = AYT + XX(1)*DAY(NST,I)
      AZT = AZT + XX(1) * DAZ(NST,I)
      ATX = ATX + ((YC(NST,I)-YCG)*DAZ(NST,I) - (ZC(NST,I)-ZCG)*DAY(NST,
     1I)) * XX(1)
      ATY = ATY + ((ZC(NST,I)-ZCG)*DAX(NST,I) - (XC(NST,I)-XCG)*DAZ(NST,
     1I)) * XX(1)
  500 ATZ = ATZ + ((XC(NST,I)-XCG)*DAY(NST,I) - (YC(NST,I)-YCG)*DAX(NST,
     1I)) * XX(1)
      GOTO 390
   32 LTT   = LATPAN(1)
      LTP   = LTT
      IF(LTT.EQ.1) LTP = 2
      S(LTP -1) = 0.0
      DO 40 I=LTP, NSSEC
   40 S(I) = S(I-1) + SQRT((YC(NST,I) - YC(NST,I-1))**2 + (ZC(NST,I) - Z
     1C(NST,I-1))**2)
      TSF   = SQRT((YC(NSSEC)-YC(1))**2 + (ZC(NSSEC)-ZC(1))**2)
      IF(LTT.EQ.1) GOTO 46
      S(1)  = S(NSSEC) + TSF
```

```
      LTTT  = LTT - 1
      IF(LTTT.EQ.1) GOTO 46
      DO 45 I= 2, LTTT
   45 S(I)   = S(I-1) + SQRT((YC(NST,I)-YC(NST,I-1))**2 + (ZC(NST,I) -
     1ZC(NST,I-1))**2)
   46 XX(LATLEN+1) = XX(1)
      DO 50 I=1,LATLEN
      LT = LATPAN(I)
   50 SS(I) =S(LT)
      SS(LATLEN+1) = TSF + S(NSSEC)
      IF(LTT.NE.1) SS(LATLEN+1) = S(LTTT) + SQRT((YC(NST,LTTT) - YC(NST,
     1      LTT))**2    +  (ZC(NST,LTTT)-ZC(NST,LTT))**2)
      SLXX(1) = XX(LATLEN)
      SLSS(1) = SS(1) - SS(LATLEN+1) + SS(LATLEN)
      DO 60 I=1, NSSEC
      T      = CODIM1( S(I), SLSS, SLXX, LATLEN+2, 1.0)
      PANSUM = PANSUM + T
      AXT = AXT + T*DAX(NST,I)
      AYT = AYT + T*DAY(NST,I)
      AZT = AZT + T*DAZ(NST,I)
      ATX = ATX + ((YC(NST,I) - YCG) * DAZ(NST,I) -(ZC(NST,I)-ZCG)*DAY(N
     1ST,I)) * T
      ATY = ATY + ((ZC(NST,I)-ZCG)*DAX(NST,I) - (XC(NST,I)-XCG)*DAZ(NST,
     1I)) * T
   60 ATZ = ATZ + ((XC(NST,I)-XCG)*DAY(NST,I) - (YC(NST,I)-YCG)*DAX(NST,
     1I)) * T
  390 CONTINUE
      PANS(NSTA) = PANSUM
      SLX(NSTA)  = (XTL(NSTA)+XTL(NSTA+1))/2.0
               T = (XTL(NSTA+1)-XTL(NSTA))/ BCHRDL
      DXC(NSTA) = T
      TS = T * SY
      T = T*SZ
      CX(NSTA) = -AXT/T
      CY(NSTA) = -AYT/T S
      CZ(NSTA) = -AZT/T
      T = ARBJ * BCHRDL
      CMX(NSTA)= -ATX/T
      CMY(NSTA)= -ATY/T
```

```
  400 CMZ(NSTA)= -ATZ/T
C     FIND TOTAL MOMENTS * LOADS
      TSXL =   0.
      TSYL =   0.
      TSZL =   0.
      TMX  =   0.
      TMY  =   0.
      TMZ  =   0.
      PRINT 923, NC
  923 FORMAT(1H1,35X,*BODY SECTIONAL LOADS FOR COMPONENT*, I5,/1H0,10X,
     1               *X/C*,8X, *CXW/WAVG*,7X,*CYH/HAVG*,07X,*CZW/WAVG*)
      DO 410 I=1, NSTAT
      SLX(I)=SLX(I)/BCHRDL
      PRINT 948, SLX(I), CX(I), CY(I), CZ(I)
  948 FORMAT(1H ,4F15.6)
      TSXL = TSXL +  CX(I)*DXC(I)
      TSYL = TSYL +  CY(I)*DXC(I)
      TSZL = TSZL + CZ(I) * DXC(I)
      TMX = TMX + CMX(I)*DXC(I)
      TMY = TMY + CMY(I)*DXC(I)
  410 TMZ = TMZ + CMZ(I)*DXC(I)
      TSXL = TSXL * SZ/ARBJ
      TSYL = TSYL*SY/ARBJ
      TSZL = TSZL*SZ/ARBJ
      T = TSXL*TSXL + TSYL*TSYL + TSZL*TSZL
      XCPC = XCG / BCHRDL + (TSXL*TMZ - TSZL*TMY)/T
      YCPC = YCG/BCHRDL + (TSZL*TMX-TSXL*TMZ)/T
      ZCPC = ZCG/BCHRDL + (TSXL*TMY-TSYL*TMX)/T
      CL = TSZL*COS(ALPHA) - TSXL*SIN(ALPHA)
      CD    = TSZL * SIN(ALPHA) + TSXL * COS(ALPHA)
      PRINT 906
  906 FORMAT(///1H0,44X,*TOTAL BODY LOADS*)
      CDI=CDI/ARBJ
      PRINT 901, TSXL, TSYL, TSZL, TMX, TMY, TMZ, CDI
  901 FORMAT(1H0,7X,*CX*,13X,*CY*, 13X,*CZ*,12X,*CMX*,12X,*CMY*,12X,*CMZ
     1*, 12X, *CDI*/1H ,7F15.6)
      PRINT 902, XCPC, YCPC, ZCPC, SX, SY, SZ
  902 FORMAT(1H0, 5X,*X/C CP*,9X,*Y/C CP*,9X, *Z/C CP*,11X,*SX*,13X,*SY*
     1, 13X, *SZ*/1H , 6F15.6)
```

```
      RETURN                                                      6   8510
C     BLAINE D. GAITHER 9/72                                      6   8520
      END                                                         6   8530
```

```
      SUBROUTINE REFLCT(SYM,LO,LA,AX,AY,AZ,X,Y,Z,LON,LAT,LATP,CPD,NSS)
      DIMENSION AX(LO,LA),AY(LO,LA),AZ(LO,LA), X(LO,LA),Y(LO,LA),Z(LO,LA
     1),LATP(LAT),CPD(LON,LAT)
      REAL LATP
      LL=NSS/2+1
      DO 1 I=1,LAT
    1 LATP(I)=(LATP(I)-1)*NSS+LL
      IF(SYM.EQ.-1.) RETURN
C     REFLECT GEOMETRY
      ILA=LA
      ILA1=ILA+1
      K=LA
      DO 10 J=1,LA
      JJ=LA+1-J
      IF(ABS(Y(1,JJ)).LT..0001) GO TO 10
      K=K+1
      DO 5 I=1,LO
      AX(I,K)=AX(I,JJ)
      AY(I,K)=-AY(I,JJ)
      AZ(I,K)=AZ(I,JJ)
      X(I,K)=X(I,JJ)
      Y(I,K)=-Y(I,JJ)
    5 Z(I,K)=Z(I,JJ)
   10 CONTINUE
      LA=K
      IF(SYM.EQ.1.0) RETURN
C     REFLECT CPS
      K=LAT
      DO 20 J=1,LAT
      JJ=LAT+1-J
      LT=LATP(JJ)
      IF(ABS(Y(1,LT)).LT..0001) GO TO 20
      K=K+1
      DO 15 I=1,LON
   15 CPD(I,K)=CPD(I,JJ)
      DO 25 K1=ILA1,LA
      IF (Z(1,K1).EQ.Z(1,LT).AND.-Y(1,K1).EQ.Y(1,LT)) GO TO 26
   25 CONTINUE
      GO TO 20
```

```
26 LATP(K)=K1
20 CONTINUE
   LAT=K
   RETURN
   END
```

```
      SUBROUTINE  BPRINT(NO, NA, X, Y, Z, VM, VT, CP, NC)                6
      DIMENSION   X(NO, NA), Y(NO, NA), Z(NO, NA), VM(NO, NA), VT(NO, NA6
     1), CP(NO, NA)                                                       6
      PRINT 1,NC                                                         6
    1 FORMAT(1H1, 24X, *BODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMP6
     1ONENT*,I3)                                                          6
      DO 10 I=1, NA                                                      6
      PRINT 2, I                                                         6
    2 FORMAT(*0LATERAL STATION *, I3)                                    6
      PRINT 3                                                            6
    3 FORMAT(5X, *NO*,10X, *XQ*, 10X, *YQ*, 10X, *ZQ*, 9X, *VM/V*, 8X,   6
     1       *VT/V*, 9X, *CP*)                                            6
      PRINT 4, ( J, X(J,I), Y(J,I), Z(J,I), VM(J,I), VT(J,I), CP(J,I),   6
     1       J=1, NO)                                                     6
    4 FORMAT(4X,I3,5X,6F12.6)                                            6
   10 PRINT 5                                                            6
    5 FORMAT(1H1)                                                        6
      RETURN                                                             6
C     BLA9NE D. GAITHER 11/72                                            6
      END                                                                6
```

```
      SUBROUTINE FFDRAG(NB,NP,YB,ZB,XKB,NBE,BSYM,YP,ZP
     1 ,XKP,NPE,PSYM,PIND,TBD,TPD,NPS)
      DIMENSION YB(1),ZB(1),XKB(1),NBE(1),BSYM(1),YP(1)
     1 ,ZP(1),XKP(1),NPE(1),PSYM(1),PIND(1)
     2 ,T2D(1),TPD(1)
      COMMON/BODY/ DUM(15000),Y(200),Z(200)
     1  ,S(200),SYM(200),YN(200),ZN(200),XK(200),D(200),NUMB(200),
     2   NUMP(5),XP(200),DY(200),DZ(200),DS(200),ETA(200),YNP(200)
     3  ,ZNP(200)
      DATA IONE/1/,MAXVOR/200/,NVOR/40/
      K=0
      TNB5=0.
      IF(NB.EQ.0) GO TO 20
CALCULATION OF BODY DATA
      DO 5 I=1,NB
    5 TNBE=TNBE+NBE(I)
      L=0
      LKB=0
      DO 15 I=1,NB
      NUB=NBE(I)
      DO 10 J=1,NUB
      N1=L+J
      N2=N1+1
      K=K+1
      XK(K)=XKB(LKB+J)
      DELY=YB(N2)-YB(N1)
      DELZ=ZB(N2)-ZB(N1)
      S(K)=SQRT(DELY**2+DELZ**2)
      EPS=.000001
      IF(S(K).GT.EPS) GO TO 2
      S(K)=0.
      YN(K)=1.
      ZN(K)=1.
      GO TO 3
    2 YN(K)=-DELZ/S(K)
      ZN(K)=DELY/S(K)
    3 SYM(K)=BSYM(I)
      Y(K)=.5*(YB(N1)+YB(N2))
   10 Z(K'=.5*(ZB(N1)+ZB(N2))
```

```
      NUM2(I)=NBE(I)
      LKB=LKB+NUB
   15 L=L NUB+1
   20 IF(NP.EQ.0) GO TO 50
CALCULATION OF PANEL DATA
      INDP=0
      IF(NPS.EQ.0) GO TO 150
      INDP=1
      GO TO 8
  150 NPS=NVOR/NPE(1)
      DO 4 I=1,NP
    4 IF(NVOR/NPE(I).LT.NPS) NPS=NVOR/NPE(I)
    6 TNPE=0.
      DO 7 I=1,NP
    7 TNPE=TNPE+NPS*NPE(I)
      IF(TNPE+TNBE.LE.MAXVCR)GO TO 8
      NPS=NPS-1
      GO TO 6
    8 L=0
      LKP=0
      DO 49 I=1,NP
      NUP=NPE(I)
      SPAN=0.
      DO 45 J=1,NUP
      N1=L+J
      N2=N1+1
      DY(J)=YP(N2)-YP(N1)
      DZ(J)=ZP(N2)-ZP(N1)
      DS(J)=SQRT(DY(J)**2+DZ(J)**2)
      YNP(J)=-DZ(J)/DS(J)
      ZNP(J)=DY(J)/DS(J)
   45 SPAN=SPAN+DS(J)
      SUM=0.
      DO 46 J=1,NUP
      ETA(J)=(SUM+.5*DS(J))/SPAN
   46 SUM=SUM+DS(J)
      DO 47 J=1,NUP
   47 XP(J)=XKP(LKP+J)
      NUZ=NUP
```

```
      IF(PIND(I).NE.1.) GO TO 44
      NUZ=NUP+1
      ETA(NUZ)=1.
      XP(NUZ)=0.
   44 SUM=0.
      DO 48 J=1,NUP
      N1=L+J
      N2=N1+1
      DELS=DS(J)/NPS
      DO 48 J1=1,NPS
      K=K+1
      S(K)=DELS
      SYM(K)=PSYM(I)
      YN(K)=YNP(J)
      ZN(K)=ZNP(J)
      SUM=SUM+.5*DELS
      ET=SUM/SPAN
      SUM=SUM+.5*DELS
      IF(INDP.EQ.1) GO TO 488
      CALL CODIM(ETA,XP,NUZ,ET,XK(K),IONE)
  488 Y(K)=YP(N1)+(J1-.5)*DELS*ZN(K)
   48 Z(K)=ZP(N1)+(J1-.5)*DELS*(-YN(K))
      NUMP(I)=NPS*NUP
      LKP=LKP+NUP
   49 L=L NUP+1
CALCULATION OF DRAG ON EACH ELEMENT
   50 NV=K
      WRITE(6,333)(XK(J),J=1,NV)
  333 FORMAT(//(10F12.5))
      DO 58 I=1,NV
      V=0.
      W=0=
      DO 56 J=1,NV
      YY=Y(I)-Y(J)
      ZZ=Z(I)-Z(J)
      CALL HSHOE(YY,ZZ,S(J),YN(J),ZN(J),AV,AW)
      IF(SYM(J).EQ.1..OR.SYM(J).EQ.-1.) GO TO 54
      YY=-Y(I)-Y(J)
      CALL HSHOE(YY,ZZ,S(J),YN(J),ZN(J),DELV,DELW)
```

```
      IF(SYM(J).EQ.0.) GO TO 52
      DELV=-DELV
      DELW=-DELW
   52 AV=AV+DELV
      AW=AW+DELW
   54 V=V AV*XK(J)
   56 W=W+AW*XK(J)
   58 D(I)=-2.*XK(I)*S(I)*(V*YN(I)+W*ZN(I))
C     CALCULATION OF TOTAL DRAG
      K=0
      IF(NB.EQ.0) GO TO 95
      DO 93 I=1,NB
      TBD(I)=0.
      N1=K+1
      N2=K+NUMB(I)
      DO 92 J=N1,N2
   92 TBD(I)=TBD(I)+D(J)
      K=N2
      IF(SYM(J).EQ.0..OR.SYM(J).EQ.-2.) TBD(I)=2.*TBD(I)
   93 CONTINUE
   95 IF(NP.EQ.0) RETURN
      DO 98 I=1,NP
      TPD(I)=0.
      N1=K+1
      N2=K+NUMP(I)
      DO 96 J=N1,N2
   96 TPD(I)=TPD(I)+D(J)
      K=N2
   98 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NFDRAG(NC,NS,NCP,NSP,XOCCP,XOCV,ETACP,XV,YV,ZV
     1,TANV,SIGMAV,CPNET,NSPP,CDSCP,CDACP,CDTCP,CDTH,CDST,CDAT,CDTT,CDTH
     2T)
      DIMENSION XOCCP(1),XOCV(1),ETACP(1),XV(1),YV(1),ZV(1),TANV(1)
     1,SI7MAV(1),CPNET(1),CDSCP(1),CDACP(1),CDTCP(1),CDTH(1)
      DIM5NSION DUMI(100),DUMO(100),P(2000),ETAV(100)
      DIMENSION X(1000,2),Y(1000),Z(1000),TAN(1000,2),CP(2000)
     1  ,SIGMA(2000,2),D(2000),CDS(100),CDA(100),CDT(100)
      DIMENSION XK(200)
      DIMENSION   XOCPC(50)
      COMMON DA(5000)
      COMMON/BODY/ B(31550)
      EQUIVALENCE (Q,DA(13)),(GAM,DA(14)),(YCG,DA(8)),(ZCG,DA(9))
     1,(CBAR,DA(5)),(SPAN,DA(6))
      EQUIVALENCE (XK(1),B(16201)),(X(1,1),B(19001)),(Y(1),B(17001))
     1,(Z(1),B(18001)),(TAN(1,1),B(15001)),(CP(1),B(21001))
     2,(SIGMA(1,1),B(23001)),(D(1),B(27001)),(CDS(1),B(29001))
     3,(CDA(1),B(29101)),(CDT(1),B(29201)),(DUMI(1),B(29301))
     4,(DUMO(1),B(29401)),(P(1),B(29501)),(XOCPC(1),B(31501))
  333 FORMAT(//(1X,10F12.5))
  332 FORMAT(//(1X,10I12))
      WRITE(6,332)NC,NS,NCP,NSP,NSPP
      WRITE(6,333)(XOCCP(I),I=1,NCP)
      WRITE(6,333)(XOCV(I),I=1,NC)
      WRITE(6,333)(ETACP(I),I=1,NSP)
      R4PI=.07957747
      EPS2=.0075968656
      EPS3=.00001
      ISYM=DA(3426)
      NV1=NS*NC
      NV2=2*NV1
      WRITE(6,333)(XV(I),I=1,NV2)
      WRITE(6,333)(YV(I),I=1,NV2)
      WRITE(6,333)(ZV(I),I=1,NV2)
      WRITE(6,333)(TANV(I),I=1,NV2)
      WRITE(6,333)(SIGMAV(I),I=1,NV2)
      WRITE(6,333)(CPNET(I),I=1,NV1)
C     CALCULATE NEW VORTEX COORDINATES
      NSPP=3
```

```
      JK=0
      NC2=2*NC
      DELY=(YV(NC2+1)-YV(1))/NSPP
      DELY1=-DELY*(NSPP/2+1)
      DO 100 I=1,NS
      DEL=DELY1
      DO 100 J=1,NSPP
      DEL=DEL+DELY
      DO 100 K=1,NC2,2
      IK1=( I-1)*NC2+K
      IK2=IK1+1
      JK=JK+1
      X(JK,1)=XV(IK1)+DEL*TANV(IK1)
      X(JK,2)=XV(IK2)+DEL*TANV(IK2)
      Y(JK)=YV(IK1)+DEL
      Z(JK)=ZV(IK1)
      TAN(JK,1)=TANV(IK1)
      TAN(JK,2)=TANV(IK2)
      SIGMA(JK,1)=SIGMAV(IK1)
  100 SIGMA(JK,2)=SIGMAV(IK2)
      NV=JK
      NSS=NS*NSPP
      WRITE(6,333)(X(I,1),X(I,2),I=1,NV)
      WRITE(6,333)(Y(I),I=1,NV)
      WRITE(6,333)(Z(I),I=1,NV)
      WRITE(6,333)(TAN(I,1),TAN(I,2),I=1,NV)
      WRITE(6,333)(SIGMA(I,1),SIGMA(I,2),I=1,NV)
CALCULATE ETAS OF VORTICES
      DELE=1./NSS
      SUM=-.5*DELE
      DO 110 I=1,NSS
      SUM=SUM+DELE
  110 ETAV(I)=SUM
      WRITE(6,333)(ETAV(I),I=1,NSS)
CALCULATE X/C OF PANEL CENTROIDS
      SLOPE=(TANV(2*NC)-TANV(1))/(XV(2*NC)-XV(1))
      TANLE=TANV(1)-.5*(XV(2)-XV(1))*SLOPE
      TANTE=TANV(2*NC)+.5*(XV(2*NC)-XV(2*NC-1))*SLOPE
      DX=DA(3450)+DA(3432)
```

```
      XLE=YV(1)*TANLE+DX
      XTE=YV(1)*TANTE+DX+DA(3453)
      CHORD=XTE-XLE
      J=0
      DO 1000 I=1,NC2,2
      J=J+1
 1000 XOCPC(J)=(.5*(XV(I)+XV(I+1))-XLE)/CHORD
CALCULATE CPNET AT VORTICES
      DO 120 I=1,NCP
      DO 130 J=1,NSP
  130 DUMI(J)=CPNET(NCP*(J-1)+I)
      CALL POL(ETACP,DUMI,NSP,ETAV,DUMO,NSS)
      DO 120 J=1,NSS
      NJ=NCP*(J-1)+I
  120 P(NJ)=DUMO(J)
      DO 140 I=1,NSS
      K=NC*(I-1)+1
      J=NCP*(I-1)+1
  140 CALL POL(XOCCP,P(J),NCP,XOCPC,CP(K),NC)
      WRITE(6,333)(CP(I),I=1,NV)
      YVV=.5*DELY
      EPS1=2.*YVV
      NV2=2*NV
      IF(ISYM.EQ.1) NSS=2*NSS
      DO 1 I=1,NV2
      IF(ABS(CP(I)).GT.EPS3) GO TO 3
    1 CONTINUE
      GO TO 44
CALCULATE DRAG DUE TO LIFT
    3 DO 43 II=1,2
      WRITE(6,331) YVV
  331 FORMAT(1X,10E12.5)
      DO 5 JJ=1,NV2
    5 D(JJ)=0.
      NSKIP1=0
    9 DO 20 J=1,NV
      IPASS=1
      XMF=1.
      NSKIP2=0
```

```
      W=0.
      DO 22 K=1,NV
      DELX=X(J,II)-X(K,1)
      YY=Y(J)
      IF(ISYM.EQ.1.AND.NSKIP1.NE.0) YY=-YY
      DELY=YY-Y(K)
      DELZ=Z(J)-Z(K)
      DELXSQ=DELX**2
      DELZSQ=DELZ**2
   10 YP=DELY+YVV
      YN=DELY-YVV
      YPSQ=YP**2
      YNSQ=YN**2
      TERM1=YPSQ+DELZSQ
      TERM2=YNSQ+DELZSQ
      TERM3=DELXSQ+YPSQ
      TERM4=DELXSQ+YNSQ
      TERM5=TERM1+DELXSQ
      TERM6=TERM2+DELXSQ
      CHECK1=DELXSQ/TERM3
      CHECK2=DELXSQ/TERM4
      F=0.
      IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.FPS2.AND.ABS(DELY
     1).GT.YVV) GO TO 32
      IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(DELX).LT.ABS(.5*(X(K,
     12)-X(K,1)))) GO TO 33
      F=(YP/SQRT(TERM5)-YN/SQRT(TERM6))*DELX/(DELXSQ+DELZSQ)
      GO TO 33
   32 F=.5*DELX*ABS(1./TERM4-1./TERM3)
   33 IF(TERM1/TERM5.LT.EPS2.AND.DELX.LT.0.) GO TO 34
      IF(SQRT(TERM1).LT.EP.3) GO TO 35
      F=F+YP/TERM1*(1.+DELX/SQRT(TERM5))
      GO TO 35
   34 F=F+SQRT(TERM1)/TERM5
   35 IF(TERM2/TERM6.LT.EPS2.AND.DELX.LT.0.) GO TO 36
      IF(SQRT(TERM2).LT.EPS3) GO TO 30
      F=F-YN/TERM2*(1.+DELX/SQRT(TERM6))
      GO TO 30
   36 F=F-SQRT(TERM2)/TERM6
```

```
   30 KK=K+NSKIP2
      W=W+XMF*CP(KK)*(X(K,2)-X(K,1))*F
      IF(IPASS.EQ.2.OR.ISYM.EQ.-1) GO TO 31
      IPASS=2
      DELY=-YY-Y(K)
      IF(ISYM.EQ.0) GO TO 10
      IF(ISYM.EQ.1) GO TO 11
      XMF=-1.
      GO TO 10
   11 NSKIP2=NV
      GO TO 10
   31 IPASS=1
      XMF=1.
   22 NSKIP2=0
      KK=J+NSKIP1
   20 D(KK)=2.*CP(KK)*(X(J,2)-X(J,1))*W*R4PI
      IF(ISYM.NE.1.OR.NSKIP1.NE.0) GO TO 21
      NSKIP1=NV
      GO TO 9
   21 IF(II.EQ.2) GO TO 40
      DO 41 I=1,NSS
      CDS(I)=0.
      DO 41 J=1,NC
      K=NC*(I-1)+J
   41 CDS(I)=CDS(I)+D(K)
      GO TO 43
   40 DO 42 I=1,NSS
      CDA(I)=0.
      DO 42 J=1,NC
      K=NC*(I-1)+J
   42 CDA(I)=CDA(I)+D(K)
   43 CONTINUE
   44 DO 45 I=1,NV2
      DO 45 J=1,2
      IF(ABS(SIGMA(I,J)).GT.EPS3) GO TO 39
   45 CONTINUE
      GO TO 500
C     CALCULATE DRAG DUE TO THICKNESS
   39 DO 46 I=1,NV2
```

```
46 D(I)=0.
   NSKIP1=0
47 DO 50 I=1,NV
   DO 60 J=1,2
   IPASS=1
   NSKIP2=0
   U=0.
   DO 70 K=1,NV
   DO 70 L=1,2
   DELX=X(I,J)-X(K,L)
   YY=Y(I)
   IF(ISYM.EQ.1.AND.NSKIP1.NE.0) YY=-YY
   DELY=YY-Y(K)
   DELZ=Z(I)-Z(K)
   T=TAN(K,L)
   XP=DELX+T*YVV
   XN=DELX-T*YVV
62 YP=DELY+YVV
   YN=DELY-YVV
   XY=DELX-T*DELY
   TSQ1=1.+T**2
   PP=SQRT(TSQ1)
   R2SQ=XY**2+DELZ**2*TSQ1
   TEST1=XN**2+YN**2
   TEST2=XP**2+YP**2
   R4=SQRT(TEST1+DELZ**2)
   R5=SQRT(TEST2+DELZ**2)
   YX=DELY+T*DELX
   YVT=YVV*TSQ1
   TERM4=1./R4-1./R5
   XYSQ=XY**2
   CHECK1=XYSQ/TEST1
   CHECK2=XYSQ/TEST2
   IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY
  1).GT.YVV) GO TO 48
   IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(XY).LT.ABS(.5*(X(K,2)
  1-X(K,1)))) GO TO 49
   XI2=(YX+YVT)/R5
   XI3=-(YX-YVT)/R4
```

```
      TERM1=(XI2+XI3)/R2SQ
      GO TO 51
   48 TERM1=.5/PP*ABS(1./TEST1-1./TEST2)
      GO TO 51
   49 TERM1=0.
   51 EUS=T/PP*TERM4+1./PP*XY*TERM1
      KK=K+NSKIP2
      U=U+SIGMA(KK,L)*EUS
      IF(IPASS.EQ.2.OR.ISYM.EQ.-1) GO TO 61
      IPASS=2
      DELY=-YY-Y(K)
      IF(ISYM.EQ.0)GO TO 62
      NSKIP2=NV
      GO TO 62
   61 IPASS=1
   70 NSKIP2=0
      KK=I+NSKIP1
      U=U*R4PI
   60 D(KK)=D(KK)-(U+1.-Q*2.*(Z(I)-ZCG)/CBAR-GAM*2.*(Y(I)-YCG)/SPAN)*
     1 SIGMA(KK,J)*SQRT(1.+TAN(KK,J)**2)
   50 D(KK)=2.*D(KK)
      IF(ISYM.NE.1.OR.NSKIP1.NE.0) GO TO 81
      NSKIP1=NV
      GO TO 47
   81 DO 80 I=1,NSS
      CDT(I)=0.
      DO 80 J=1,NC
      K=NC*(I-1)+J
   80 CDT(I)=CDT(I)+D(K)
  500 CONTINUE
      CDST=0.
      CDAT=0.
      CDTT=0.
C     CALCULATION OF TOTAL DRAG
      DO 200 I=1,NSS
      CDST=CDST+CDS(I)
      CDAT=CDAT+CDA(I)
  200 CDTT=CDTT+CDT(I)
      CDST=CDST*2.*YVV
```

```
      CDAT=CDAT*2.*YVV
      CDTT=CDTT*2.*YVV
      CDTHT=CDAT-CDST
C     CALCULATE SPANWISE DISTRIBUTION OF DRAG
      CALL POL(ETAV,CDS,NSS,ETACP,CDSCP,NSP)
      CALL POL(ETAV,CDA,NSS,ETACP,CDACP,NSP)
      CALL POL(ETAV,CDT,NSS,ETACP,CDTCP,NSP)
      DO 190 I=1,NSP
  190 CDTH(I)=CDACP(I)-CDSCP(I)
      WRITE(6,333)(CDS(I),I=1,NSS)
      WRITE(6,333)(CDA(I),I=1,NSS)
      WRITE(6,333)(CDT(I),I=1,NSS)
      WRITE(6,333)(CDSCP(I),I=1,NSP)
      WRITE(6,333)(CDACP(I),I=1,NSP)
      WRITE(6,333)(CDTCP(I),I=1,NSP)
      WRITE(6,333)(CDTH(I),I=1,NSP)
      WRITE(6,333) CDST,CDAT,CDTT,CDTHT
C     CALCULATE TRAILING EDGE K VALUES
      DO 180 I=1,NSS
      XK(I)=0.
      DO 180 J=1,NC
      K=NC*(I-1)+J
  180 XK(I)=XK(I)+CP(K)*(X(K,2)-X(K,1))
      WRITE(6,333)(XK(I),I=1,NSS)
      RETURN
      END
```

```
      FUN3TION POLINT(X,Y,NPTS,RLLIM,RULIM)
C
C     X         -  INDEPENDENT VARIABLES
C     Y         -  DEPENDENT VARIABLES
C     NPTS      -  NUMBER OF DEPENDENT AND INDEPENDANT VARIABLES
C     0 .LT. RLLIM .LT. RULIM .LT.1
C
      DIM5NSION X(NPTS), Y(NPTS), PX(22), PY(22)
      IF(NPTS-20) 11, 11, 1
    1 PRINT 6
    6 FORMAT(*0TOO MANY POINTS GIVEN TO POLINT*)
      STOP
   11 PX(1) = ACOS(1.0-2.0*RLLIM)
      PY(1)= 0.0
      PX(NPTS+2) = ACOS(1.0-2.0*RULIM)
      PY(NPTS+2) = 0.0
      DO 20 I=1, NPTS
      PHI = ACOS(1.0-2.0*X(I))
      PX(I+1) = PHI
      PY(I+1) = Y(I)*.5*SIN(PHI)
   20 CONTINUE
      CALL INTNC(PX(1),PX(NPTS+2), NPTS+2, PX, PY, POLINT, IERR)
      RETURN
      END
```

```
      FUN3TION TANALF(                      DZT,DZC,EPSILO,DA,ETA,X,TB)
C
C
C     DZT       - DELTA X T
C     DZC       - DDELTA X C
C     ESILO     - EPSILON
C     X         - X VALUE AT THIS POINT
C     TB        - 1.FOR BOTTOM SURFACE           -1.FOR TOP SURFACE
C     DA        - ARRAY OF CONTROL SURFACE INFO.
      DIMENSION DA(1)
C     IS THERE A CONTROL SURFACE AT THIS POINT
      IJ = 1
C     IS THIS CONTROL POINT AT OUR ETA
    1 IF(DA(IJ).EQ.0.0) GOTO150
      IF(DA(IJ+2).LE.ETA .AND. DA(IJ+3).GE. ETA) GOTO 30
    2 IJ = IJ + 8
      IF(IJ.LT.80) GOTO 1
      PRINT 156, IJ, DA
  156 FORMAT(I3,8G14.6)
      STOP 445
   30 T3= DA(IJ+4)+(DA(IJ+5)*DA(IJ+4))/(DA(IJ+3)-DA(IJ+2))*(ETA-DA(IJ+2)
     1)
      IF( DA(IJ) .EQ.1..AND. T3 .GE. X)GOTO 100
      IF( DA(IJ) .EQ.-1..AND.T3 .LE. X)GOTO 75
      GOTO 2
   75 TA = DA(IJ+1)
      GOTO 101
  100 TA =-DA(IJ+1)
  101 T =(TB*DZT - DZC)     + EPSILO
      TA = TAN(TA)
      TAN1LF =(TA+T)/(1.0-T*TA)
      RETURN
  150 TANALF = (TB*DZT-DZC)     + EPSILO
      RETURN
      END
```

```
      SUBROUTINE HSHOE(Y,Z,S,YN,ZN,V,W)
      DATA R4PI/.07957747/
      DATA EPS/.000001/
      IF(S.GT.EPS) GO TO 5
      V=0.
      W=0.
      RETURN
    5 HS=.5*S
      YP=Y*ZN-Z*YN
      ZP=Z*ZN+Y*YN
      YPP=YP+HS
      YPN=YP-HS
      RP=YPP**2+ZP**2
      RN=YPN**2+ZP**2
      EV=ZP*(1./RP-1./RN)*R4PI
      EW=(YPN/RN-YPP/RP)*R4PI
      V=EV*ZN+EW*YN
      W=EW*ZN-EV*YN
      RETURN
      END
```

```
      SUBROUTINE POL(XI,YI,NI,XO,YO,NO)
      DIMENSION XI(1),YI(1),XO(1),YO(1)
      DIM5NSION TI(50),FI(50),TO(100)
      DATA PI/3.1415927/
      NI1=NI+1
      NI2=NI+2
      TI(1)=0.
      DO 10 I=2,NI1
      TI(I)=ACOS(1.-2.*XI(I-1))
   10 FI(I)=YI(I-1)*2.*SQRT(XI(I-1)*(1.-XI(I-1)))
      FI(1)=0.
      TI(NI2)=PI
      FI(NI2)=0.
      DO 15 I=1,NO
   15 TO(I)=ACOS(1.-2.*XO(I))
      CALL CODIM(TI,FI,NI2,TO,YO,NO)
      DO 20 I=1,NO
   20 YO(I)=YO(I)/(2.*SQRT(XO(I)*(1.-XO(I))))
      RETURN
      END
```

```
      SUBROUTINE INTNC (XL, XU, NX, X, Y, Z, IERR )
C   F R AN4ERSON 056 291 072  BLDG. 2  STATION 15
C                 INTEGRATION ROUTINE, NEWTON-COTES METHOD
C                 CALL INTNC (XL, XU, NX, X, Y, Z, IERR)
C               XL LOWER LIMIT OF INTEGRATION
C               XU UPPER LIMIT OF INTEGRATION
C               NX NUMBER OF ITEMS  IN X AND Y ARRAYS
C               X  ARRAY OF ABSCISSAS (INDEPENDENT VARIABLE)
C               Y  ARRAY OF ORDINATES (DEPENDENT VARIABLE)
C               Z  VALUE OF DEFINITE INTEGRAL
C               XI ARRAY OF ABSCISSAS USED IN INTEGRATION
C               YI ARRAY OF ORDINATES USED IN INTERPOLATION
C               IERR ERROR INDICATOR, ZERO VALUE INDICATES PROPER
C                    SOLUTION,  NONZERO INDICATES ERROR
C               IXU INDICATOR FOR UPPER LIMIT
      DIMENSION X(1), Y(1), XI(5), YI(5)
      IXU = 0
      IERR = 0
C                 TEST TO BE WITHIN RANGE
    1 IF (X(1) - XL)10,10,5
    5 WRITE          ( 6, 7) XL  , X(1)
    7 FORMAT ( 1H0 E15.8, 66H, LOWER LIMIT OF INTEGRATION IS LESS THAN F
     1IRST ABSCISSA OF ARRAY, E17.8 )
      GO TO 110
   10 IF (XU - X(NX)) 15,15,12
   12 WRITE          ( 6, 14) XU, X(NX)
   14 FORMAT (1H0 E15.8, 68H, UPPER LIMIT OF INTEGRATION IS GREATER THAN
     1 LAST ABSCISSA OF ARRAY,   E17.8)
      GO TO 110
C                 INITIALIZE
   15 Z = 0.0
      XK = 1.0
C                 FIND FIRST INTERVAL
      DO 30 I = 1, NX
      IF (X(I) - XL) 30,45,40
   30 CONTINUE
C                 SETUP FOR FIRST INTERVAL
   40 DX = X(I) - XL
      XI(1) = XL
```

```
      XI(5) = X(I)
      YI(5) = Y(I)
      NI = 4
      K1 = 1
      GO TO 60
C                 LOWER LIMIT SAME AS A TABULAR VALUE
   45 I = I + 1
C                 SETUP FOR SUCCEEDING INTERVALS
   50 XI(1) = X(I-1)
      YI(1) = Y(I-1)
      XI(5) = X(I)
      YI(5) = Y(I)
      NI = 3
      K1 = 2
      DX = X(I) - X(I-1)
C                 INTERMEDIATE POINTS IN INTERVAL
   60 XI(2) = XI(1) + .25 * DX
      XI(3) = XI(1) + .5 * DX
      XI(4) = XI(1) + .75 * DX
C                 OBTAIN INTERPOLATED ORDINATES
   70 CALL CODIS1 (NI, XI(K1), YI(K1), X, Y, NX, XK )
C                 NEWTON-COTES FIVE POINT FORMULA
   80 Z = Z +(DX /90.0)*(7.0 * (YI(1)+ YI(5) ) + 32.0 * (YI(2) + YI(4))
     1+ 12.0 * YI(3) )
C                 PREPARE FOR NEXT INTERVAL
   90 I = I + 1
C                 TEST FOR LAST TABULAR VALUE
      IF (NX - I) 100,94,94
C                 TEST FOR LAST INTERVAL
   94 IF (XU - X(I)) 120,120,50
  110 IERR = 1
C                 TEST FOR COMPLETION
  100 RETURN
  120 IF (IXU) 100,130,100
C                 SETUP FOR LAST INTERVAL
  130 IXU = 1
      XI(5) = XU
      XI(1) =  X(I-1)
      YI(1) = Y(I-1)
```

```
      NI = 4                                                              6  5420
      K1 = 2                                                              6  5430
      DX = XU - X(I-1)                                                    6  5440
      GO TO 60                                                            6  5450
      END                                                                 6  5460

      SUBROUTINE DUMP                                                     6  5470
      PRINT 6                                                             6  5480
    6 FORMAT(*0SUBROUTINE DUMP*)                                          6  5490
      STOP                                                                6  5500
      END                                                                 6  5510
```

```
      SUBROUTINE CODIS1(N1,X,Y,XI,YI,N2,XK)
C   F R ANDERSON 056 291 072  BLDG. 2  STATION 15
C     CONTROLLED DEVIATION INTERPOLATION SUBROUTINE....CODIS1
C
C     CALLING SEQUENCE
C
C     CALL CODIS1(N1,X,Y,XI,YI,N2,XK)
C          N1 = NO. OF ARGUMENTS X
C           X = ARGUMENTS-ABSCISSA VALUES.
C           Y = INTERPOLATED ORDINATES.
C          XI = ARRAY OF THE ABSCISSAE.
C          YI = ARRAY OF THE ORDINATES.
C          N2 = NO. OF POINTS ON CURVE.
C          XK = END INTERVAL CONTROL CONSTANT ( 0 TO 1.0 )
C
      DIMENSION X(1),Y(1),XI(1),YI(1), D(2),A(2),B(2),C(2)
  100 IN =0
      DO 800 N = 1,N1
      IF(N2-2)110,115,120
  110 Y(N) = YI(N2)
      GO TO 800
  115 Y(N)= (YI(2)-YI(1))/(XI(2)-XI(1))* (X(N)-XI(1))+YI(1)
      GO TO 800
  120 J = 1
  125 IF(XI(J)-X(N))130,140,150
  130 J= J+1
      IF(J-N2)125,125,145
  140 Y(N) = YI(J)
      GO TO 800
  145 Y(N)= (YI(N2)-YI(N2-1))/(XI(N2)-XI(N2-1))*(X(N)-XI(N2-1))+YI(N2-1)
      GO TO 800
  150 IF(J-2)115,155,160
  155 K = 3
      JJ= 1
      GO TO 185
  160 IF(J-N2)170,165,145
  165 K = N2-1
      JJ= 2
      GO TO 185
```

```
170 IF(J-IN)180,300,180
180 JJ = 3
    K = J
185 DO 200 M = 1,2
    X1 = XI(K-1)-XI(K)
    X2 = XI(K)-XI(K-2)
    X3 = XI(K-2)-XI(K-1)
    Y1 = YI(K-1)-YI(K)
    Y2 = YI(K)-YI(K-2)
    Y3 = YI(K-2)-YI(K-1)
    XX1 = XI(K-2)**2
    XX2= XI(K-1)**2
    XX3= XI(K)**2
    D(M) = XX1*X1 +XX2*X2+ XX3*X3
    A(M) =(YI(K-2)*X1 +YI(K-1)*X2 +  YI(K)*X3)/D(M)
    B(M) =(XX1*Y1 + XX2*Y2+XX3*Y3)/D(M)
    C(M) = YI(K-2)- A(M)*XX1 -B(M)*XI(K-2)
200 K = K+1
300 P1 =  X(N)*(A(1)*X(N)+B(1)) +C(1)
    P2 =  X(N)*(A(2)*X(N)+B(2)) +C(2)
    AL = (X(N)-XI(J-1))/(XI(J)-XI(J-1))
    S  = YI(J)*AL + YI(J-1)*(1.0-AL)
    GO TO (320,330,350),JJ
320 P2 = P1
    AL = (X(N)-XI(1))/(XI(2)-XI(1))
    S = AL*YI(2) + (1.0-AL)*YI(1)
    P1= S + XK *(P2-S)
    GO TO 350
330 P1 = P2
    AL = (X(N)-XI(N2-1))/ (XI(N2)-XI(N2-1))
    S = AL* YI(N2) +(1.0-AL)*YI(N2-1)
    P2 = S + XK*(P1-S)
350 E1 =  ABS(P1-S)
    E2 =  ABS(P2-S)
    IN = J
    IF(E1+E2)700,700,750
700 Y(N) = S
    GO TO  800
750 BT =  (E1*AL)/(E1*AL+(1.0-AL)*E2)
```

```
      Y(N)=BT*P2+(1.0-BT)*P1                                  6   6300
  800 CONTINUE                                                6   6310
  900 RETURN                                                  6   6320
      END                                                     6   6330
```

```
      OVERLAY(OVL,07,0)
      PROGRAM TRAIL
C     MAIN PROGRAM FOR FREE TRAILING VORTICES
      DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI( 500),YTVI(500)
     1          ,ZTVI(500),UTV(1000),VTV(1000),WTV(1000),V1(100),V2(100)
     2          ,V3(100),NIP(100),NTVE(100)
      EQUIVALENCE(B(15001),UTV),    (B(16001),VTV),    (B(17001),WTV)
     1          ,(B(18001),XTV),    (B(19001),YTV),    (B(20001),ZTV)
     2          ,(B(21001),XTVI),   (B(21501),YTVI),   (B(22001),ZTVI)
     3          ,(B(22501),V1),     (B(22601),V2),     (B(22701),V3)
     4          ,(B(22801),NIP),    (B(22901),NTVE)
     5          ,(B(23001),NV)
      EQUIVALENCE(XVS,B(12871))
      DIMENSION XVS(100)
      COMMON/BODY/ B(25000)
      COMMON/SCRAT/ XQ(1000),YQ(1000),ZQ(1000)
      COMMON DA(5000)
      EQUIVALENCE (DA(7), XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
     1  ,(DA (11),BETA), (DA(12),PSTAR),(DA(13),QSTAR),(DA(14),RSTAR)
C
COMPUTE FREE TRAILING VORTEX POINTS, INCLUDING INITIAL POINTS.
C
      XMIN=XVS(100)
      XMAX=XTVI(1)
      WRITE(6,70) XMIN,XMAX
 70   FORMAT(6HOTRAIL/(1P3E20.6))
      WRITE(6,65) (NIP(J),J=1,NV)
 65   FORMAT(11HONIP ARRAY=10I5)
      K=0
      DO 75 J=1,NV
      NN=NIP(J)
      DO 75 I=1,NN
      K=K+1
      WRITE(6,70) XTVI(K),YTVI(K),ZTVI(K)
      IF(XMAX.GT.XTVI(K)) GO TO 75
      XMAX=XTVI(K)
 75   CONTINUE
      DXX=XMAX-XMIN
      DELX=DXX/20.0
```

```
      XMAX=XMIN+1.25*DXX
      WRITE(6,5) XMAX,DXX,XMIN,DELX
    5 FORMAT(20H0XMAX,DXX,XMIN,DELX=1P4E20.6)
      K=0
      DO 50 I=1,NV
      N1=0
      IF(I-1) 15,15,10
   10 IM1=I-1
      DO 11 II=1,IM1
   11 N1=NIP(II)+N1
   15 NP = NIP(I)
      DO 20 J=1,NP
      K=K+1
      NJ=N1+J
      XTV(K)=XTVI(NJ)
      YTV(K)=YTVI(NJ)
   20 ZTV(K)=ZTVI(NJ)
      DO 30 J=1,20
      J1=J
      K=K+1
      XTV(K)=XTV(K-1)+DELX
      YTV(K)=YTV(K-1)
      ZTV(K)=ZTV(K-1)
      IF(XTV(K).GT.XMAX) GO TO 35
   30 CONTINUE
   35 CONTINUE
   50 NTVE(I)=NIP(I)+J1
      WRITE(6,80)(XTV(I),YTV(I),ZTV(I),I=1,K)
   80 FORMAT( 19H0TRAIL. XTV,YTV,ZTV/(1P3E20.6))
C
COMPUTE POINTS AT WHICH VELOCITIES WILL BE FOUND IN SUB.TRAVEL.
C
      K=0
      DO 100 I=1,NV
      M=NTVE(I)
      DO 90 J=1,M
      K=K+1
      K1=K+1
      IF(J.EQ.M) GO TO 85
```

```
      XQ(K)=0.5*(XTV(K1)+XTV(K))
      YQ(K)=0.5*(YTV(K1)+YTV(K))
      ZQ(K)=0.5*(ZTV(K1)+ZTV(K))
      GO TO 90
   85 XQ(K)=XTV(K)
      YQ(K)=YTV(K)
      ZQ(K)=ZTV(K)
   90 CONTINUE
  100 CONTINUE
      CALL TRAVEL(XQ,YQ,ZQ,UTV,VTV,WTV,K)
      WRITE(6,105)(XQ(I),YQ(I),ZQ(I),UTV(I),VTV(I),WTV(I),I=1,K)
  105 FORMAT( 28HOTRAIL. XQ,YQ,ZQ,UTV,VTV,WTV/(1P6E20.6))
C    TEMPORARY CBAR,BREF.    ********************************************
      CBAR=10.0
      BREF=1.0
C
      DO 110 I=1,K
      XX=XQ (I)-XCG
      YY=YQ (I)-YCG
      ZZ=ZQ (I)-ZCG
      VX=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/BREF)
      VY=-BETA-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
      VZ=ALPHA+2.0*(PSTAR*YY/BREF+QSTAR*XX/CBAR)
      WRITE(6,109) I,VX,VY,VZ
  109 FORMAT(I5,1P3E16.4)
      UTV(I)=UTV(I)+VX
      VTV(I)=VTV(I)+VY
  110 WTV(I)=WTV(I)+VZ
      CALL INTEG(XTV,YTV,ZTV,UTV,VTV,WTV,NV,NTVE)
      WRITE(6,106)(XTV(I),YTV(I),ZTV(I),I=1,K)
  106 FORMAT(23HOINTEGRATED XTV,YTV,ZTV/(1P3E20.6))
      END
      SUBROUTINE INTEG(X,Y,Z,U,V,W,NTV,NTVE)
      DIMENSION X(1),Y(1),Z(1),U(1),V(1),W(1),NTVE(1)
      K=0
      DO 10 I=1,NTV
      NE=NTVE(I)
      DO 5 J=2,NE
      L2=J+K
```

```
      L1=L2-1
      DELX=(X(L2)-X(L1))/U(L1)
      Y(L2)=Y(L1)+DELX*V(L1)
      Z(L2)=Z(L1)+DELX*W(L1)
    5 CONTINUE
   10 K=K+NE
      RETURN
      END
      SUBROUTINE TRAVEL(XI,YI,ZI,UI,VI,WI,NPTS)
      DIMENSION XI(1),YI(1),ZI(1),UI(1),VI(1),WI(1)
      EQUIVALENCE(B(1),XV),    (B(5001),YV),    (B(1001),ZV)
      DIMENSION XV(5000),YV(5000),ZV(5000)
      COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2
      COMMON DA(5000)
      COMMON/MATRIX/ MDUM(1000),XSOL(200)
      COMMON/BODY/ B(25000)
      EQUIVALENCE (B(1),XVR), (B(201),YVR), (B(401),ZVR), (B(601),XVO)
     1           ,(B(801),YVO),    (B(1001),ZVO),    (B(2871),SXMT)
     2           ,(B(3871),SYMT), (B(4871),SZMT),   (B(5871),DYS)
     3           ,(B(6871),DZS),   ( B(7871),TS),    (B(8871),XSS)
     4           ,(B(9871),YSS),   (B(10871),ZSS)
     5           ,(B(11871),SIGMA),    (B(12871),XVS),   (B(12971),YVS)
     6           ,(B(13071),ZVS)
      DIMENSION XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1          ,YVO(10,20),ZVO(10,20),SXMT(1000),SYMT(1000),SZMT(1000)
     2          ,DYS(1000),DZS(1000),TS(1000),XSS(1000),YSS(1000)
     3           ,ZSS(1000),SIGMA(1000),XVS(100),YVS(100),ZVS(100)
      DIMENSION BODYG(15000),PANELG(13170)
      EQUIVALENCE(B,BODYG,PANELG)
      COMMON/SCRAT/ XQ(1000),YQ(1000),ZQ(1000),AXB(200),AYB(200)
     1  ,SUM(3),TRSUMX(100),TRSUMY(100),TRSUMZ(100),AZB(200)
     2  ,SUM1(3)
      COMMON/NUMBER/ NVPTS(5),NCPTS(5),NLN(5),NLT(5),LTC(5),LNC(5)
     1 ,NCT,NB,NBODS,NVL(5),NVT(5),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2  ,LSEG(5),TSEG(5),LFUNC(5),TFUNC(5)
     3 ,LNDIVB(5),LTDIVB(5),NSPP(5),ROOTP(5),OUTERP(5)
     4  ,SYMM(5)
      COMMON/PANINF/PANSYM(10)
      EQUIVALENCE(SYMP,DA(3426)),(SYMB,DA(19))
```

```
      EQUIVALENCE(DA(2),PANS)
      PI=3.1415926
      PI4=12.5663704
      NPANS=PANS
      REWIND 18
      LNDIV=1
      DO 2 I=1,NPTS
      UI(I)=0.0
      VI(I)=0.0
    2 WI(I)=0.0
      IF(NBODS.EQ.0) GO TO 190
      DO 65 KK=1,NBODS
      READ(18) BODYG
      SYMB=SYMM(KK)
      LTDIV=LTDIVB(KK)
      NTVV=NVT(KK)
      NBVV=NVL(KK)
      DO 63 M=1,NPTS
      X=XI(M)
      Y=YI(M)
      Z=ZI(M)
      U=0.0
      V=0.0
      W=0.0
      N=0
      DO 60 J=1,NTVV
      DO 60 I=1,NBVV
      N=N+1
      TOT1=0.0
      TOT2=0.0
      TOT3=0.0
      DO 55 K=1,4
      DO 8 KS=1,3
    8 SUM(KS)=0.0
      GO TO (10,20,30,39),K
   10 IF(I.EQ.1) GO TO 12
      DO 11 KS=1,3
   11 SUM(KS)=-SUM1(KS)
      GO TO 50
```

```
   12 II1=1
      IF1=1
      II2=(J-1)*LTDIV
      DO 14 L=1,LTDIV
      II2=II2+1
      IF2=II2+1
   14 CALL VORTEX(SUM)
      GO TO 50
   20 II2=1+J*LTDIV
      IF2=II2
      II1=(I-1)*LNDIV
      DO 24 L=1,LNDIV
      II1=II1+1
      IF1=II1+1
   24 CALL VORTEX(SUM)
      SX=SUM(1)
      SY=SUM(2)
      SZ=SUM(3)
      GO TO 50
   30 II1=IF1
      II2=II2+1
      DO 34 L=1,LTDIV
      II2=II2-1
      IF2=II2-1
      IF(I.NE.NBVV) GO TO 33
      DO 32 KS=1,3
   32 SUM(KS)=0.0
      GO  TO 34
   33 CALL VORTEX(SUM)
   34 CONTINUE
      DO 35 KS=1,3
   35 SUM1(KS)=SUM(KS)
      GO TO 50
   39 IF(J.EQ.1) GO TO 42
      SUM(1)=-TRSUMX(I)
      SUM(2)=-TRSUMY(I)
      SUM(3)=-TRSUMZ(I)
      GO TO 50
   42 II2=IF2
```

```
      III=III+1
      DO 44 L=1,LNDIV
      III=III-1
      IF1=III-1
44    CALL VORTEX(SUM)
50    TOT1=TOT1+SUM(1)
      TOT2=TOT2+SUM(2)
      TOT3=TOT3+SUM(3)
55    CONTINUE
      TRSUMX(I)=SX
      TRSUMY(I)=SY
      TRSUMZ(I)=SZ
      AXB(N)=TOT1/PI4
      AYB(N)=TOT2/PI4
      AZB(N)=TOT3/PI4
60    CONTINUE
      DO 62 II=1,N
      U=U+AXB(II)*XSOL(II)
      V=V+AYB(II)*XSOL(II)
62    W=W+AZB(II)*XSOL(II)
      UI(M)=U+UI(M)
      VI(M)=V+VI(M)
63    WI(M)=W+WI(M)
65    CONTINUE
190   IF(NPANS.EQ.0) GO TO 405
      DO 200 KK=1,NPANS
      READ(18) PANELG
      SYMP=PANSYM(KK)
      NSPACE=NSPP(KK)
      NTVV=NVT(NBODS+KK)
      NBVV=NVL(NBODS+KK)
      IF(KK-1) 191,191,192
191   NPPV=0
      GO TO 193
192   KK1=K-1
      NB1=NVL(NBODS+KK1)
      NT1=NVT(NBODS+KK1)
      NPPV=NT1*NB1
193   CONTINUE
```

```
      DO 198 M=1,NPTS
      X=XI(M)
      Y=YI(M)
      Z=ZI(M)
      U=0.0
      V=0.0
      W=0.0
      KVOR=0
      DO 195 I=1,NTVV
      IS=I-NSPACE
      DO 195 J=1,NBVV
      J1=J+1
      KVOR=KVOR+1
      IF(IS.LE.0) GO TO 75
      IT=(I-1)*NBVV*2
      IT1=IT+1+(J-1)*2
      IT12=IT1+2
      T1=TS(IT12)
      T2=TS(IT1)
   75 CONTINUE
      DO 5 K1=1,3
    5 SUM(K1)=0.0
      DO 41  K=1,4
      IF(J.EQ.NBVV.AND.K.EQ.4) GO TO 41
      IF(K.GT.1) GO TO 21
      IF(IS.LE.0) GO TO 76
      X2=XVO(IS,J) -T2*SQRT(DYS(IT1)**2+DZS(IT1)**2)*0.5
      Y2=YVO(IS,J) -0.5*DYS(IT1)
      Z2=ZVO(IS,J) -0.5*DZS(IT1)
      IF(J.EQ.NBVV) GO TO 305
      X1=XVO(IS,J1)-T1*SQRT(DYS(IT12)**2+DZS(IT12)**2)*0.5
      Y1=YVO(IS,J1)-0.5*DYS(IT12)
      Z1=ZVO(IS,J1)-0.5*DZS(IT12)
      X3=X1
      Y3=Y1
      Z3=Z1
      GO TO 40
  305 X1=XVS(IS)
      Y1=YVS(IS)
```

```
      Z1=ZVS(IS)
      X3=X1
      Y3=Y1
      Z3=Z1
      GO TO 40
76    CONTINUE
      X1=XVR(I,J+1)
      Y1=YVR(I,J+1)
      Z1=ZVR(I,J+1)
      X2=XVR(I,J)
      Y2=YVR(I,J)
      Z2=ZVR(I,J)
      GO TO 40
21    X1=X2
      Y1=Y2
      Z1=Z2
      IF(K-3) 25,36,38
25    IF(I.GT.NSPACE) GO TO 251
      X2=XVR(I+1,J)
      Y2=YVR(I+1,J)
      Z2=ZVR(I+1,J)
      GO TO 40
251   X2=XVO(IS,J)+0.5*SQRT(DYS(IT1)**2+DZS(IT1)**2)*T2
      Y2=YVO(IS,J)+0.5*DYS(IT1)
      Z2=ZVO(IS,J)+0.5*DZS(IT1)
      GO TO 40
36    IF(I.GT.NSPACE) GO TO 301
      X2=XVR(I+1,J+1)
      Y2=YVR(I+1,J+1)
      Z2=ZVR(I+1,J+1)
      GO TO 40
301   IF(J.EQ.NBVV) GO TO 303
      X2=XVO(IS,J1)+0.5*SQRT(DYS(IT12)**2+DZS(IT12)**2)*T1
      Y2=YVO(IS,J1)+0.5*DYS(IT12)
      Z2=ZVO(IS,J1)+0.5*DZS(IT12)
      GO TO 40
303   X2=XVS(IS+1)
      Y2=YVS(IS+1)
      Z2=ZVS(IS+1)
```

```
      GO TO 40
38    IF(I.GT.NSPACE) GO TO 351
      X2=XVR(I,J+1)
      Y2=YVR(I,J+1)
      Z2=ZVR(I,J+1)
      GO TO 40
351   X2=X3
      Y2=Y3
      Z2=Z3
40    IF(J.EQ.MBVV.AND.K.EQ.4) GO TO 4111
401   CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,X,Y,Z)
41    CONTINUE
      GO TO 417
4111  CONTINUE
      Y2H=Y2
      Y1H=Y1
      SYMPAS=-1.0
C     LEFT  SEMI-INFINITE VORTEX LINE.
410   T1=SQRT((Y2-Y)**2+(Z2-Z)**2)
      IF(T1-0.00001) 412,411,411
411   CONTINUE
      T2=(X2-X)/SQRT((X2-X)**2+(Y2-Y)**2+(Z2-Z)**2)
      QT=0.25*(1.0-T2) /(PI*T1)
      SUM(2)=SUM(2)+SYMPAS*QT*(Z2-Z)/T1
      SUM(3)=SUM(3)-SYMPAS*QT*(Y2-Y)/T1
C     RIGHT SEMI-INFINITE VORTEX LINE.
412   CONTINUE
      T1=SQRT((Y1-Y)**2+(Z1-Z)**2)
      IF(T1-0.00001) 414,413,413
413   CONTINUE
      T2=(X1-X)/SQRT((X1-X)**2+(Y1-Y)**2+(Z1-Z)**2)
      QT=0.25*(1.0-T2) /(PI*T1)
      SUM(2)=SUM(2)-SYMPAS*QT*(Z1-Z)/T1
      SUM(3)=SUM(3)+SYMPAS*QT*(Y1-Y)/T1
414   CONTINUE
      IF(SYMPAS.EQ.-1.0.AND.SYMP.EQ.0) GO TO 415
      GO TO 416
415   Y1=-Y1
      Y2=-Y2
```

```
      SYMPAS=1.0
      GO TO 410
416   Y1=Y1H
      Y2=Y2H
417   IF(IS.GT.0) GO TO 46
      TERM=XSOL(KVOR+NPPV)
      GO TO 48
46    TERM=0.0
      K1=(I-1)*NBVV
      DO 47 I2=1,J
47    TERM=TERM+XSOL(K1+I2+NPPV)
48    U=U+SUM(1)*TERM
      V=V+SUM(2)*TERM
195   W=W+SUM(3)*TERM
      UI(M)=UI(M)+U
      VI(M)=VI(M)+V
198   WI(M)=WI(M)+W
200   CONTINUE
      RETURN
      DO 400 K=1,NPTS
      X=XI(K)
      Y=YI(K)
      Z=ZI(K)
      U=0.0
      V=0.0
      W=0.0
      MA=0
      MS=0
      DO 300 I=1,NTVV
      DO 300 J=1,NBVV
      DO 300 IS=1,2
      MS=MS+1
      KSOL=1
      T=TS(MS)
      DZ=DZS(MS)
      DY=DYS(MS)
300   CALL PVSK(T,DY,DZ,X,Y,Z,MS,MA,I,J,KSOL)
      DO 350 KV=1,MS
      U=U+UXMT(KV)*SIGMA(KV)
```

```
      V=V+SYMT(KV)*SIGMA(KV)
350   W=W+SZMT(KV)*SIGMA(KV)
      UI(K)=UI(K)+U
      VI(K)=VI(K)+V
400   WI(K)=WI(K)+W
405   CONTINUE
      RETURN
      END
      SUBROUTINE VORTEX(SUM)
      COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)
      COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
      COMMON DA(5000)
      EQUIVALENCE (DA(19),SYM)
      COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2
      DIMENSION SUM(1)
      XI=XV(II1,II2)
      YI=YV(II1,II2)
      ZI=ZV(II1,II2)
      XF=XV(IF1,IF2)
      YF=YV(IF1,IF2)
      ZF=ZV(IF1,IF2)
      XFQ=XF-X
      ZFQ=ZF-Z
      XIQ=XI-X
      ZIQ=ZI-Z
      SYMLOO=1.0
10    YFQ=YF-Y
      YIQ=YI-Y
      DELX=XF-XI
      DELY=YF-YI
      DELZ=ZF-ZI
      RXS1=YFQ*DELZ-ZFQ*DELY
      RXS2=ZFQ*DELX-XFQ*DELZ
      RXS3=XFQ*DELY-YFQ*DELX
      RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)
      TERM1=SQRT(DELX**2+DELY**2+DELZ**2)
      TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)
      TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)
      TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ
```

```
      RATIO = TERM4/TERM1**2
      COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)
      COSB = TERM4/(TERM1*TERM2)
      CC=COSB-COSA
      HX=XFQ-RATIO*DELX
      HY=YFQ-RATIO*DELY
      HZ=ZFQ-RATIO*DELZ
      H=SQRT(HX*HX+HY*HY+HZ*HZ)
      IF(H-0.00001) 11,12,12
   11 COEF=0.0
      GO TO 13
   12 CONTINUE
      HRXS=H*RXS
      COEF=SYMLOO*CC/HRXS
   13 CONTINUE
      SUM(1)=COEF*RXS1 + SUM(1)
      SUM(2)=COEF*RXS2 + SUM(2)
      SUM(3)=COEF*RXS3 + SUM(3)
      IF(SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN
      SYMLOO=-1.0
      YI=-YI
      YF=-YF
      GO TO 10
      END
      SUBROUTINE VORPAN(SUM,XI,YI,ZI,XF,YF,ZF,X,Y,Z)
      COMMON DA(5000)
      EQUIVALENCE(DA(3426),SYM)
      DIMENSION SUM(1)
      R4PI=0.07957747
      YIH=YI
      YFH=YF
      XFQ=XF-X
      ZFQ=ZF-Z
      XIQ=XI-X
      ZIQ=ZI-Z
      SYMLOO=1.0
   10 YFQ=YF-Y
      YIQ=YI-Y
      DELX=XF-XI
```

```
      DELY=YF-YI
      DELZ=ZF-ZI
      RXS1=YFQ*DELZ-ZFQ*DELY
      RXS2=ZFQ*DELX-XFQ*DELZ
      RXS3=XFQ*DELY-YFQ*DELX
      RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)
      TERM1=SQRT(DELX**2+DELY**2+DELZ**2)
      TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)
      TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)
      TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ
      RATIO = TERM4/TERM1**2
      COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)
      COSB = TERM4/(TERM1*TERM2)
      CC=COSB-COSA
      HX=XFQ-RATIO*DELX
      HY=YFQ-RATIO*DELY
      HZ=ZFQ-RATIO*DELZ
      H=SQRT(HX*HX+HY*HY+HZ*HZ)
      IF(H-0.00001) 11,12,12
   11 COEF=0.0
      GO TO 13
   12 CONTINUE
      HRXS=H*RXS
      COEF=R4PI*SYMLOO*CC/HRXS
   13 CONTINUE
      SUM(1)=COEF*RXS1 + SUM(1)
      SUM(2)=COEF*RXS2 + SUM(2)
      SUM(3)=COEF*RXS3 + SUM(3)
      YI=YIH
      YF=YFH
      IF(SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN
      SYMLOO=-1.0
      YI=-YI
      YF=-YF
      GO TO 10
      END
      SUBROUTINE PVSK(T,DYY,DZZ,XC,YC,ZC,MS,MA,IV,JV,KSOL)
      COMMON DA(5000)
      EQUIVALENCE(DA(3426),SYM)
```

```
      COMMON/PANINF/ PANSYM(10)
      COMMON/SCRAT/XSOL(5000),ROUND(5000),AX(5000),AY(5000),AZ(5000)
      COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
     1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSURV(100),CHORD(100)
     2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
     3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
     4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
     5  ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
     6  ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
      COMMON/PANEL/ NPAN,IPSYM,IUC,NBVVP,NTVVP,LNCFP,LTCFP,LNCPP,LTCPP
     1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
     2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
     3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
      COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
     1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
     2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
     3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
C
C   KSOL=1  FOR SOURCE PTS. ONLY
C   KSOL=2  FOR BOTH SOURCE PTS. AND VORTEX POINTS.
C   MS=SUBSCRIPT OF SOURCE PT.
C   XC,YC,ZC  --- CONTROL PT.
C   IV = LATERAL VORTEX SUBSCRIPT.
C   JV = LONGITUDINAL VORTEX SUBSCRIPT.
C  INSERT SPECIFICATION STATEMENTS HERE.
      REAL I1,I2,I3,I4
      YVV=0.5*SQRT(DYY**2+DZZ**2)
      YV2=2.0*YVV
      PI=3.141592654
      YV=YVV
      GO TO (10,20),KSOL
   10 YK=YSS(MS)
      ZK=ZSS(MS)
      XK=XSS(MS)
      GO TO 25
   20 IF(MS-2*(MS/2).EQ.0) GO TO 10
      YK=YVO(IV,JV)
      ZK=ZVO(IV,JV)
      XK=XVO(IV,JV)
```

```
25    CONTINUE
      SUMX=0.0
      SUMY=0.0
      SUMZ=0.0
      UTSUM=0.0
      VTSUM=0.0
      WTSUM=0.0
      SIGN=1.0
      DZ=ZC-ZK
      X=XC-XK
50    CONTINUE
      DY=YC-SIGN*YK
      RY=DYY/YV2
      RZ=DZZ/YV2
      Y=RY*DY+RZ*DZ
      Z=-RZ*DY+RY*DZ
      R12=(Y+YV)**2+Z**2
      R22=(X-T*Y)**2+Z**2*(1.0+T*T)
      R32=(Y-YV)**2+Z**2
      R4= SQRT((X-T*YV)**2+(Y-YV)**2+Z**2)
      R5= SQRT((X+T*YV)**2+(Y+YV)**2+Z**2)
      I1=(X+T*YV)/R5
      I2=(Y+T*X+YV*(1.0+T**2))/R5
      I3=-(Y+T*X-YV*(1.0+T**2))/R4
      I4=(X-T*YV)/R4
      IF(ABS(Z).GT.YV2) GO TO 42
      Z=0.0
      R6D=(X+T*YV)**2+(Y+YV)**2
      R7D=(X-T*YV)**2+(Y-YV)**2
      RXTY=(X-T*Y)**2
      R6=RXTY/R6D
      R7=RXTY/R7D
      IF(R6.GE.0.0075968656) GO TO 41
      IF(R7.GE.0.0075968656) GO TO 41
      IF(ABS(Y).LE.YV) GO TO 41
      TERM1=ABS(1.0/R7D-1.0/R6D)*0.5/P
      GO TO 43
41    IF(ABS(Y).GT.YV) GO TO 42
      IF(ABS(X-T*Y).GE.0.25*ABS(PLL(MA))) GO TO 42
```

```
      TERM1=0.0
      GO TO 43
   42 TERM1=(I2+I3)/R22
   43 CONTINUE
      TERM2=(I1+1.0)/R12
      TERM3=(I4+1.0)/R32
      TERM4=1.0/R4-1.0/R5
      P=SQRT(1.0+T*T)
      EUS=(T*TERM4+(X-T*Y)*TERM1)/P
      EVS=(TERM4-T*(X-T*Y)*TERM1)/P
      EWS=P*Z*TERM1
      US=0.25*EUS/PI
      VS=0.25*EVS/PI
      WS=0.25*EWS/PI
      UT=US
      VT=VS*RY-WS*RZ
      WT=VS*RZ+WS*RY
      UTSUM=UT+UTSUM
      VTSUM=VT+VTSUM
      WTSUM=WT+WTSUM
      IF(KSOL.EQ.1) GO TO 45
      EU=Z*TERM1
      EV=Z*(-T*TERM1+TERM2-TERM3)
      EW=-(X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
      UV=0.25*EU/PI
      VV=0.25*EV/PI
      WV=0.25*EW/PI
      UI=UV
      VI=RY*VV-RZ*WV
      WI=RZ*VV+RY*WV
      SUMX=UI+SUMX
      SUMY=VI+SUMY
      SUMZ=WI+SUMZ
   45 CONTINUE
C     FOR SYMMETRY, GET IMAGE CONTRIBUTION.
      IF(SYM.NE.0.0) GO TO 60
      IF(SIGN.LT.0.0) GO TO 60
      SIGN=-1.0
      DZZ=-DZZ
```

```
      T=-T                                                       8  6630
      GO TO 50                                                   8  6640
   60 CONTINUE                                                   8  6650
      SXMT(MS)=UTSUM                                             8  6660
      SYMT(MS)=VTSUM                                             8  6670
      SZMT(MS)=WTSUM                                             8  6680
      IF(KSOL.EQ.1) RETURN                                       8  6690
      AX(MA)=SUMX                                                8  6700
      AY(MA)=SUMY                                                8  6710
      AZ(MA)=SUMZ                                                8  6720
      RETURN                                                     8  6730
      END                                                        8  6740
```

```
      OVERLAY(OVL,10,0)
      PROGRAM HARDAP
      COMMON/SCRAT/A(20),DA(5000),ATACH(5,10)
      INTEGER UNIT,UNIT2
      DATA UNIT2/21/
      DATA UNIT/10/
      REWIND UNIT
      REWIND UNIT2
      PRINT 3
      READ 5, A
      IF(A(1).EQ.4H     .AND.A(2).EQ.4H     .AND.A(3).EQ.4H    1) GOTO 60
      CALL OUTIN (A)
      GOTO 99
    1 READ 5, A
   60 IF(EOF, 5) 2, 372
  372 PRINT 6, A
      WRITE(UNIT, 5) A
      GOTO 1
    2 ENDFILE UNIT
      REWIND UNIT
      CALL DECRD(DA, UNIT)
      NB = DA(1)
      IF(NB.EQ.0) GOTO 20
      DO 10 I=1, NB
   10 CALL DECRD(DA, UNIT)
   20 NP = DA(2)
      IF(NP.EQ.0) GOTO 40
      DO 30 I=1, NP
      CALL DECRD(DA, UNIT)
      ATACH(1,I) = DA(3420)
      ATACH(2,I) = DA(3444)
      ATACH(3,I)=DA(3445)
      ATACH(4,I)=DA(3446)
   30 ATACH(5,I)=DA(3447)
   40 REWIND UNIT
      WRITE(UNIT2) ATACH
      WRITE(6,51) UNIT2
   51 FORMAT(*0UNIT2=*I3)
      REWIND UNIT2
```

```
   99 CONTINUE
    5 FORMAT(20A4)
    6 FORMAT(1H ,20A4)
    3 FORMAT(1H1, 30X,*LISTING OF INPUT DATA CARDS*/1H0)
C     BLAINE D. GAITHER 11/72
      END
```

```
      SUBROUTINE DECWRT(DA,IARG,MAX,IUNIT)
      DIMENSION DA(1)
      ISTART = IARG
  444 IF(MAX - ISTART - 4) 1, 1, 2
    1 MSTART = -ISTART
      WRITE(IUNIT, 111) MSTART, (DA(I), I=ISTART,MAX)
      PRINT 111, MSTART, (DA(I), I=ISTART, MAX)
  111 FORMAT(I12,5G12.5)
      RETURN
    2 MSTART = ISTART + 4
      IF(DA(ISTART).EQ.0.0.AND.DA(ISTART+1).EQ.0.0.AND.DA(ISTART+2).EQ.0
     1.0.AND.DA(ISTART+3).EQ.0.0.AND.DA(ISTART+4).EQ.0.0) GOTO 3
      WRITE( IUNIT, 111) ISTART, (DA(I), I=ISTART, MSTART)
      PRINT 111, ISTART, (DA(I), I=ISTART, MSTART)
    3 ISTART = ISTART +5
      GOTO 444
      END
```

```
      SUBROUTINE OUTIN(TITLE)                                           OUT 0110
C                                                                       OUT 0070
C     CODED BY B. D. GAITHER                                            OUT 0080
C     AT SOME LATER DATE ALL TAPE I/O SHOULD BE DONE BY BUFFER IN AND   OUT 0090
C     OUT,  THEN ALL ZEROING LOOPS AND CALLS TO DECRD AND DECWRT COULD  OUT 0100
C     BE REMOVED.   AT PRESENT THE MAIN ROUTINES EXPECT DECRD STYLE INPUOUT 0140
C                                                                       OUT 0150
      INTEGER TITLE(20), NRADX(4), NFORX(4), OUNIT, OUNIT2,SCRTCH       OUT 0160
      REAL DA(5000), XAF(30), XOAFLE(20), YOAFLE(20), ZOAFLE(20),       OUT 0170
     1AFSWCL(20), TZORD(30)    ,  WAFORD(410)   ,XVXFXF(120) , ZLCCS(120) OUT 0190
     2 , FCSA(120), YOHS(30,120), ZOHS(30,120)
     3,XOP    , YOP    , ZOP    ,   PX(30)   , PODRAD(30)   , FINXL,
     5FINYL    , FINZL    , LCHORD    , FINXH    , FINYH    , FINZH    ,   OUT 0200
     6 HCHORD    , XLC    ,YLC    , ZLC    , CONCRD    , XHC    , YHC    , ZHC OUT 0210
     7  , CONCOH    , CANPCL(10) ,  CANPLN(10) ,  CANS(10)              OUT 0220
     8,PCHORD(10)    , FAFHT(10)   ,ATA(29),ZC(440),PPD(150)            OUT 0230
     9, PANARE(1580),BODARE(3405)                                       OUT 0240
     A, ATACH(5,10), NPPDP, NTTXCS
     B,NXSBOT,BCT(50),LCT(670),RB(800)                                  OUT 0260
     C,TT(29),ETA(29),NATA                                              OUT 0270
     D,BMFCXS(49)                                                       OUT 0280
     E,ZLCCT(120), SR(30), S(39), ZCSA(39), SZ(120,30),SY(120,30)
C                                                                       OUT 0290
      DOUBLE PRECISION PI, RAD, RAD 1 DG                                OUT 0300
C                                                                       OUT 0310
      COMMON /PATACH/ ATACH                                             OUT 0320
C                                                                       OUT 0330
      EQUIVALENCE(REFA,DA(16)),(PSI,DA(9))                              OUT 0340
     1,(ATA(1),DA(3631)),(ANAS,DA(3630)),(ZC(1),DA(3660))               OUT 0350
     2,(XAF(1),DA(3601)),(XCNUM,DA(3600))                               OUT 0360
     3,(REFAF,DA(3421))                                                 OUT 0370
     4,(XPO,DA(3432)),(YPO,DA(3433)),(ZPO,DA(3434))                     OUT 0380
     5,(AFSIM,DA(3426))                                                 OUT 0390
     6,(XAF(1),PCHORD(1)),(WAFORD(1),DA(4190)),(WAFORD(1),FAFHT(1))     OUT 0400
     7 , (PANARE(1),DA(3420)), (BODARE(1), DA(15)), (XCL,FINXL),        OUT 0410
     8 (YCL,FINYL), (ZCL, FINZL), (CONCRD, LCHORD), (XHC, FINXH),       OUT 0420
     9  (YHC,FINYH), (ZHC, FINZH), (CONCOH, HCHORD),( CANPCL(1), PCHORD(OUT 0430
     A1)),(CANPLN(1),FAFHT(1)),(PPD(1),DA(3450))                        OUT 0440
     B,(NPPDP,DA(3443)),(NTTXCS,DA(4130))                               OUT 0450
```

```
     C,(BOX,DA(27)),(BOY,DA(28)),(BOZ,DA(29)),(MXSBOT,DA(40)),          OUT 0460
     D(LCT(1),DA(130)),(RB(1),DA(830)),(BCT(1),DA(41))                   OUT 0470
     E,(TT(1),DA(4131)),(ETA(1),DA(4161)),(NATA,DA(4160))                OUT 0480
     F,(BMFCXN,DA(1600)),(BMFCXS(1),DA(1601))                            OUT 0490
     G,(BCTII,DA( 20)),(SN,DA(130)), (ZLCCT(1), DA(1800)),
     H(S(1), DA(131)), (BSYM, DA(19)), (PCONT, DA(3429))
C                                                                        OUT 0510
      DATA IUNIT/5/,OUNIT/10/, OUNIT2/20/,DA/5000*-0.0/, SCRTCH/18/      OUT 0520
     1, PI/ 3.1415 92653 58979 32384 62643 D+0/,                         OUT 0530
     2 RAD 1 DG/.0174 53292 51994 32957 69237 D+0/                       OUT 0540
     3, ZLCCS/120*-0.0/
C                                                                        OUT 0550
      NAMELIST /JLIST/ J0, J1, J2, J3, J4, J5, J6, NWAF, NWAFOR, NFUS, NOUT 0560
     1RADX,NFORX,NP, NPODOR, NF, NFINOR, NCAN, NCANOR                    OUT 0570
     2/WLIST/ XAF, XOAFLE, YOAFLE, ZOAFLE, AFSWCL, SPAN, ATA, PPD         OUT 0580
     3,WAFORD                                                            OUT 0590
     4/BLIST/ XVXFXF, ZLCCS, FCSA                                        OUT 0600
     5/PLIST/ XOP, YOP, ZOP, PX, PODRAD                                  OUT 0610
     6/FLIST/ FINXL, FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH, HCHORD,  OUT 0620
     7 PCHORD, FAFHT                                                     OUT 0630
     8/PLOTLT/ PHI,THETA, PSII                                           OUT 0640
     9/CLIST/ XLC, YLC, ZLC, CONCRD, XHC, YHC, ZHC, CONCHO, CANPCL,      OUT 0650
     ACANPLN, CANS                                                       OUT 0660
     B/ATTACH/ATACH                                                      OUT 0670
C                                                                        OUT 0680
C     READ IDENTIFICATION CARD AND ECHO                                  OUT 0690
      PRINT 6                                                            OUT 0700
    6 FORMAT(1H1, 30X,*BEGIN CONVERSION OF LANGLEY DATA*)                OUT 0710
      PRINT 2, TITLE                                                     OUT 0740
    2 FORMAT(1X,20A4)                                                    OUT 0750
C                                                                        OUT 0760
C     READ CONTROL INTEGERS                                              OUT 0770
      READ 3, J0, J1, J2, J3, J4, J5, J6, NWAF, NWAFOR, NFUS, (NRADX(I),OUT 0780
     1NFORX(I), I=1, 4, 1), NP, NPODOR, NF, NFINOR, NCAN, NCANOR         OUT 0790
    3 FORMAT(24I3)                                                       OUT 0800
      PRINT JLIST                                                        OUT 0810
C                                                                        OUT 0820
C     READ REFERANCE AREA CARD                                           OUT 0830
      IF(J0.EQ.1) READ 4, REFA                                           OUT 0840
```

```
    4 FORMAT(10F7.0)
      REFAA=REFA
      REFAF=REFA
C
C     PUNCH UNIVERSAL INFO
      CALL DECWRT(DA,1,14,OUNIT)
C
C     ***************************************************************
C
C     READ WING DATA
C
      IF(IABS(J1).NE.1) GOTO 101
C
C     NUMBER OF PERCENT CHORD LOCATIONS
      XCNUM = NWAFOR
C
C     SET SYMMETRY OF WING
      AFSIM = 0.0
C
C     SET PANEL CONTOUR INDICATOR
      PCONT = 1.0
C
C     READ PERCENT CHORD LOCATIONS
      READ 4, (XAF(I), I=1, NWAFOR)
      DO 324 I=1, NWAFOR
      XAF(I)= XAF(I)/100.0
  324 TT(I) = XAF(I)
C
C     READING DATA CARDS
      READ 5, (XOAFLE(I), YOAFLE(I), ZOAFLE(I), AFSWCL(I), I=1, NWAF)
    5 FORMAT(4F7.0)
C
C     NUMBER OF ETA(ATA) STATIONS
      ANAS = NWAF
      NATA = ANAS
C
C     FIND AIRFOIL LENGTHS  (SPAN IS THEIR SUM)
      SPAN = 0.0
      ATA(1) = SPAN
```

```
      DO 102 I=2, NWAF
      ATA(I)=SQRT((YOAFLE(I)-YOAFLE(I-1))**2+(ZOAFLE(I)-ZOAFLE(I-1))**2)
      SPAN = ATA(I) + SPAN
  102 ATA(I)=SPAN
C
C     LOAD(AIRFOIL LENGTH / SPAN)INTO ARRAY ATA
      DO 104 I=1,NWAF
      ATA(I)=ATA(I)/SPAN
  104 ETA(I)=ATA(I)
C
C     READ DELTA Z VALUES AND CONVERT TO  Z/ CHORD LENGTH
      IF(J1.NE.1) GOTO 222
      I=1
      DO 31 J=1, NWAF
      IF(AFSWCL(J).NE.0) GOTO 517
      PRINT 228, J
  228 FORMAT(*0THE CHORD LENGTH OF WING AIRFOIL*,I3,* IS ZERO, IT HAS BE
     1EN SET TO  .01*/)
      AFSWCL(J)=.01
  517 READ 4, (TZORD(II),II=1, NWAFOR)
      PRINT 400, (TZORD(II), II=1, NWAFOR)
  400 FORMAT(*0TZORD*,3(/10G12.5))
      DO 31 IT = 1, NWAFOR
      ZC(I)=TZORD(IT)/AFSWCL(J)
   31 I=I+1
C
C     PANEL (WING) ORIGIN
  222 XPO = 0.0
      YPO = 0.0
      ZPO = 0.0
C
C     PANEL PERIMETER DESCRIPTION (LEADING  EDGE)
      J=1
      DO 150 I=1, NWAF
      PPD(J)=XOAFLE(I)
      PPD(J+1)=YOAFLE(I)
      PPD(J+2)=ZOAFLE(I)
      PPD(J+3) = AFSWCL(I)
  150 J = J + 4
```

```
C                                                                       OUT 1600
C     SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS                  OUT 1610
      NPPDP=NWAF                                                        OUT 1620
C                                                                       OUT 1630
C     SET NUMBER OF X/C STATIONS ENTRIES IN THE THICKNESS TABLE PER AIRFOUT 1640
      NTTXCS=NWAFOR                                                     OUT 1650
C                                                                       OUT 1660
C     READ WING AIRFOIL ORDINATE CARDS (HALF THICKNESS)                 OUT 1670
      JJ = 0                                                            OUT 1680
      DO 22 J=1, NWAF                                                   OUT 1690
      IT = JJ+1                                                         OUT 1700
      JJ = NWAFOR+JJ                                                    OUT 1710
   22 READ 4, (WAFORD(I), I=IT, JJ)                                     OUT 1720
      DO 516 I=1, JJ                                                    OUT 1730
  516 WAFORD(I)=WAFORD(I)/100.                                          OUT 1740
      PRINT WLIST                                                       OUT 1750
C                                                                       OUT 1760
C     *****************************************************************OUT 1770
C                                                                       OUT 1780
C     START FUSELAGE DATA CARDS                                         OUT 1790
C                                                                       OUT 1800
  101 IF(J2 .EQ. 0  .OR.  NFUS .EQ. 0) GOTO 171                         OUT 1810
C                                                                       OUT 1820
C     SET BODY COORDINATE TABLE INPUT INDICATOR TO INDICATE A CIRCULAR  OUT 1830
C     BODY OR ARBITRARY BODY                                            OUT 1840
      BCTII =0.0
      BSYM  = 1.0
      IF( J2.EQ.1) BCTII= 1.0                                           OUT 1860
      LLL = 0
      LL = 1
      DO 100 IX=1, NFUS                                                 OUT 1910
      NFO = NFORX(IX)                                                   OUT 1920
C                                                                       OUT 1930
      LLL=LLL+NFO                                                       OUT 1940
C     READ X STATIONS                                                   OUT 1950
      READ 4, (XVXFXF(I), I=LL, LLL)                                    OUT 1960
      XVXFXF(LL) = XVXFXF(LL)+1.0E-7
C                                                                       OUT 1970
C     READ CAMBER(IF ANY)                                               OUT 1980
```

```
      IF(J2.EQ.-1.AND.J6.EQ.0) READ 4, (ZLCCS(I), I=LL, LLL)           OUT 1990
      IF(J2.NE.-1)GOTO 200                                             OUT 2000
C                                                                      OUT 2010
C     READ CROSS SECTIONAL AREAS(IF ANY)                               OUT 2020
      READ 4, (FCSA(I), I=LL, LLL)                                     OUT 2030
      GOTO 100                                                         OUT 2040
  200 IF (J2.NE.1) GOTO 100                                            OUT 2050
C                                                                      OUT 2060
C     READ Y S  AND Z S  OF ARBITRARY BODY                             OUT 2070
      NFF=NRADX(IX)                                                    OUT 2080
      DO 7044 M=LL, LLL                                                OUT02090
      READ 4, (YOHS(I,M), I=1, NFF)
      READ 4, (ZOHS(I,M), I=1, NFF)
      PRINT 401, (YOHS(I,M), I=1, NFF)
      PRINT 402, (ZOHS(I,M), I=1, NFF)
 7044 CONTINUE
  401 FORMAT(*0YOHS*,3(/10G12.5))                                      OUT 2150
  402 FORMAT(5H0ZOHS,3(/10G12.5))                                      OUT 2160
  100 LL=LL+NFO                                                        OUT 2260
C                                                                      OUT 2270
C     FIND AND SET TOTAL NUMBER OF X STATIONS                          OUT 2280
      J=NFORX(1)+NFORX(2)+NFORX(3)+NFORX(4)                            OUT 2290
      NXS = J
C                                                                      OUT 2500
C     PUNCH FUSELAGE AND WING                                          OUT 2510
      CALL DECWRT( DA, 15, 3419, OUNIT)                                OUT 2520
      PRINT BLIST                                                      OUT 2530
  171 IF(IABS(JI).NE.1) GOTO 715                                       OUT 2540
      CALL DECWRT( DA, 3420, 5000,OUNIT)                               OUT 2550
C                                                                      OUT 2560
C     ZERO BODY AREA OF STORAGE                                        OUT 2570
  715 DO 229 I=1, 3405                                                 OUT 2580
  229 BODARE(I)=-0.0                                                   OUT 2590
C                                                                      OUT 2600
C     ZERO PANEL  SECTION OF STORAGE                                   OUT 2610
      DO 226 I=1, 1580                                                 OUT 2620
  226 PANARE(I) =-0.0                                                  OUT 2630
C                                                                      OUT 2640
C     ************************************************************************OUT 2650
```

```
C
C     READ POD DATA
C
      IF (J3.NE.1.OR.NP.EQ.0)    GOTO 300
C
C     REWIND SCRATCH UNIT TO PUT PODS ON
      REWIND SCRTCH
      DO 290 IX=1, NP
      READ 4, XOP     , YOP     , ZOP
      READ 4, (PX(I)     , I=1,NPODOR)
      READ 4,(PODRAD(I)     , I=1,NPODOR)
      PRINT PLIST
C
C     IS THIS A POD OR A NACELLE
      IF(PODRAD(1).LE.0)GOTO 123
C
C     ITS A NACELLE
C     SET REFERANCE AREA CARDS
      REFAF=REFAA
C
C     SET PANEL CONTOUR INDICATOR
      PCONT = 1.0
C
C     SET PANEL ORIGIN
      XPO=XOP
      YPO=YOP
      ZPO=ZOP
C
C     FIND CHORD LENGTHS
      CHORD=PX(NPODOR)
C
C     PANEL PERIMETER DESCRIPTION(STEP BY TEN DEGREES)
      J=1
      DO 764 K=1, 361, 10
      RAD = RAD 1 DG * FLOAT(K-1)
      PPD(J)=PX(1)
      PPD(J+1)=DSIN(RAD)*PODRAD(1)
      PPD(J+2)=DCOS(RAD)*PODRAD(1)
      PPD(J+3)=CHORD
```

```
  764 J=J + 4                                                          OUT 3020
C                                                                      OUT 3030
C     SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS                 OUT 3040
      NPPDP=37                                                         OUT 3050
C                                                                      OUT 3060
C     PERCENT CHORD LOCATIONS(X/CHORD LENGTH)                          OUT 3070
      XCNUM=NPODOR                                                     OUT 3080
      DO 746 I=1, NPODOR                                               OUT 3090
  746 PCHORD(I)=PX(I)/CHORD                                            OUT 3100
C                                                                      OUT 3110
C     SET ETA(ATA) STATIONS (AS WELL AS THEIR NUMBER)                  OUT 3120
      ANAS=2                                                           OUT 3130
      ATA(1)=0.0                                                       OUT 3140
      ATA( 2)=1.0                                                      OUT 3150
C                                                                      OUT 3160
C     CAMBER LINE = DELTA Z S (X S) / AIRFOIL STREAMWIZE CHORD LENGTH  OUT 3170
      DO 476 I=1, NPODOR                                               OUT 3180
      ZC(I)=PODRAD(I)/CHORD                                            OUT 3190
  476 ZC(NPODOR+I)=ZC(I)                                               OUT 3200
C                                                                      OUT 3210
C     PUNCH PANEL                                                      OUT 3220
      CALL DECWRT(DA,3420,5000,OUNIT)                                  OUT 3230
C     ZERO PANEL AREA                                                  OUT 3240
      DO 227 I=1, 1580                                                 OUT 3250
  227 PANARE(I)=-0.0                                                   OUT 3260
      GOTO 290                                                         OUT 3270
C                                                                      OUT 3280
C     ITS A POD                                                        OUT 3290
C                                                                      OUT 3300
C     SET BODY ORIGIN                                                  OUT 3310
  123 BOX=XOP                                                          OUT 3320
      BOY=YOP                                                          OUT 3330
      BOZ=ZOP                                                          OUT 3340
C                                                                      OUT 3350
C     SET BODY COORDINATE TABLE INPUT INDICATOR                        OUT 3360
      BCTII = 0.0
      BSYM =1.
C                                                                      OUT 3380
C     SET NUMBER OF X STATIONS IN BODY CO - ORDINATE TABLE             OUT 3390
```

```
      NXSBOT = NPODOR
      BMFCXN = NXSBOT
C                                                                       OUT 3420
C     FILL IN BODY CO - ORDINATE TABLE                                  OUT 3430
      DO 321 I=1, NPODOR                                                OUT 3440
      BMFCXS(I)=PX(I)                                                   OUT 3450
  321 BCT(I)=PX(I)                                                      OUT 3460
      K=1                                                               OUT 3480
      DO 456 I=1, NPODOR                                                OUT 3490
C                                                                       OUT 3500
C     FILL IN LATERAL CO - ORDINATE TABLE                               OUT 3510
C                                                                       OUT 3560
C     SET RADII                                                         OUT 3570
      RB(K)=PODRAD(I)                                                   OUT 3580
      RB(K+1)=PODRAD(I)                                                 OUT 3590
  456 K=K+2                                                             OUT 3600
      LCT(1) = 2.0
      LCT(2)=0.0
      LCT(3)=180.0
C                                                                       OUT 3610
C     SET REFERENCE AREA                                                OUT 3620
      REFA=REFAA                                                        OUT 3630
C                                                                       OUT 3640
C     PUNCH BODY(POD)                                                   OUT 3650
      CALL DECWRT(DA,15,3419,SCRTCH)                                    OUT 3660
C     ZERO BODY AREA                                                    OUT 3670
      DO 722 I=1, 3405                                                  OUT 3680
  722 BODARE(I)=-0.0                                                    OUT 3690
  290 CONTINUE                                                          OUT 3700
C                                                                       OUT 3710
C     ******************************************************************OUT 3720
C                                                                       OUT 3730
C     READ FIN DATA                                                     OUT 3740
C                                                                       OUT 3750
  300 IF(J4.NE.1 .OR.NF.EQ.0) GOTO390                                   OUT 3760
      DO 380 IX=1, NF                                                   OUT 3770
      READ 4, FINXL    , FINYL    , FINZL    , LCHORD    , FINXH    ,   OUT 3780
     1FINYH    , FINZH    , HCHORD                                      OUT 3790
      READ 4,(PCHORD(I)    , I=1, NFINOR)                               OUT 3800
```

```
      READ 4, (FAFHT(I)    , I=1, NFINOR)                                 OUT 3810
      DO 225 I=1,      NFINOR                                             OUT 3820
      PCHORD(I)=PCHORD(I)/100.                                            OUT 3830
  225 FAFHT(I)=FAFHT(I)/100.0                                             OUT 3840
C                                                                         OUT 3350
C     PERCENT CHORD LOCATIONS (NUMBER OF ORDINATES USED TO DESCRIBE EACHOUT 3860
C     AIR FOIL SECTION(THICKNESS TABLE X/C STATIONS))                     OUT 3870
      XCNUM = NFINOR                                                      OUT 3880
      NTTXCS=XCNUM                                                        OUT 3890
      DO 379 I=1, NFINOR                                                  OUT 3900
  379 TT(I)=PCHORD(I)                                                     OUT 3910
      PRINT FLIST                                                         OUT 3920
      ASSIGN 380 TO LABEL                                                 OUT 3930
      GOTO 1000                                                           OUT 3940
  380 CONTINUE                                                            OUT 3950
C                                                                         OUT 3960
C     *****************************************************************OUT 3970
C                                                                         OUT 3980
C     READ CANARD DATA                                                    OUT 3990
C                                                                         OUT 4000
  390 IF(J5.NE.1.OR.NCAN.EQ.0) GOTO 999                                   OUT 4010
      DO 500 IX =1, NCAN                                                  OUT 4020
      READ 4, XLC     ,YLC     ,ZLC     ,CONCRD    ,XHC     ,YHC     ,ZHC OUT 4030
     1,CONCOH                                                             OUT 4040
      J=IABS(NCANOR)                                                      OUT 4050
      READ 4, (CANPCL(I)    , I=1,J)                                      OUT 4060
      READ 4, (CANPLN(I)    , I=1, J)                                     OUT 4070
      DO 223 I=1, J                                                       OUT 4080
      CANPLN(I)=CANPLN(I)/100.0                                           OUT 4090
  223 CANPCL(I)=CANPCL(I)/100.0                                           OUT 4100
      IF(NCANOR .GE. 0) GOTO 7094                                         OUT 4110
C                                                                         OUT 4120
C     HERE IF CAMBERED(NOT SYMETRICAL)                                    OUT 4130
C     SET PANEL CONTOUR INDICATOR
      PCONT = 1.0
      READ 4, (CANS(I), I=1, J)                                           OUT 4140
      DO 712 I=1, J                                                       OUT 4150
  712 CANS(I)=CANS(I)/100.0                                               OUT 4160
C                                                                         OUT 4170
```

```
C     SET CAMBER
      DO 1052 I=1, J
 1052 ZC(I)=(CANPLN(I)-CANS(I))/2.0
C
C     ADJUST THICKNESS TO BE RELATIVE TO CAMBER LINF
      DO 1050 I=1, J
 1050 CANPLN(I)=(CANPLN(I)+CANS(I))/2.0
C
C     PERCENT CHORD LOCATIONS(NUMBER OF ORDINATES USED TO DESCRIBE EACH
C     SECTION(THICKNESS TABLE STATIONS))
 7094 XCNUM= J
      NTTXCS=XCNUM
C     ZERO UNIVERSAL DATA STORAGE
      DO 7093 I=1, J
 7093 TT(I)=CANPCL(I)
      PRINT CLIST
      ASSIGN 500 TO LABEL
      GOTO 1000
  500 CONTINUE
C
C     ***************************************************************
C
C     READ PLOT DATA
C
  999 READ 7,PHI,THETA,PSII
    7 FORMAT(7X,3F5.0)
      PRINT PLOTLT
C
C
C     REWIND OUTPUT FILE FROM COMPUTER CONVERSION
      REWIND OUNIT
      ENDFILE SCRTCH
      REWIND SCRTCH
C
C     ***************************************************************
C
C     USER EDIT SECTION
C
C     EDIT UNIVERSAL INFO
```

```
C     ZERO UNIVERSAL AREA OF STORAGE
      PRINT 8
    8 FORMAT(1H1,30X,*START EDIT*)
      DO 710 I=1, 14
  710 DA(I)=-0.0
      CALL DECRD(DA,OUNIT)
      CALL DECRD(DA,IUNIT)
      CALL DECWRT(DA,1,14,OUNIT2)
C     ZERO BODY AREA STORAGE
      DO 711 I=1, 3405
  711 BODARE(I)=-0.0
C
C     EDIT BODIES IF ANY
      NPOD = 0
      IF(J2 .EQ. 0) GOTO 505
      CALL DECRD(DA,OUNIT)
      CALL DECRD(DA,IUNIT)
      J = NXS
      IF( J .GT. 44) GOTO 201
C
C     FILL IN DATA FOR BODY THAT FITS AS IS
      NXSBOT = NXS
      BMFCXN = NXSBOT
      DO 7030 I=1, J
      BCT(I) = XVXFXF(I)
      ZLCCT(I) = ZLCCS(I)
 7030 BMFCXS(I) = BCT(I)
      IF( J2 .NE. -1) GOTO 202
C
C     SET ANGLE OF CIRCULAR BODY
      LCT(1)=2.0
      LCT(2)=0.0
      LCT(3)=180.0
      L=1
      DO 7040 I=1, J
C
C     FIND RADIUS OF CIRCULAR BODY
      RB(L) =DSQRT(FCSA(I)/PI)
      RB(L+1) = RB(L)
```

```
 7040 L = L + 2
      GOTO  205
  202 MM    = 1
      N     = 0
      LLL   = 0
      LL    = 1
      DO 110 IX=1, NFUS
      NFF   = NRADX(IX)
      NZ    = NFF
      NFO   = NFORX(IX)
      LLL   = LLL + NFO
      DO 10 M=LL, LLL
      LCT(MM)= NFF
      DO 7090 I=1, NFF
C
C     SET Y AND Z VALUES
      RB(N+I) = ZOHS(NZ,M)
      LCT(MM+I) = YOHS(NZ,M)
 7090 NZ =  NZ - 1
      N     = N + NFF
   10 MM    = MM + 1 + NFF
  110 LL    = LL + NFO
      GOTO  205
C
C     INTERPOLATE FOR OVER SPECIFIED DATA
  201 J = NXSBOT
      BMFCXN = NXSBOT
      DO 217 I=1, J
  217 BMFCXS(I) = BCT(I)
      IF( J2 .NE. -1) GOTO 204
C
C     SET ANGLE OF CIRCULAR BODY
      LCT(1) = 2.0
      LCT(2) = 0.0
      LCT(3) = 180.0
      J=NXSBOT
      L=1
C
C     FIND RADIUS OF CIRCULAR BODY
```

```
      DO 203 I=1, J
      RB(L) =DSQRT(CODIM1(BCT(I), XVXFXF, FCSA, NXS, 0.0) / PI)
      RB(L+1) = RB(L)
C
C     FIND  CAMBER
      ZLCCT(I) = CODIM1(BCT(I), XVXFXF, ZLCCS, NXS, 0.0)
  203 L      = L + 2
      GOTO  205
  204 CONTINUE
C
C     FIND  S  BAR(CIRCUMFERENCE)
      LL     = 1
      LLL    = 0
      N      = SN
      DO 206 IX=1, NFUS
      NFO    = NFORX(IX)
      LLL    = LLL + NFO
      NFF    = NRADX(IX)
C
C     FILL AND NON DIMENSIONALIZE S MATRIX
      DO 208 M=LL, LLL
      SB(1) = 0.0
      DO 207 I=2, NFF
  207 SB(I) = SB(I-1) + SQRT((YOHS(I,M) - YOHS(I -1,M))**2 +
     1           (ZOHS(I,M)-ZOHS(I-1,M))**2)
     2            + 1.0E-7
      T      = SB(NFF)
      DO 209 I=2, NFF
  209 SB(I) = SB(I) / T
      DO 210 I=1, N
      FCSA(I) = CODIM1( S(I), SB, YOHS(1,M), NFF, 0.0)
  210 ZCSA(I) = CODIM1( S(I), SB, ZOHS(1,M), NFF, 0.0)
      IN     = N
      DO 215 I=1, N
      YOHS(I,M) = FCSA(IN)
      ZOHS(I,M) = ZCSA(IN)
  215 IN     = IN - 1
  208 CONTINUE
  206 LL = LL + NFO
```

```
      DO 211 I=1, N
      DO 211 M=1, NXS
      SZ(M,I) = ZOHS(I,M)
  211 SY(M,I) = YOHS(I,M)
      J       = NXSBOT
      L       = 1
      K       = 0
      DO 212 I=1, J
      LCT(L) = N
      DO 213 M=1, N
      LCT(L+M)= CODIM1(BCT(I), XVXFXF, SY(1,M), NXS, 0.0)
  213 RB(K+M) = CODIM1(BCT(I), XVXFXF, SZ(1,M), NXS, 0.0)
      L = L + 1 + N
  212 K = K + N
      DO 214 I=1, J
  214 ZLCCT(I) = CODIM1(BCT(I), XVXFXF, ZLCCS, NXS, 0.0)
  205 CONTINUE
      CALL DECWRT(DA,15,3419,OUNIT2)                                     OUT 4740
C     ZERO BODY AREA STORAGE                                             OUT 4750
  720    DO 721 I=1, 3405                                                OUT 4760
  721 BODARE(I)=-0.0                                                     OUT 4770
C     EDIT PODS( WITH ZERO RADIUS OF NOSE)                               OUT 4780
  505 CALL DECRD(DA, SCRTCH)                                             OUT 4790
      IF(SCRTCH) 501,501,499                                             OUT 4800
  499 CALL DECRD(DA, IUNIT)                                              OUT 4810
      CALL DECWRT(DA,15,3419,OUNIT2)                                     OUT 4820
      NPOD = NPOD + 1                                                    OUT 4830
      GOTO 720                                                           OUT 4840
C                                                                        OUT 4850
C     EDIT PANELS (IF ANY)                                               OUT 4860
  501 CALL DECRD(DA,OUNIT)                                               OUT 4870
      IF( OUNIT ) 502, 502, 503                                          OUT 4880
  503 CALL DECRD(DA, IUNIT)                                              OUT 4890
      CALL DECWRT(DA, 3420, 5000, OUNIT2)                                OUT 4900
C     ZERO PANEL AREA OF STORAGE                                         OUT 4910
      DO 713 I=1, 1580                                                   OUT 4920
  713 PANARE(I)=-0.0                                                     OUT 4930
      GOTO 501                                                           OUT 4940
C                                                                        OUT 4950
```

```
C     REWIND FILES
  502 OUNIT = IABS(OUNIT)
      REWIND OUNIT
      REWIND OUNIT2
C
C     ******************************************************************
C
C     ADDITION SECTION
C
C     COPY PAST OLD BODY (IF ANY)
      PRINT 9
    9 FORMAT(1H1, 30X,*START ADDITIONS*)
      DO 714 I=1, 14
  714 DA(I)=-0.0
      CALL DECRD(DA, OUNIT2)
      CALL DECWRT(DA, 1, 14, OUNIT)
      I = DA(1)
      IF (J2 .EQ. 0) GOTO 507
      I = I - 1
      CALL DECRD (DA, OUNIT2)
      CALL DECWRT(DA, 15, 3419, OUNIT)
C     ZERO BODY AREA STORAGE
  513 DO 716 J=1, 3405
  716 BODARE(J)=-0.0
  507 IF(NPOD) 504, 504, 508
  508 CALL DECRD(DA,OUNIT2)
      CALL DECWRT(DA,15,3419,OUNIT)
      I=I-1
      NPOD = NPOD - 1
      GOTO 513
C
C     ADD NEW BODIES (IF ANY)
  504 IF(I) 518, 518, 506
  506 DO 509 J=1, I
      CALL DECRD(DA, IUNIT)
      CALL DECWRT(DA, 15, 3419, OUNIT)
C     ZERO BODY AREA STORAGE
      DO 717 K=1, 3405
  717 BODARE(K)=-0.0
```

```
  509 CONTINUE                                                  OUT 5350
C                                                               OUT 5360
C     COPY REST OF PANELS                                       OUT 5370
  518 IF(DA(2).EQ.0) GOTO 514                                   OUT 5380
      I = DA(2)                                                 OUT 5390
      DO 510 J=1,I                                              OUT 5400
C     ZERO PANEL AREA OF STORAGE                                OUT 5410
      DO 718 K=1, 1580                                          OUT 5420
  718 PANARE(K)=-0.0                                            OUT 5430
      CALL DECRD(DA,OUNIT2)                                     OUT 5440
      IF (OUNIT2) 512, 512, 511                                 OUT 5450
  511 CALL DECWRT(DA, 3420, 5000, OUNIT)                        OUT 5460
      ATACH(1,J) = DA(3420)                                     OUT 5470
      ATACH(2,J) = DA(3444)                                     OUT 5480
      ATACH(3,J) = DA(3445)                                     OUT 5490
      ATACH(4,J) = DA(3446)
      ATACH(5,J) = DA(3447)
  510 CONTINUE                                                  OUT 5500
C                                                               OUT 5510
C     OUNIT NOW CONTAINS FULLY UPDATED DATA, READY FOR FRED      OUT 5520
C     OUNIT2 IS FREE                                            OUT 5530
  514 REWIND OUNIT                                              OUT 5540
      REWIND OUNIT2                                             OUT 5550
      PRINT ATTACH                                              OUT 5560
      RETURN
C                                                               OUT 5580
C     ADD NEW PANELS (IF ANY)                                   OUT 5590
  512 OUNIT2 = IABS(OUNIT2)                                     OUT 5600
      DO 515 K=J, I                                             OUT 5610
C     ZERO PANEL AREA OF STORAGE                                OUT 5620
         DO 719 L=1, 1580                                       OUT 5630
  719 PANARE(L)=-0.0                                            OUT 5640
      CALL DECRD( DA, IUNIT)                                    OUT 5650
      CALL DECWRT(DA, 3420, 5000, OUNIT)                        OUT 5660
      ATACH(1,K) = DA(3420)                                     OUT 5670
      ATACH(2,K)=DA(3444)                                       OUT 5680
      ATACH(3,K) = DA(3445)
      ATACH(4,K) = DA(3446)
      ATACH(5,K) = DA(3447)
```

```
  515 CONTINUE                                                          OUT 5700
      GOTO 514                                                          OUT 5710
C                                                                       OUT 5720
C     ******************************************************************OUT 5730
C                                                                       OUT 5740
C     PANEL DATA STORER                                                 OUT 5750
C                                                                       OUT 5760
C                                                                       OUT 5770
C                                                                       OUT 5780
C     CO - ORDINATE OF LEADING EDGE OF FIRST AIRFOIL                    OUT 5790
 1000 XPO = 0.0                                                         OUT 5800
       YPO =0.0                                                         OUT 5810
      ZPO = 0.0                                                         OUT 5820
C                                                                       OUT 5830
C     SET REFERENCE AREA                                                OUT 5840
      REFAF=REFAA                                                       OUT 5850
C                                                                       OUT 5860
C     PANEL PERIMETER DESCRIPTIONS                                      OUT 5870
       PPD(1)= FINXL                                                    OUT 5880
       PPD(2)= FINYL                                                    OUT 5890
       PPD(3)= FINZL                                                    OUT 5900
      PPD(4) = LCHORD                                                   OUT 5910
      PPD(5) =FINXH                                                     OUT 5920
       PPD(6) = FINYH                                                   OUT 5930
      PPD(7) = FINZH                                                    OUT 5940
      PPD(8) = HCHORD                                                   OUT 5950
C                                                                       OUT 5960
C     SET NUMBER OF PANEL PERIMETER DISCRIPTION POINTS                  OUT 5970
      NPPDP=2                                                           OUT 5980
C                                                                       OUT 5990
C                                                                       OUT 6000
C     NUMBER OF ETA(ATA) STATIONS                                       OUT 6010
       ANAS = 2                                                         OUT 6020
      NATA=ANAS                                                         OUT 6030
C                                                                       OUT 6040
C     ETA STATIONS                                                      OUT 6050
      ETA(1)=0.0                                                        OUT 6060
      ETA(2)=1.0                                                        OUT 6070
      ATA(1)=0.0                                                        OUT 6080
```

```
      ATA(2)=1.0                                                        OUT 6090
C                                                                       OUT 6100
C      PUNCH PANEL                                                      OUT 6110
      CALL DECWRT(DA,3420, 5000, OUNIT)                                 OUT 6120
C                                                                       OUT 6130
C     ZERO PANEL SECTION OF STORAGE                                     OUT 6140
      DO 224 I=1, 1580                                                  OUT 6150
  224 PANARE(I)=-0.0                                                    OUT 6160
C                                                                       OUT 6170
C     RETURN TO CALLER                                                  OUT 6180
       GOTO LABEL,(380,500)                                             OUT 6190
      END                                                               OUT 6200
```

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